28.2 The Photoelectric Effect

- Light incident on some metallic surfaces causes electrons to be emitted from the surfaces.

↓

Photoelectrons & photoelectric effect

→ Experimental setup

Collector + Emitter

EM waves (light)

Vacuum tube

Variable power supply

Diagram showing collector, emitter, and variable power supply.
→ when the tube is in the dark \( \Rightarrow \) no current (the ammeter \( A \) reads 0)

→ when the light of appropriate \( \lambda \) shines on \( E \), current is detected

→ Experimental data: fixed \( \lambda \)

\[
\text{(photo) current} \quad \text{applied voltage}
\]

\( -\Delta V_s \)

high \( I \)

low \( I \)

\( \text{stopping potential for photoelectrons} \)

→ stopping potential does NOT depend on the light intensity \( I \)

→ at a fixed wavelength \( \lambda \) of light, higher light intensity \( I \) results in a larger photocurrent
potential $\Delta V$

$\Delta V > 0 \Rightarrow$ the current at a max value

$\Delta V < 0 \text{ OR } \Delta V = -\Delta V_{s}$

stopping potential

current decreases & at $\Delta V = -\Delta V_{s} \Rightarrow$ the photocurrent is zero

$\Rightarrow$ why does the current stop?

$e \Delta V_{s} = K_{\text{max}}$

Negative potential does a negative work on photoelectrons. When $\Delta V = -\Delta V_s$, the negative potential stops the fastest $e^-$. 
(2) Time lag between incidence of light & ejection of electrons

Exp: $10^{-9}$s after illumination (almost instantaneous) for ALL values of $I$

Classical prediction: because $E$ of absorbed light $\propto \Delta t$, $\propto I$, for very small $I$, $\Delta t$ should be correspondingly larger $\Rightarrow$ larger time lag for smaller $I$ $\Rightarrow$ INCORRECT

(3) Ejection of electrons versus light frequency $f$

Exp: there is a cutoff frequency $f_c$, such that $f < f_c \Rightarrow$ NO photoelectric effect
Classical prediction:
Electrons ejected at any frequency of light, $f$, as long as either $I$ is large enough or $\Delta t$ is large enough \( \implies \) INCORRECT

(4) $K_{\text{max}}$ versus light frequency $f$

Exp: $K_{\text{max}}$ increases with $f$ for $f > f_c$

Classical prediction:
$K_{\text{max}}$ does NOT depend on $f$ \( \implies \) INCORRECT
All four predictions of the classical physics are INCORRECT.

Successful explanation of the photoelectric effect: Einstein in 1905 (Nobel Prize in physics in 1921)

→ Einstein adopted & extended Planck's concept of quantization of EM waves:
  light of frequency $f$ can be considered as a stream of quanta $\Rightarrow$ PHOTONS

→ each photon has an energy $E$

$$E = hf = \frac{hc}{\lambda}$$

roof $f$ frequency

Planck's constant
- photons move in vacuum with the speed of light $c$

$$c = 3 \times 10^8 \text{ m/s}$$

- photoelectric effect involves energy transfer via one-photon/one-electron event

- each metal is characterized by a work function $\phi$ which is equal to the minimal energy of bound electrons

$$E = \phi = hf$$

free $e^-$

$$U = 0$$

$$U = -\phi$$

incoming energy of a photon

$$hf = \phi + K_{\text{max}}$$

potential energy of bound $e^-$
Einstein's explanation

(1) $K_{\text{max}}$ versus $I$

$K_{\text{max}}$ is independent of $I$ because of 1 photon - 1 electron event: each photon $hf = \Phi + K_{\text{max}}$ but higher $I$ means a larger # of electrons, thus stronger photocurrent

(2) Time lag

Even for small $I$, photons hit the emitter $\Rightarrow$ photoelectron $\Rightarrow$ no time lag ($\sim 10^{-9}$ s)

(3) Ejection versus $f$

If $hf < \Phi$, then photons do NOT have enough energy to free the electron from the metal $\Rightarrow$ NO effect
(4) \( K_{\text{max}} \) versus \( f \rightarrow \) linear function

\[ K_{\text{max}} = hf - \phi \]

- Einstein's model
- exp data

\[ f_{G1} = \frac{\Phi_1}{h} \rightarrow \lambda_c,1 = \frac{c}{f_{G1}} \]
\[ f_{G2} = \frac{\Phi_2}{h} \rightarrow \lambda_c,2 = \frac{c}{f_{G2}} \]
\[ f_{G3} = \frac{\Phi_3}{h} \rightarrow \lambda_c,3 = \frac{c}{f_{G3}} \]

\[ \lambda_c = \frac{hc}{\phi} \]

\( hc = 1240 \text{ eV} \cdot \text{nm} \)
\( 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \)
Photomultiplier Tubes

- $e^-$ are accelerated across potential differences

\[ \text{one photon at the input results in millions of electrons at the output} \]

Other applications: CCD (charge-coupled device)
Quiz: Suppose that the light intensity $I$ is fixed but we increase the wavelength of incoming light such that $\lambda < \lambda_c$ ($\lambda_c$ corresponds to the cutoff frequency $f_c$).

Do we see the photoelectric effect?

(a) Yes 
(b) No

What happens to the stopping potential $AV_s$?

(a) Increases (moves to the left) 
(b) Decreases (moves to the right)
X-Ray Production

- photoelectric effect: photon gives ALL energy to excite an electron out of photocathode → photoelectron

- Can an electron give up its energy to create a photon? ⇒ YES

Unlike a photon which can be created or absorbed as a unit, an electron can give only part of its energy (kinetic).

When an electron with large energy passes through matter, part of its K is radiates as photons (e- slows down) → bremsstrahlung (= breaking radiation) or inverse photoelectric effect
$E_f = E_i - hf$  

**conservation of energy**

X-rays are produced by the breaking radiation

$\lambda \approx 0.01 \text{nm to } 1 \text{nm}$

**Applications:**

- medical diagnosis & therapy
- fundamental research in structure of matter
Experiment:

- X-rays

- $e^-$ evaporated from the filament by thermionic emission

- Thermionic $e^-$ accelerated to a high $k$ by a high voltage

Diagram:

- Target
- Filament
- Vacuum tube

Graph:

- X-ray spectral intensity ($I$)
- $e^-$ gives all kinetic energy $K$ to create a photon
- $\lambda_{\text{min}}$
$\lambda_{\text{min}}$ .... wavelength of the most energetic photon of x-ray emission

$$n f_{\text{max}} = e_0 V$$ — voltage

$$\frac{hc}{\lambda_{\text{min}}} = e_0 V \Rightarrow \lambda_{\text{min}}^{-1} = \frac{e_0 V}{hc}$$

$$\lambda_{\text{min}} = \frac{hc}{e_0 V} = \frac{1.24 \times 10^{-6} \text{ V.m}}{\sqrt{V}}$$ — voltage

Additional note: because voltages used to produce x-rays are large ~ 20, 30, 50 kV $\Rightarrow$ the work function of the metal (a few eV, typically) can be neglected!

$\phi \approx 4 \text{ eV} < e_0 \cdot V = 2 \cdot 10^4 \text{ eV}$

$E-4$
11.5. Atomic Spectra & the Bohr Theory of Hydrogen

In contrast to the spectrum of x-rays (continuous), the atomic spectrum is **discrete** and depends on the type of atom.

Atomic spectra are two types:

1. emission
2. absorption

(i) emission spectrum

[Diagram of a discharge tube, slit, diffraction grating, and screen with wavelengths λ₁, λ₂, λ₃, λ₄.]
emission spectrum on the screen

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are characteristic of a particular atom(s) in the gas

(2) absorption spectrum

white light

H gas

slit
diffr. grating

screen
Absorption spectrum = continuous spectrum with discrete dark absorption bands (lines)

H-atom characterized by several spectral lines in the visible spectrum of EM waves:

\[ \frac{1}{\lambda} = R_\text{H} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \]

\( n = 3, 4, 5, \ldots \)

Rydberg constant

Rydberg equation for the Balmer series of H-atom spectrum

\[ R_\text{H} = 1.097 \times 10^7 \text{ m}^{-1} \]
Why is the spectrum of hydrogen atom discrete?

The Bohr model of Hydrogen Atom is based on four postulates:

1. The electron moves around the proton in circular orbits; each orbit represents a stationary state with a fixed energy; analogous to motion of planets around the Sun if one orbit considered.

2. Only selected discrete orbits are stable (quantization).
(3) Radiation is emitted or absorbed ONLY when an electron makes a TRANSITION from one discrete orbit to the other:

\[ E_i - E_f = h\nu \]

\[ E_f = E_i + h\nu \]

(4) Discretization (quantization) of orbits is determined by quantization of electron's orbital angular momentum:

\[ m_e\nu r = n\hbar \]

\[ n = 1, 2, 3, \ldots \]

\[ \hbar = \frac{\hbar}{2\pi} \]

\[ \hbar = 6.63 \times 10^{-34} \text{ J s} \]

Planck's constant
Bohr successfully merged classical and new quantum concepts to explain the H-atom spectrum.

Energy levels in Bohr's model:

Total energy \( E = K + U_e \)

- **Kinetic Energy**: \( K = \frac{1}{2} m_e v^2 \)
- **Potential Energy (Electric)**: \( U_e = -k_e \frac{e^2}{r} \)

Balance of forces in a radial direction:
- **Attractive Electric Force**:\
- **Centripetal Force**:\

\( k_e = (4\pi\varepsilon_0)^{-1} \)
\[
\frac{k e e^2}{r^2} = \frac{m e e^2}{\kappa}
\]

\[\Rightarrow \nu = \sqrt{\frac{k e e^2}{m e r}}\]

\[K = \frac{1}{2} m \nu^2 = \frac{1}{2} \frac{m e e^2}{m e r} = \frac{1}{2} \frac{k e e^2}{r}\]

\[E = K + U_e = \frac{1}{2} \frac{k e e^2}{r} - \frac{k e e^2}{r}\]

\[E = -\frac{1}{2} \frac{k e e^2}{r}\]

Total energy

Attractive, binding energy of the e\(^{-}\) around the proton (+e charge)
Crucial step: quantization of the orbital momentum

\[ m_e v_n^2 r_n = n \hbar \]
\[ v_n^2 = \frac{k_e e^2}{m_e r_n} \quad (\text{page 11-7}) \]

\[ m_e^2 v_n^2 r_n^2 = n^2 \hbar^2 \]
\[ m_e \frac{k_e e^2}{m_e r_n} r_n^2 = n^2 \hbar^2 \]

\[ r_n = \frac{n^2 \hbar^2}{m_e k_e e^2} \quad n = 1, 2, 3, \ldots \]

The radius of orbits is quantized & grows with \( n \) as \( n^2 \), \( r_n \propto n^2 \).

For \( n = 1 \), \( r_1 = a_0 \) Bohr radius

\[ a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.0529 \text{ nm} \]
Easy to remember:

a diameter of H-atom in the GROUND \((n=1)\) state:

\[ 2a_0 \approx 1 \, \text{Å} = 10^{-10} \, \text{m} \]

Inserting \( r_n = r_n (n^2) \) into energy \( E \rightarrow E_n \)

\[ E_n = -\frac{k_e e^2}{2a_0 n^2} \quad n=1,2,3,\ldots \]

OR

\[ E_n = -\frac{1}{n^2} \times 13.606 \, \text{eV} \]

ionization energy

\( \text{e.g. energy needed to ionize the H-atom (to free the } \text{e}^- \text{ from the atom)} \)
Energy levels:

\[ E_1 = -13.6 \text{eV} \]
\[ E_2 = -\frac{1}{4} \times 13.6 \text{eV} = -3.40 \text{eV} \]
\[ E_3 = -\frac{1}{9} \times 13.6 \text{eV} = -1.51 \text{eV} \]
How do we understand the H-atom emission spectrum?

\[ hf = E_i - E_f \]

\[ f = \frac{E_i - E_f}{\hbar} = \frac{k e e^2}{2a_0 \hbar} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

Lyman series \( n_f = 1 \)

Balmer series \( n_f = 2 \)

Paschen series \( n_f = 3 \)

He, Li\(^{2+}\), Be\(^{3+}\), ...