Phase change due to reflection

Lloyd's mirror

2 paths SP & S'P
\[ \delta = \phi_2 - \phi_1 \] interferes
BUT
the positions of bright & dark fringes are REVERSED
because coherent sources S & S' differ in phase by 180° \((\pi)\)

(A) \[ 180^\circ (\pi) \] change

(B) NO phase change

\[ n_1 < n_2 \]

\[ n_1 > n_2 \]

\[ n_1, n_2 \ldots \text{index of refraction} \]
Interference in thin films

- oil film on water
- soap bubble reflections

Consider a film of uniform thickness \( t \) & index of refraction \( n \)

\[ \text{air} \quad \text{film, } n \quad \text{air} \]

\[ \text{air} \quad \text{film, } n \quad \text{air} \]

- ray 1: reflected ray 2 undergoes a phase change 180° (\( n > 1 \), for air \( n_{\text{air}} = 1 \))

- ray 3: reflected ray 4 (\& 4') ⇒ no phase change \( n_{\text{air}} < n \)

- inside the film \( \lambda_n = \frac{\lambda}{n} \) (changed wavelength)
Rule for the constructive interference:

\[ 2t = (m + \frac{1}{2}) \lambda_n \]  \( (m = 0, 1, 2, \ldots) \)

Or

\[ 2nt = (m + \frac{1}{2}) \lambda \]  \( (m = 0, 1, 2, 3, \ldots) \)

Rule for the destructive interference:

\[ 2nt = m \lambda \]  \( (m = 0, 1, 2, 3, \ldots) \)

But if \( n_1 < n_2 < n_3 \) or \( n_1 > n_2 > n_3 \)

\[
\begin{align*}
n_1 & \quad 1 \rightarrow 2 \quad 180^\circ \\
n_2 & \quad 3 \rightarrow 4 \\
n_3 & \quad 1 \rightarrow 2 \\
\end{align*}
\]

\[ 2nt = m \lambda \]  \( \text{constructive} \)

\[ 2nt = (m + \frac{1}{2}) \lambda \]  \( \text{destructive} \)

\[ \text{REVERSED criteria} \]

27-8
Quiz: You spill two liquids onto water. They both form thin films on the water surface but film 1 appears dark & film 2 appears bright in reflected light. Explain!

Assume: $n_{\text{film1}} > n_{\text{air}}$ & $n_{\text{film2}} > n_{\text{air}}$

(a) $n_{\text{film1}} > n_{\text{H}_2\text{O}}$ & $n_{\text{film2}} > n_{\text{H}_2\text{O}}$

(b) $n_{\text{film1}} < n_{\text{H}_2\text{O}}$ & $n_{\text{film2}} < n_{\text{H}_2\text{O}}$

(c) $n_{\text{film1}} < n_{\text{H}_2\text{O}}$ & $n_{\text{film2}} > n_{\text{H}_2\text{O}}$

(d) $n_{\text{film2}} < n_{\text{H}_2\text{O}}$ & $n_{\text{film1}} > n_{\text{H}_2\text{O}}$
Diffraction Patterns

EM waves pass through small openings & around obstacles & by sharp edges

Consider light passing through a narrow opening, slit, & projected onto a screen:

- incoming wave
- slit
- intensity of light
- screen
- diffraction pattern

... dark
... bright
... dark
Mathematical model for single-slit Fraunhofer diffraction pattern

Path difference: 1, 3, 5 & 2, 4
\[ \frac{1}{2}a \sin \theta \]

Huygens principle: each portion of the slit acts as a source of waves

\[ \text{Interference} \]
Where is the first dark band?

\[ \frac{a}{2} \sin \theta = \frac{\lambda}{2} \quad \text{(phase change} \quad \pi, 180^\circ) \]

\[ \sin \theta = \frac{\lambda}{a} \]

General condition for destructive interference

\[ \sin \theta_{\text{dark}} = m \frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots \]

(but NOT \( m = 0 \))

\[ y_2 \quad \sin \theta_2 = \frac{2 \lambda}{a} \quad y_2 = L \tan \theta_2 \]

\[ y_1 \quad \sin \theta_1 = \frac{\lambda}{a} \quad y_1 = L \tan \theta_1 \]

\[ -y_1 \quad \sin \theta_{-1} = -\frac{\lambda}{a} \quad y_{-1} = L \tan \theta_{-1} \]

\[ -y_2 \quad \sin \theta_{-2} = -\frac{2 \lambda}{a} \quad y_{-2} = L \tan \theta_{-2} \]

27-12
Resolution of Single-Slit & Circular Apertures

when an instrument or your eye can distinguish between two objects (light sources) \( \Rightarrow \) the two objects are RESOLVED
Rayleigh's criterion: the central maximum of the diffraction pattern of one source falls on the first minimum of the diffraction pattern of the other source.

\[ \text{the two sources are just RESOLVED} \]

Minimum angular separation \( \Theta_{\text{min}} \):

\[ \sin \Theta_{\text{min}} = \frac{\lambda}{a} \]

- \( \lambda \) \text{ wavelength of EM wave (light)}
- \( a \) \text{ width of a slit}

\( \lambda \ll a \) (most situations)

\[ \sin \Theta \approx \Theta \]

\[ \Theta_{\text{min}} = \frac{\lambda}{a} \quad \text{[in radians!]} \]

The angle between 2 sources, \( S_1 \) & \( S_2 \), has to be \( > \Theta_{\text{min}} \):

\[ \Theta > \Theta_{\text{min}} \Rightarrow \text{RESOLVED.} \]
Circular aperture of diameter $D$

$$\theta_{\text{min}} = 1.22 \frac{\lambda}{D}$$

for circular apertures

For example:

$D$ ... diameter of a pupil

OR

diameter of the astronomical telescope

Quiz: You are observing a binary star (2 stars close by). Which $\lambda$ of light you should choose to MAXIMIZE resolution?

(a) red  (b) yellow  (c) green  (d) blue
The Diffraction Grating = device for analysing light sources consists of large # of equally spaced slits, e.g. 5000 slits per cm

\[
d = \frac{10^{-2} \text{m}}{5000} = 2 \times 10^{-6} \text{m}
\]

= \(2 \mu\text{m}\)
Warning: here the size of the slit $a$ is not the parameter which determine the maxima of the diffraction → interference pattern but rather the spacing $d$ BETWEEN slits.

$\delta$ ... path difference

$\delta = d \cdot \sin \theta$

Condition for constructive interference:

$d \cdot \sin \theta_{\text{bright}} = m \cdot \lambda$

$m = 0, \pm 1, \pm 2, \ldots$
Quiz: Red light of \( \lambda = 700 \text{nm} \) is used to form interference pattern on a screen a distance \( L \) away. The spacing between slits is \( d \). We mark the bright maxima on the screen, then replace the red light source with UV source with \( \lambda_1 = \frac{1}{2} \lambda = 350 \text{nm} \). If we want to match the marks on the screen with the new bright maxima of \( \lambda_1 \) source, we have to:

(a) move the screen to \( 2L \)

(b) \( \frac{1}{2} \) to \( \frac{1}{2} L \)

(c) double the distance \( d_1 = 2d \) between the slits

(d) half the distance \( d_1 = \frac{1}{2} d \) between the slits

\( 27-18 \)
Diffraction of X-rays by Crystals

The wavelength $\lambda$ of any EM wave can be determined if a grating of the spacing $\sim \lambda$ is available.

X-rays $\lambda \approx 10^{-10} \text{ m} = 0.1 \text{ nm}$

$10^{-10} \text{ m}$ is the order of inter-atom spacing in a crystal.

Max von Laue suggested that a crystal can act as a 3D grating for X-rays and yield a diffraction pattern.

X-ray diffraction nowadays elucidates structure of matter.
Why is the Laue pattern complicated?

Consider 3D structure of a crystal, for example NaCl:
Bragg's law

The path difference between the upper plane & lower plane reflection:

$$2d \sin \theta = m \lambda$$

Bragg's law

$$m = 1, 2, 3, \ldots$$
Holography

Dennis Gabor
Nobel Prize 1971

(A) Producing a hologram

(B) Viewing a hologram
hologram acts as a grating
Quantum Physics

Dual nature of light?
wave vs. particle

Blackbody Radiation & Planck's Theory

→ any object at temperature \( T > 0 \) emits energy called thermal radiation

→ what is a black body?

A model: cavity

incoming light gets TOTALLY absorbed

→ the characteristics of radiation emitted from the hole ⇒ temperature of the walls
distribution of different wavelengths of cavity (blackbody) radiation [exp. studied in late 19th century]

Intensity

\[ \lambda \text{ [\textmu m]} \]

\[ \lambda_{\text{max}} \]
Experimentally observed features of the distribution:

→ The total power of emitted radiation increases with temperature ⇒ Stefan's law

\[ P = \sigma A e T^4 \]

- \( P \): power
- \( \sigma \): Stefan's constant ≈ 5.67 \( \times \) 10\(^{-8}\) W/m\(^2\)K\(^4\)
- \( A \): area
- \( e \): emissivity (≈ 1 for blackbody)
- \( T \): temperature

Stefan's constant \( \sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \)

→ The peak of the wavelength distribution, \( \lambda_{\text{max}} \), shifts to shorter wavelengths as the temperature increases ⇒ Wien's displacement law

\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m.K} \]
Theoretical explanation for blackbody radiation

→ classically:
  large $\lambda \rightarrow$ agree
  small $\lambda \Rightarrow$
  ultraviolet catastrophe
  $I \rightarrow \infty$ as $\lambda \rightarrow 0$

small $\lambda \Rightarrow$ UV light

→ explanation by Max Planck:
  onset of quantum physics

Planck imagined oscillators at the surface of b.b. (related to fluctuating charges in molecules) & made

2 assumptions

(i) energy of the oscillator is quantized:

$$E_n = n \cdot h \cdot f$$

$E_n$ → Planck constant

integer → frequency
(2) Oscillators emit or absorb energy in discrete units, quanta of radiation.

Oscillator's energy states: quantum states

\[ E_5 = 5hf \]
\[ E_4 = 4hf \]
\[ E_3 = 3hf \]
\[ E_2 = 2hf \]
\[ E_1 = hf \]

Emission transitions:
4 \rightarrow 3

Absorption transitions:
3 \rightarrow 4

\[ E_4 - E_3 \]

\[ 28-5 \]
Planck presented his theory which yielded the wavelength distribution in remarkable agreement with experimental curves.

Why do we not see quantum effects on a daily basis?

- in a macroscopic world, quantum jumps (say in energy) are so small that our senses perceive continuous behavior

- quantum effect observed on the submicroscopic level of atoms & molecules

Quiz: In the constellation Orion, stars Rigel & Betelgeuse glow in blue & red light, respectively. Which one has a higher surface temperature?
EXAMPLE: Thermal radiation from the Human Body

The temperature of the surface of the human body is 35°C. Calculate $\lambda_{\text{max}}$, the peak wavelength, of the radiation emitted by the human body, & estimate the total power $P$.

$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m K}$

$\rightarrow$ Wien's displacement law

$T_{\text{HB}} = 35^\circ\text{C} = 308 \text{ K}$

$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m K}}{308 \text{ K}} = 9.41 \times 10^{-6} \text{ m} = 9.41 \mu\text{m}$

$\rightarrow$ infrared region of spectrum

$\rightarrow$ Stefan's law: $P = \sigma A T^4$

$\rightarrow$ area of HB: $A \approx 2 \text{ m}^2$

height

2m \times 0.3m \times 0.2m \rightarrow depth