Hertz's Discoveries

Heinrich R. Hertz (in 1888) generated & detected EM waves in a lab for the first time using LC circuit.

- LC oscillations initiated by short voltage pulses

\[ f = \frac{1}{2\pi\sqrt{LC}} \sim 100 \text{ MHz } (10^8 \text{ s}^{-1}) \]

- Transmitter produces EM waves

- Receiver, several meters away, receives energy from transmitter if the resonance of receiver is matched to \( f \) of transmitter.
Energy carried by EM waves

\[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

\( \vec{S} \) describes the rate of energy flow in an EM wave. It is the rate of energy flow per unit surface area perpendicular to the direction of the EM wave.

\( \vec{S} \) units \( \left[ \frac{1}{s/m^2} = \frac{W}{m^2} \right] \)

Example: \( |\vec{S}| \) for a plane EM wave

\[ S = |\vec{S}| = \frac{1}{\mu_0} EB = \frac{1}{\mu_0 c} \frac{E^2}{2} = \frac{c}{\mu_0} B^2 \]

\[ B = \frac{E}{c} \quad \Rightarrow \quad E = cB \]

\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \Rightarrow \quad \varepsilon_0 \mu_0 = \frac{\mu_0}{\varepsilon_0} \]

\[ \rightarrow \quad \text{at any time: } \quad S = \frac{1}{\mu_0 c E^2} = \frac{c}{\mu_0 B^2} \]
→ time average of $S$ over one or more cycles is INTENSITY $I$

$$I = \frac{1}{2} \frac{1}{\epsilon_0 \mu_0} E_{\text{max}}^2 = \frac{1}{2} \frac{c}{\mu_0} B_{\text{max}}^2$$

$$E(x,t) = E_{\text{max}} \cos (kx - \omega t)$$
$$B(x,t) = B_{\text{max}} \cos (kx - \omega t)$$

$$\langle \cos^2 (kx - \omega t) \rangle_{\text{one cycle}} = \frac{1}{2}$$

→ energy per unit volume $\mu_E$ & $\mu_B$

$$\mu_E = \frac{1}{2} \epsilon_0 E^2$$
$$\mu_B = \frac{1}{2} \frac{1}{\mu_0} B^2$$

at any instant $t$

$$\mu_E = \mu_B$$

TOTAL $\mu = \mu_E + \mu_B = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$

→ average energy per unit volume

$$\mu_{\text{avg}} = \langle \mu_E \rangle_{\text{cycle}} + \langle \mu_B \rangle_{\text{cycle}}$$

$$\mu_{\text{avg}} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 = \frac{1}{2 \mu_0} B_{\text{max}}^2$$
relationship between intensity $I$ and the average energy per unit volume (av. energy density)

$$I = S_{avg} = c \cdot \overline{\mu_{avg}}$$

Quiz: Which quantity varies with time?

(a) intensity of an EM wave $I$

(b) average energy density $\overline{\mu_{avg}}$

(c) magnitude of the Poynting vector $|\vec{S}|$
Momentum and Radiation Pressure

EM waves carry linear momentum & energy ⇒ EM wave that hits a surface exerts pressure on the surface.

EM wave hits the surface at normal incidence, gets absorbed:

\[ P = \frac{U}{c} \quad \text{(complete absorption)} \]

\[ P = \frac{F}{A} = \frac{1}{A} \frac{dP}{dt} = \frac{1}{cA} \frac{dU}{dt} \]

\[ \frac{1}{A} \frac{dU}{dt} = S \quad \text{the magnitude of the Poynting vector} \]

\[ P = \frac{2S}{c} \quad \text{for complete reflection} \]

\[ P = \frac{S}{c} \quad \text{for complete absorption (black body)} \]
Quiz: Consider apparatus for measuring radiation pressure which is small even for direct sunlight \( \approx 5 \times 10^{-6} \text{ N/m}^2 \).

What happens if we replace the black disk with a radius \( R \) by a disk with a radius \( 2R \)? Which quantity remains the same?

1. \( x \) radiation pressure on the disk
2. \( x \) radiation force on the disk
3. \( (b) \) radiation momentum delivered to the disk in a time interval \( t \)
Ch. 27: Wave Optics

Wave optics = diffraction & interference
wave nature of light & principle of superposition
(addition of $\mathbf{E}$ & $\mathbf{B}$ in the EM wave)

Conditions for interference

→ at least 2 sources of waves with IDENTICAL wavelengths
→ individual waves must maintain a CONSTANT PHASE relationship
≡ COHERENT WAVES
[ordinary light sources → random changes in $\Delta t \sim 10^{-9}$ s → INCOHERENT]

Young's double-slit experiment

A) NO diffraction
B) diffraction
diffraction $\Rightarrow$ interference $\Rightarrow$ bright & dark parallel bands (= FRINGES)

wavefronts of $S_1$

wavefronts of $S_2$

bright

dark
Conditions for constructive & destructive interference

double slit

\[ s_1 \text{ and } s_2 \]

\[ r_1 \text{ and } r_2 \] paths of waves from \( s_1 \) \& \( s_2 \)

\[ \delta = r_2 - r_1 \] path difference

L \( \gg \) d \( \Rightarrow \) paths are parallel
\[ \delta = r_2 - r_1 = d \sin \theta \]

→ constructive interference

\[ d \cdot \sin \theta_{\text{bright}} = m \lambda \]
\[ m = \{0, \pm 1, \pm 2, \pm 3, \ldots \} \]

→ destructive interference

\[ d \cdot \sin \theta_{\text{dark}} = (m + \frac{1}{2}) \lambda \]
\[ m = \{0, \pm 1, \pm 2, \pm 3, \ldots \} \]

→ displacement \( y \) on the screen

\[ y = L \tan \theta \]

→ small angles: \( \sin \theta \approx \tan \theta \approx \theta \)

\[ y_{\text{bright}} \approx L \frac{m \lambda}{d} \]
\[ m = \{0, \pm 1, \pm 2, \ldots \} \]
Quiz: Which of the following will result in narrower fringes closer together?

(a) increasing the wavelength of light $\lambda$

(b) increasing the screen distance $L$

(c) increasing the slit spacing $d$

Example: laser light (coherent)

$d = 0.030 \text{ mm}$
$L = 1.2 \text{ m}$
2nd order fringe ($m = 2$)
$Y_2 = 5.1 \text{ cm}$

(a) $Y_2 = L \left( \frac{2\lambda}{d} \right)$
$\Rightarrow \lambda = \frac{1}{2} \frac{dY_2}{L}$
$\approx 640 \text{ nm}$

(b) $Y_2 - Y_0 = \frac{5.1 \text{ cm}}{2}$
$Y_1 - Y_0 \approx \frac{1}{2} (Y_2 - Y_0)$
$\approx 2.6 \text{ cm}$