Ch. 14 (14.1-14.5 & 18.7, Sound Waves)

Superposition & Standing Waves

The principle of superposition states that two traveling waves in a medium combine in such a way that the position of any element in the medium is the vector sum of positions due to individual waves.

INTERFERENCE

(a) Constructive

(b) Destructive

(c) Interference

(d) Interference

What is a mathematical description of interference?
Q12: Two waves interfere destructively, which statement is correct?
(a) The energy of the two initial waves is 1/2.
(b) The amplitude of the resulting wave is smaller than the amplitudes of the two initial waves.
(c) The amplitude of the resulting wave is greater than the amplitude of the initial wave.
(d) Interference is associated with an energy gain.
Standing Waves

Two transverse sinusoidal waves with the same amplitude, frequency, and wavelengths travel in opposite directions in a medium:

\[ y_1 = A \sin(\omega x - \omega t) \quad \text{and} \quad y_2 = A \sin(\omega x + \omega t) \]

According to superposition principle:

\[ y = y_1 + y_2 = A \left[ \sin(\omega x - \omega t) + \sin(\omega x + \omega t) \right] = 2A \sin(\omega x) \cos(\omega t) \]

The result is not a travelling wave because it lacks the forms \( f(x - vt) \) or \( f(x + vt) \) of simple harmonic motion with amplitude that depends on \( x \) and \( t \).

Every element of the medium, vibrates in simple harmonic motion with the same angular frequency. The maximum amplitude (amplitude):

\[ A_{\text{max}} = \frac{A}{2} \]

where \( A \) is the amplitude of the individual waves.
The minimal amplitude (mode): 

\[ \sin(kx) = \sin(x) \rightarrow kx = x_n, 2x_n, 3x_n, \ldots \]

Adjacent nodes are separated by \( \frac{x}{2} \) for adjacent modes. 

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Example: If an element of the string is moving up, its velocity is positive [upward].

For either of the two figures (A) and (B), the instantaneous velocity of all elements on the string is equal to zero? 

(A) 

\[ \text{(A)} \] 

(B) 

\[ \text{(B)} \]
Standing waves in strings

A string stretched between two rigid supports and driven by an incoming wave (λ₁) will interfere with the reflected wave (λ₂ = L - λ₁) to form a standing wave.

Normal modes of a string:

- **n=0**: L = λ₁ = λ₂ = 2L
  - 2 nodes at the two ends
  - 0 antinodes at the center
- **n=1**: L = λ₁ = λ₂ = L
  - 3 nodes: 1 at center and 2 modes
- **n=2**: L = λ₁ = λ₂ = 3L/2
  - 5 nodes: 1 at center and 3 modes

Wavelengths are discrete and determined by the length L of the string and its boundary conditions.

Frequency of normal modes: $f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$
Fundamental frequency $f_0$:

$\frac{1}{2L} \sqrt{\frac{T}{\mu}}$

Frequencies of higher modes are INTEGRAL MULTIPLES of the fundamental frequency $f_0$:

$f_n = n \cdot f_0 \quad (2f_0, 3f_0, 4f_0, \ldots)$

... first harmonic

$f_2 = 2f_0$ ... second harmonic ...

(A) A standing wave is set up on a string which is fixed at both ends. Which statement is true?

a) The number of antinodes is always one.

b) The number of nodes is equal to the number of antinodes plus 1.

c) The wavelength $\lambda$ is equal to the length $L$, divided by $n$, where $n$ is an integer.

d) The number of nodes is equal to the number of antinodes.
The speed of sound wave in air depends on:

\[ v = 331 \text{ m/s} + (0.6 \text{ m/s \cdot ^\circ C}) T[^\circ C] \]

Quiz: the wavelength of the sound is reduced by a factor of 2?,

\[ \lambda_2 = \frac{\lambda_1}{2} \]

What happens to the frequency \( f \) and speed \( v \)?

(a) \( f_2 = f \) and \( v_2 = \frac{1}{2}v_1 \) because \( \nu_2 =\nu_1 \), \( \lambda_2 = \frac{\lambda_1}{2} \) such that \( f_2 = f_1 \lambda_2 \)

(b) \( f_2 = \frac{1}{2}f_1 \) and \( v_2 = v_1 \)

(c) \( f_2 = 2f_1 \) and \( v_2 = v_1 \)
Standing Waves in Air Columns

- Compression = pressure antinode
- Instruments like brasses or woodwinds produce sound waves using a column of air as a result of interference between two longitudinal waves travelling in opposite directions.
- The closed end of the column corresponds to a displacement node and a pressure antinode.
- The open end of the column corresponds to a displacement antinode and a pressure node.

\[ L = L + \frac{\pi}{n} \]

Change in the character of the medium.

Physical length

resonant length
Air columns, open at both ends

First harmonic
\[ L = \frac{\lambda_1}{2} = \frac{\lambda_2}{2} \]
\[ \frac{L}{\lambda_1} = \frac{L}{\lambda_2} = \frac{L}{2} \]

Second harmonic
\[ L = \lambda_1 = \lambda_2 = L \]
\[ \frac{L}{\lambda_1} = \frac{L}{\lambda_2} = 1 \]

Third harmonic
\[ L = \frac{3}{2} \lambda_1 = \frac{3}{2} \lambda_2 = \frac{3}{2} L \]
\[ \frac{L}{\lambda_1} = \frac{L}{\lambda_2} = \frac{3}{2} \]

Air columns, closed at one end

First harmonic
\[ L = \lambda_1 = L \]
\[ \frac{L}{\lambda_1} = 1 \]

Second harmonic
\[ L = \frac{3}{2} \lambda_1 = \frac{3}{2} L \]
\[ \frac{L}{\lambda_1} = \frac{3}{2} \]

Third harmonic
\[ L = \frac{5}{4} \lambda_1 = \frac{5}{4} L \]
\[ \frac{L}{\lambda_1} = \frac{5}{4} \]
Standing waves in a pipe are excited at a fundamental frequency $f$, at a room temperature. Someone heats up the room with a pipe. What happens with the frequency $f$? Consider that $v = \lambda f$ ($v$ is the speed of sound)

(a) $f$ increases (the pipe goes dull)
(b) $f$ decreases (the pipe goes flat)

Question: The pipe is open at both ends and is excited at a fundamental frequency $\lambda$. Someone closes the pipe at one end, and the pipe is excited at a fundamental frequency $\lambda'$. Which is correct?

(a) $\lambda' = 2\lambda$
(b) $\lambda' = \frac{1}{2}\lambda$
(c) $\lambda' = 4\lambda$
Ch. 24: Electromagnetic Waves

- visible light
- infrared waves from Earth’s surface
- microwaves
- radio-frequency waves
- prediction of Maxwell’s equations

Maxwell’s Equations (1 equations & formula)

(i) Gauss Law for electricity

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \]

- integral around a closed surface
- simple example: spherical surface A around a point charge

(ii) Gauss Law for magnetism

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

- there is no magnetic "charge" or monopoles
- flux of magnetic field density through a closed surface
Faraday's Law of Induction:

\[ \oint \mathbf{E} \cdot d\mathbf{s} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \]

- Time derivative of the magnetic flux \( \Phi_B \) through the surface \( S \) enclosed by the path.

Example: Consider homogeneous magnetic field \( \mathbf{B} \) pointing out of the transparency. A U-shaped wire with a movable crossbar...

A current moving with \( \mathbf{E}_A \) produces an electric field \( \mathbf{E}_B \) perpendicular (the electric current induced in the wire.)
\[ \int_{-\pi}^{\pi} f(x) \, dx = \frac{2\pi a}{b} \]

\[ R = \frac{a}{b} \]

\[ \text{Area} = \pi a^2 \]

\[ I = \frac{a^2}{b^2} \]
Example (without the additional current)
Example with the DISPLACEMENT current

\[ I_d = \varepsilon \frac{dA}{dt} \]

What is the displacement current? Consider a changing capacitor.

1. Choose \( S_1 \)
   \[ B \cdot dA = B \cdot 2A = \mu_0 I + \mu_0 I_d \]
   \( I_d = 0 \) (no displacement current)

2. Choose \( S_2 \)
   \[ B \cdot dA = B \cdot 2A = \mu_0 I + \mu_0 I_d \] (If \( I_d \neq 0 \) there is current between the two capacitor plates)
The generalized Ampère's law states that
(1) \( f(t) \) give the same results.

Calculate the displacement current \( \mathbf{J}_d \):
\[
\mathbf{J}_d = \nabla \times \mathbf{E}_L
\]
\[
\mathbf{E}_L = \frac{\mathbf{B}}{\mu_0}
\]
\[
\mathbf{B} = \mu_0 \mathbf{H}
\]

We showed that in this example the displacement current is the same as the current through the wire.

The Ampère's law in generalized form states that magnetic fields are produced by conduction & changing electric field currents.

1. Inertial force law on a particle of charge \( q \):
\[
f = q(E + \mathbf{v} \times \mathbf{B})
\]
\[
\frac{\delta E}{\delta x} = -\frac{p^2}{m} \frac{\delta x}{\delta x} - \frac{2m}{\hbar^2} \frac{\delta^2 x}{\delta t^2} \frac{\delta x}{\delta x} = -\frac{p^2}{m} - \frac{2m}{\hbar^2} \frac{\delta^2 x}{\delta t^2} \]

\[
\frac{\delta x}{\delta t} = \Gamma \frac{\delta E}{\delta x} \]

\[
\Gamma = \frac{1}{\sqrt{\hbar^2 + \frac{p^2}{m}}} \]

\[
\text{speed of light}
\]
Important relationship:
\[ \frac{1}{v} = \frac{1}{c} \quad \text{or} \quad \frac{v}{c} = \frac{1}{c} \]

\[ c = 2.998 \times 10^8 \text{ m/s} \]

Doppler Effect for Light:
Because EM waves do not require any medium for propagation, only relative speed between the source and observer is relevant to the detected frequency \( f' \) relative to \( f \) at rest:

(a) \( f' = f \left( \frac{v - c}{v - c} \right) \) source & observer approaching

(b) \( f' = f \left( \frac{v - c}{v - c} \right) \) source & observer moving away from each other

\[ \lambda = \frac{c}{v} \rightarrow \lambda' = \frac{c}{v'} \]

and shift \( \Delta \) for galaxies moving away from us.
LC circuit as a simple harmonic oscillator

- Initial charge on capacitor when switch S is closed at t=0
- Energy of the LC circuit at t=0
  \[ E = \frac{1}{2} \cdot C \cdot V^2 \] (mechanical electric potential energy)
- At \( t = \frac{\pi}{2}\), the capacitor discharges
- Current \( I \) through \( L \) is the maximal electric potential energy transferring eventually to maximal electric current

\[ E = \frac{1}{2} I^2 L \]

- The process then reverses and repeats
Apply Kirchhoff's loop rule: The sum of the voltage differences across each element around a closed circuit loop is zero:

\[ \frac{Q}{C} + L \frac{dI}{dt} \equiv 0 \Rightarrow \frac{dQ}{dt} = -L \frac{dI}{dt} \]

\[ \Rightarrow \frac{dQ}{dt} = -\frac{1}{LC} \frac{dI}{dt} \quad \text{or} \quad \frac{e}{C} \Rightarrow \frac{C}{L} \quad \text{resonance frequency of the LC circuit} \]

Question: A capacitor in an LC circuit is discharging. Which statement is correct?

(a) There is an electric current between the plates of the capacitor.
(b) There is a static charge on the plates.
(c) There is a constant electric field between the plates of the capacitor.
(d) There is an electric field between the plates of the capacitor, varying in time.