PHYS-201: FUNDAMENTALS OF PHYSICS III
Midterm Exam # 1
January 28, 2009
CURTIS-341, 9:00-9:50 AM

RULES:

1. Do not turn over this sheet until instructed to do so.
2. No talking is allowed during the exam. Cell phones need to be turned off.
3. If you don’t understand a problem, raise your hand and we will assist you.
4. Do not detach pages from this packet. Indicate clearly where the answer is located. Numerical results need to be expressed in correct units.
5. This is a closed book exam. No books or lecture notes allowed.

Problem 1: [/40 points] ________________

Problem 2: [/20 points] ________________

Problem 3: [/20 points] ________________

Problem 4: [/20 points] ________________

Total: [/100 points] ________________
Simple Harmonic Oscillator and Damping

Problem 1. An object with a mass 250 g is hung from a spring with a force constant 100 N/m. The length of the spring with no object hanging on it is 15 cm. The acceleration due to gravity is \( g = 9.80 \text{ m/s}^2 \).

(A) What is the equilibrium length of the spring when the object is hung on it

(B) The object on the spring is pulled down and released to freely oscillate without friction. Calculate the angular frequency \( \omega_0 \) and the period \( T_0 \) of this simple harmonic motion.

(C) What is the angular frequency of the object on the spring after we immerse the oscillator in a beaker of oil with a damping constant \( b = 0.1 \text{ N s/m} \)?

(D) Calculate the time in which the total mechanical energy of the damped harmonic oscillator in (C) decreases to half its initial value.

\[
\text{(A) } m = 0.25 \text{ kg} \\
k = 100 \text{ N/m} \\
L_0 = 0.15 \text{ m} \\
\Rightarrow L = L_0 + \frac{mg}{k} = 0.15 \text{ m} + \frac{0.245 \text{ m}}{0.25} = 0.175 \text{ m} (17.5 \text{ cm})
\]

\[
\text{(B) } \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m} \cdot 0.14 \text{ kg}}{0.25 \text{ kg}}} = 20 \text{ s}^{-1} \\
T_0 = \frac{2\pi}{\omega_0} = 0.314 \text{ s}
\]

\[
\text{(C) } \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \sqrt{400 \text{ s}^{-2} - \frac{(0.4 \text{ N s/m})^2}{100 \text{ N/m} \cdot 0.1 \text{ kg}}} = \sqrt{199.999 \text{ s}^{-2}}
\]

\[
\text{(D) } E = \frac{1}{2} kA^2 e^{-\frac{bt}{m}}, \quad E_0 = \frac{1}{2} kA^2 \\
E = E_0 e^{-\frac{bt}{m}}, \quad \frac{1}{2} E_0 = E_0 e^{-\frac{bt}{m}} \\
\tau_h = \frac{b}{k} \ln 2 = 1.73 \text{ s}
\]
**Problem 2.** Consider a transverse sinusoidal wave on a rope traveling in the negative \( x \)-direction with a speed \( v = 5 \text{ m/s} \). The amplitude of the motion in the \( y \)-direction is 10 cm and the period \( T = 0.25 \text{ s} \). The wave is such that at \( t = 0 \) the \( y \)-displacement of the rope at the position \( x = 0 \) is equal to zero.

(A) Calculate the angular frequency \( \omega \) and the angular wave number \( k \) of the wave.

(B) Calculate the maximum transverse speed and maximum acceleration of an element of the rope.

\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{0.25 \text{ s}} = 25.1 \text{ s}^{-1} \]

\[ k = \frac{2\pi}{\lambda} = \frac{2\pi}{v} = \frac{\omega}{v} = \frac{25.1 \text{ s}^{-1}}{5 \text{ m/s}} = 5.05 \text{ m}^{-1} \]

\[ y(x, t) = A \sin (kx + \omega t) \]

\[ v_{\text{max}} = \left( \frac{dy}{dt} \right)_{\text{max}} = A \omega = 0.1 \text{ m} \cdot 25.1 \text{ s}^{-1} = 2.51 \text{ m/s} \]

\[ a_{\text{max}} = \left( \frac{d^2y}{dt^2} \right)_{\text{max}} = A \omega^2 = 0.1 \text{ m} \cdot (25.1 \text{ s}^{-1})^2 = 63 \text{ m/s}^2 \]
Superposition and Standing Waves

Problem 3. A horizontal 10 m—long string with a linear mass density $\mu = 16 \text{ g/m}$ has one end attached to a wall and the other draped over a pulley and attached to a hanging object with a mass of 5 kg. The string is plucked and starts vibrating. The acceleration due to gravity is $g = 9.80 \text{ m/s}^2$.

(A) Calculate the fundamental frequency (first harmonic) and the second harmonic frequency of the string vibration.

(B) What is the new fundamental frequency of the vibration if the string is cut to half and the mass of the hung object is doubled?

\[
L = 10 \text{ m} \\
\mu = 0.016 \text{ kg/m} \\
m = 5 \text{ kg}
\]

\[
\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}} = 55.3 \text{ m/s}
\]

\[
\lambda_1 = 2L = 20 \text{ m}
\]

\[
f_1 = \frac{\nu}{\lambda_1} = 2.88 \text{ s}^{-1}
\]

\[
A)
\]

\[
\lambda_1 = 2L = 20 \text{ m}
\]

\[
f_1 = \frac{\nu}{\lambda_1} = 2.88 \text{ s}^{-1}
\]

\[
B)
\]

\[
A' = \sqrt{2} \cdot \nu = 78.2 \text{ m/s}
\]

\[
\lambda_1' = 2 \cdot \frac{L}{2} = 10 \text{ m} = \frac{1}{2} \lambda_1
\]

\[
f_1' = \frac{\nu'}{\lambda_1'} = 2.88 \sqrt{2} \text{ s}^{-1}
\]
**Electromagnetic Waves**

**Problem 4.** Consider a radiotelephone with a LC-circuit of a fixed inductance \( L = 2 \mu H \) [1 H = 1 V·s/A] and a variable capacitance \( C \). We want to use this radiotelephone to receive the signal from a radio transmitter at a distance of 5 km from the radiotelephone. The radio transmitter is broadcasting equally in all directions at a frequency \( f = 8 \text{ MHz} \) with an average power of \( P_{\text{avg}} = 250 \text{ kW} \).

(A) What capacitance \( C \) on the radiotelephone do we need to use to receive the signal?

(B) Calculate the average magnitude of the Poynting vector, \( S_{\text{avg}} \), at a distance of 5 km (where the radiotelephone is located) from the radio transmitter.

\[
(A) \quad f = 8 \times 10^6 \text{s}^{-1}
\]

\[
P_{\text{avg}} = 2.5 \times 10^5 \text{ W}
\]

\[
L = 2 \times 10^{-6} \text{ Vs/A}
\]

\[
d = 5 \times 10^3 \text{ m}
\]

\[
f = \frac{1}{2\pi \sqrt{LC}}
\]

\[
\Rightarrow C = \frac{1}{L \left(\frac{2\pi f}{c}\right)^2} = 1.98 \times 10^{-10} \text{ As/V}
\]

\[
(B) \quad I = S_{\text{avg}} = \frac{P_{\text{avg}}}{4\pi d^2} = \frac{2.5 \times 10^5 \text{ W}}{4\pi \frac{25 \times 10^6 \text{ m}^2}{25 \times 10^6 \text{ m}^2}} = 7.96 \times 10^{-4} \text{ W/m}^2
\]