
You should be familiar with these equations by now (after our time with the Bohr atom and relativity). The energy is given by (Eq’s 28.3 and 11.15)

\[ E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{700 \text{ nm}} = 1.77 \text{ eV}. \]  

For momentum you can either use the relativistic energy-momentum equation (Eq. 9.22)

\[ E^2 = p^2c^2 + m_0^2c^4 \]  

\[ p_{\text{photon}} = \frac{E}{c} \]  

or the de Broglie formula (Eq. 28.10)

\[ p = \frac{h}{\lambda} = \frac{6.63 \cdot 10^{-34} \text{ J} \cdot \text{s}}{700 \text{ nm}} = 9.46 \cdot 10^{-28} \text{ kg} \cdot \text{m/s}. \]

Problem 28.15. X-rays having an energy of 300 keV undergo Compton scattering from a target. The scattered rays are detected at 37.0° relative to the incident rays. Find (a) the Compton shift at this angle, (b) the energy of the scattered x-ray, and (c) the energy of the recoiling electron.

(a) From the Compton shift equation (Eq. 28.8)

\[ \lambda' - \lambda_0 = \frac{h}{m_ec} (1 - \cos \theta) \]  

\[ \Delta \lambda = 2.43 \text{ pm}(1 - \cos 37.0^\circ) = 489 \text{ fm}. \]

(b) The wavelength of the incoming photon was

\[ \lambda_0 = \frac{hc}{E_0} = \frac{1240 \text{ eV} \cdot \text{nm}}{300 \text{ keV}} = 4.13 \text{ pm}. \]

The scattered wavelength is thus

\[ \lambda' = \lambda_0 + \Delta \lambda = (4.13 + 0.489) \text{ pm} = 4.62 \text{ pm}, \]

and the energy of the scattered photon is

\[ E' = \frac{hc}{\lambda'} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.62 \text{ pm}} = 268 \text{ keV}. \]

(c) All the energy lost by the photon must go into the recoiling electron so

\[ E_e = E_0 - E' = (300 - 268) \text{ keV} = 31.7 \text{ keV}. \]

Problem 28.16. A 0.110 nm photon collides with a stationary electron. After the collision, the electron moves forward and the photon recoils backward. Find the momentum and the kinetic energy of the electron.

The photon scatters by 180°, so from the Compton shift equation

\[ \lambda' = \lambda_0 + \frac{h}{m_ec}(1 - \cos 180^\circ) = \lambda_0 + \frac{2h}{m_ec} = (110 + 4.85) \text{ pm} = 115 \text{ pm}. \]

The kinetic energy of the electron is given by the change in photon energy (just like problem 28.15).

\[ K = E_0 - E' = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = 11.3 \text{ keV} - 10.8 \text{ keV} = 476 \text{ eV}. \]

We conserve momentum to find the electron’s momentum, using \( p_{\text{photon}} = E/c \).

\[ p_i = \frac{E_0}{c} = p_f = p_e - \frac{E'}{c} \]

\[ p_e = \frac{E_0 + E'}{c} = (11.3 + 10.8) \text{ keV}/c = 22.1 \text{ keV}/c = 1.18 \cdot 10^{-23} \text{ kg} \cdot \text{m/s}. \]