Chapter 19: DC Circuits

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• Direct Current (DC) circuits are important
  • often used in digital electronic devices and almost anything that is battery powered.
• The physics of DC currents is straightforward
  • Kirchoff’s loop rule
  • Kirchoff’s junction rule
  • Understanding the individual components
    • Resistors
    • Capacitors
    • …
Kirchoff’s loop rule

\[ \sum \Delta V_i = 0 \quad \text{Conserving energy} \]
Kirchoff’s junction rule

\[ \sum_{\text{junction}} I_i = 0 \quad \text{Conserving charge, steady state} \]
Elements in series

Two elements are in series if you cannot make a loop crossing one of them without also crossing the other.
Elements in parallel

Two elements are in parallel if you can make a loop crossing both of them which crosses no other elements.
Resistors in series

By Kirchoff’s junction rule, $I_a = I_b$ (because there are no intervening junctions).

$$V_T = V_a + V_b = IR_a + IR_b = I(R_a + R_b)$$

$$V_T = IR_T$$

So the effective resistance is given by

$$R_T = R_a + R_b$$
Resistors in parallel

\[ R_a, \ V_a \]

\[ V_T, \ R_T \]

\[ R_b, \ V_b \]

By Kirchoff’s loop rule, \( V_a - V_b = 0 \) so \( V_a = V_b = V_T \).

By Kirchoff’s junction rule, \( I_T = I_a + I_b \)

\[
V_T = I_a R_a = I_b R_b \quad I_b = I_a \frac{R_a}{R_b}
\]

\[
I_T = I_a + I_b = I_a \left( 1 + \frac{R_a}{R_b} \right)
\]

\[
V_T = I_a R_a = \frac{I_T}{1 + \frac{R_a}{R_b}} R_a = I_T \left( \frac{1}{R_a} + \frac{1}{R_b} \right)^{-1} = I_T R_T
\]

So the effective resistance is give by \( R_T = \left( \frac{1}{R_a} + \frac{1}{R_b} \right)^{-1} \).
Capacitors in series

\[
C_a, \ V_a \quad C_b, \ V_b
\]

\[\begin{array}{ccc}
\text{a} & \text{b} \\
\hline \hline
V_T, \ C_T
\end{array}\]

By Kirchoff’s junction rule, \(Q_a = Q_b = Q_T\) (because there are no intervening junctions).

\[
V_T = V_a + V_b = Q_a/C_a + Q_b/C_b = Q_T\left(1/C_a + 1/C_b\right)
\]

\[
V_T = Q_T/C_T
\]

So the effective capacitance is given by

\[
C_T = \left(\frac{1}{C_a} + \frac{1}{C_b}\right)^{-1}
\]
Capacitors in parallel

By Kirchoff’s loop rule, \( V_a - V_b = 0 \) so \( V_a = V_b = V_T \).
By Kirchoff’s junction rule, \( Q_T = Q_a + Q_b \)

\[
Q_T = Q_a + Q_b = C_a V_T + C_b V_T = (C_a + C_b) V_T
\]
\[
Q_T = C_T V_T
\]

So the effective capacitance is given by \( C_T = C_a + C_b \).
From Kirchoff’s loop rule \( \sum_{\text{loop}} V_i = \mathcal{E} - IR - \frac{Q}{C} = 0 \).

From the definition of current \( I = \frac{dQ}{dt} \).

So

\[
\mathcal{E} - \frac{Q}{C} = R \frac{Q}{t}
\]

\[
dt = (\mathcal{E} - \frac{Q}{C})^{-1} RdQ
\]

\[
Q = C\mathcal{E} \left(1 - e^{-t/RC}\right)
\]

\[
V_C = \mathcal{E} \left(1 - e^{-t/RC}\right)
\]