Problem 40. Two capacitors, $C_1 = 5.00 \mu F$ and $C_2 = 12.0 \mu F$, are connected in series, and the resulting combination is connected to a $\Delta V = 9.00 \text{ V}$ battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

(a) The wire connecting the inner plates of $C_1$ and $C_2$ contains no net charge, so we know that any charge on the inner plate of $C_1$ must have come from the inner plate of $C_2$. Because these charges are equal and opposite, the total charge $Q$ on each capacitor separately is the same for both ($Q_1 = Q_2$). So using the definition of capacitance for both cases we have

\[
\Delta V_1 = Q / C_1 \\
\Delta V_2 = Q / C_2
\]

\[
\Delta V = \Delta V_1 + \Delta V_2 = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{Q}{C_{eq}}
\]

So

\[
C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{1}{5.00 \cdot 10^{-6} \text{ F}} + \frac{1}{12.0 \cdot 10^{-6} \text{ F}} \right)^{-1} = 3.53 \mu F
\]

(c) Plugging back into equation 3 we have

\[
Q = \Delta V \cdot C_{eq} = 3.53 \mu F \cdot 9.00 \text{ V} = 31.8 \mu C
\]

(b) And plugging into equations 1 and 2 we have

\[
\Delta V_1 = \frac{31.8 \cdot 10^{-6} \text{ C}}{5.00 \cdot 10^{-6} \text{ F}} = 6.35 \text{ V}
\]

\[
\Delta V_2 = \frac{31.8 \cdot 10^{-6} \text{ C}}{12.0 \cdot 10^{-6} \text{ F}} = 2.65 \text{ V}
\]

Problem 43. Consider the circuit shown in Figure P20.43, where $C_1 = 6.00 \mu F$, $C_2 = 3.00 \mu F$, and $\Delta V = 20.0 \text{ V}$. Capacitor $C_1$ is first charged with $Q_1$ by the closing of switch $S_1$. Switch $S_1$ is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of $S_2$. Calculate $Q_1$ and the final charge on each capacitor ($Q'_1$ and $Q'_2$).

The first situation with $S_1$ closed and $S_2$ open is just a standard capacitor charging problem. Using the definition of capacitance

\[
Q_1 = C_1 \Delta V = 6.00 \cdot 10^{-6} \text{ F} \cdot 20.0 \text{ V} = 120 \mu C
\]

After disconnecting the battery and connecting the two capacitors, we have a net charge of $Q_1$ in the upper wire that we can distribute as we desire between $C_1$ and $C_2$. Because charge is conserved, we know

\[
Q_1 = Q'_1 + Q'_2
\]

We also know that at equilibrium the voltage across each capacitor must be equal (because if there was a voltage difference between the upper plates of the two capacitors, it would push current through the upper wire until the voltage difference dissipated, etc.). So

\[
\Delta V'_1 = \frac{Q'_1}{C_1} = \Delta V'_2 = \frac{Q'_2}{C_2}
\]

Now we have two equations relating our two unknowns $Q'_1$ and $Q'_2$. Solving equation 11 for $Q'_2$ and plugging into equation 10 we get

\[
Q'_2 = \frac{C_2}{C_1} Q'_1
\]

\[
Q_1 = \left( 1 + \frac{C_2}{C_1} \right) Q'_1
\]

\[
Q'_1 = \frac{Q_2}{1 + C_2/C_1} = \frac{120 \mu C}{1.5} = 80 \mu C
\]

\[
Q'_2 = 0.5 \cdot 80 \mu C = 40 \mu C
\]
Problem 47. (a) A $C = 3.00 \mu F$ capacitor is connected to a $\Delta V_a = 12.0 \text{ V}$ battery. How much energy $U_a$ is stored in the capacitor? (b) If the capacitor had been connected to a $\Delta V_b = 6.00 \text{ V}$ battery, how much energy would have been stored?

Simply plugging into the formula for energy stored in a capacitor we have

$$ U_a = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \cdot 10^{-6} \text{ F}) \cdot (12.0 \text{ V})^2 = 216 \mu \text{J} \quad (16) $$

$$ U_b = \frac{1}{2}(3.00 \cdot 10^{-6} \text{ F}) \cdot (6.0 \text{ V})^2 = 54 \mu \text{J} \quad (17) $$

Problem 51. Show that the force between two plates of a parallel-plate capacitor each have an attractive force given by

$$ F = \frac{Q^2}{2\epsilon_0 A} \quad (18) $$

The electric field generated by the plate $A$ is given by $E_A = Q/2\epsilon_0 A$ (which we derived for P19.62, along with $\sigma = Q/A$). So the force on plate $B$ due to plate $A$ is given by

$$ F = QE_A = \frac{Q^2}{2\epsilon_0 A} \quad (19) $$

Problem 54. (a) How much charge $Q_c$ can be placed on a capacitor with air between the plates before it breaks down if the area of each plate is $A = 5.00 \text{ cm}^2$? (b) Find the maximum charge assuming polystyrene is used between the plates instead of air.

From Chapter 19, the voltage difference due to a constant electric field $E$ over a displacement $d$ is given by $\Delta V = E \cdot d$. So for two plates a distance $d$ apart, the breakdown voltage is given by

$$ V_c = E_c d \quad (20) $$

where $E_c$ is the dielectric strength of the material.

The capacitance of a parallel-plate capacitor is given by

$$ C = \frac{\kappa\epsilon_0 A}{d} \quad (21) $$

Combining these two formula with the definition of capacitance we have

$$ E_c d = V = \frac{Q}{C} = \frac{Qd}{\kappa\epsilon_0 A} \quad (22) $$

Looking up the values for air and polystyrene in Table 20.1 on page 699 of the text we see:

<table>
<thead>
<tr>
<th>Name</th>
<th>Dielectric constant $\kappa$</th>
<th>Dielectric strength $E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.00059</td>
<td>$3 \cdot 10^6 \text{ V/m}$</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>$24 \cdot 10^6 \text{ V/m}$</td>
</tr>
</tbody>
</table>

So plugging into our formula for the charge

$$ Q_a = 1.00 \cdot (3 \cdot 10^6 \text{ V/m}) \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4} \text{ m}^2 = 1.33 \cdot 10^{-8} \text{ C} \quad (23) $$

$$ Q_b = 2.56 \cdot (24 \cdot 10^6 \text{ V/m}) \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4} \text{ m}^2 = 2.72 \cdot 10^{-7} \text{ C} \quad (24) $$

Problem 73. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is $\kappa = 3.00$ and whose dielectric strength is $E_c = 2.00 \cdot 10^8 \text{ V/m}$. The desired capacitance is $C = 0.250 \mu F$, and the capacitor must withstand a maximum potential difference of $V_c = 4000 \text{ V}$. Find the minimum area $A$ of the capacitor plates.

Using equation 20, we have

$$ d \geq \frac{V_c}{E_c} \quad (25) $$

Where equality represents a breakdown at $V_c$ and larger $d$ give us more protection with larger breakdown voltages. From equation 21 we have

$$ A = \frac{dC}{\kappa\epsilon_0} \quad (26) $$

From which we can see that the smaller $d$ is, the smaller $A$ can be, and we pick $d = V_c/E_c$, the smallest possible value we can. Then the smallest area is given by

$$ A = \frac{V_cC}{E_c\kappa\epsilon_0} = \frac{(4000 \text{ V}) \cdot (0.25 \cdot 10^{-6} \text{ F})}{(2.00 \cdot 10^8 \text{ V/m}) \cdot 3.00 \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{N m}^2)} = 0.188 \text{ m}^2 \quad (27) $$