Problem 34. Two long, parallel conductors, separated by \( r = 10.0 \text{ cm} \), carry current in the same direction. The first wire carries current \( I_1 = 5.00 \text{ A} \), and the second carries \( I_2 = 8.00 \text{ A} \). (a) What is the magnitude of the magnetic field \( B_1 \) created by \( I_1 \) at the location of \( I_2 \)? (b) What is the force per unit length exerted by \( I_1 \) on \( I_2 \)? (c) What is the magnitude of the magnetic field \( B_2 \) created by \( I_2 \) at the location of \( I_1 \)? (d) What is the force per unit length exerted by \( I_2 \) on \( I_1 \)?

(a) From Ampere’s law, the \( B \) field generated by a long, thin current is

\[
B = \frac{\mu_0 I}{2\pi r}
\]  

Plugging in \( I_1 \), we have

\[
B_1 = \frac{\mu_0 I_1}{2\pi r} = 10.0 \mu\text{T}
\]

This \( B \) field depends on your distance from \( I_1 \), but because the wires are parallel, the \( B \) field from \( I_1 \) is constant along \( I_2 \). We can use the right hand rule to determine that \( B_1 \) is perpendicular to both \( I_1 \) and \( r \).

(b) From \( F_B = qv \times B \) we have the force on a current carrying wire in a uniform magnetic field as

\[
F_B = I l \times B
\]

Combining these two equations, we have the force per unit length of \( I_1 \) on \( I_2 \) as

\[
\frac{F_{B12}}{l} = I_2 B_1 = \frac{\mu_0 I_1 I_2}{2\pi r} = 80.0 \mu\text{N}
\]

where there is no sin\( \theta \) term in the cross product, because \( B_1 \) is perpendicular to \( I_2 \). By drawing the situation and doing some right hand rules, you can convince yourself that this force is attractive.

(c) Because the situation in (c) is identical to (a) with \( I_1 \leftrightarrow I_2 \), we simply relabel eqn. 2

\[
B_2 = \frac{\mu_0 I_2}{2\pi r} = 16.0 \mu\text{T}
\]

(d) Eqn. 4 is identical under the relabeling, so we have another attractive force at the same magnitude

\[
\frac{F_{B21}}{l} = 80 \mu\text{N}
\]

as we would expect from Newton’s third law (for every action there is an equal and opposite reaction).

Problem 37. Four long, parallel conductors carry equal currents of \( I = 5.00 \text{ A} \). Figure P22.37 is an end view of the conductors. The current direction is into the page at points A and B and out of the page at points C and D. Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square of edge length \( a = 0.200 \text{ m} \).

First, let us pick a coordinate system by choosing unit vectors. Let \( \hat{i} \) be down and to the left, \( \hat{j} \) be down and to the right, and \( \hat{k} \) be straight down.

Using the right-hand rule, we determine the direction of the magnetic field at \( P \) generated by each wire to be

\[
\vec{B}_A = \hat{i}
\]

\[
\vec{B}_B = \hat{j}
\]

\[
\vec{B}_C = \hat{i}
\]

\[
\vec{B}_D = \hat{j}
\]

The magnitude of each \( B \) is given by

\[
B = \frac{\mu_0 I}{2\pi r}
\]

And since the currents have the same magnitude, and each corner is equidistant from the square center, each magnetic field contribution will have the same magnitude. The distance \( r \) is given by

\[
r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}
\]

We still have to add our vector fields, which gives

\[
\vec{B}_P = \vec{B}_A + \vec{B}_B + \vec{B}_C + \vec{B}_D = 2B(\hat{i} + \hat{j}) = \frac{2\mu_0 I}{2\pi r} \cdot \sqrt{2} \hat{k} = \frac{\sqrt{2} \mu_0 I}{\pi r} \hat{k} = \frac{2\mu_0 I}{\pi a} \hat{k} = 20 \mu\text{T}
\]
Problem 43. Niobium metal becomes superconducting when cooled below 9K. Its superconductivity is destroyed when the surface B field exceeds \( B_{\text{max}} = 0.100 \, \text{T} \). Determine the maximum current in a \( d = 2.00 \, \text{mm} \) diameter niobium wire can carry and remain superconducting, in the absence of any external B field.

For long, cylindrical wires, the magnetic field a distance \( r \) from the center of the wire is
\[
B = \frac{\mu_0 I}{2\pi r}
\] (14)

As long as you are outside the wire.

Therefore, the magnetic field at the surface is maximized when
\[
B_{\text{max}} = \frac{\mu_0 I_{\text{max}}}{2\pi r}
\] (15)
\[
I_{\text{max}} = \frac{(2\pi B_{\text{max}}) / \mu_0 = 500 \, \text{A}}{}
\] (16)

Problem 48. In Bohr’s 1913 model of the hydrogen atom, the electron is in a circular orbit of radius \( r = 5.29 \cdot 10^{-11} \, \text{m} \), and its speed is \( v = 2.19 \cdot 10^6 \, \text{m/s} \). (a) What is the magnitude of the magnetic moment \( \mu \) due to the electron’s motion? (b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of \( \mu \)?

(a) The magnetic moment is defined on page 742 as
\[
\mu = I A
\] (17)

The area swept out by our electron is just
\[
A = \pi r^2
\] (18)

The current is the amount of charge circling the nucleus in a unit time. Because
\[
\Delta x = v \Delta t
\] (19)

The time \( \tau \) taken for an entire circuit is
\[
\tau = \frac{\Delta x}{v} = \frac{2\pi r}{v}
\] (20)

The current is then given by
\[
I = \frac{\Delta q}{\Delta t} = \frac{q_e v}{2\pi r}
\] (21)

Plugging \( I \) and \( A \) into our moment equation
\[
\mu = \frac{q_e v}{2\pi r} \cdot \pi r^2 = (q_e vr) / 2 = 9.27 \cdot 10^{-24} \, \text{A m}^2
\] (22)

The direction of the current is opposite the direction of the electron (because the electron has negative charge), so the direction of \( \mu \) is down.

Problem 57. A positive charge \( q = 3.20 \cdot 10^{-19} \, \text{C} \) moves with a velocity \( \mathbf{v} = (2\hat{i} + 3\hat{j} - \hat{k}) \, \text{m/s} \) through a region where both a uniform magnetic field and a uniform electric field exist. (a) Calculate the total force \( \mathbf{F} \) on the moving charge (in unit-vector notation), taking \( \mathbf{B} = (2\hat{i} + 4\hat{j} + \hat{k}) \, \text{T} \) and \( \mathbf{E} = (4\hat{i} - \hat{j} - 2\hat{k}) \, \text{V/m} \). (b) What angle \( \theta \) does the force vector \( \mathbf{F} \) make with \( \hat{i} \)?

(a) From Chapter 19,
\[
\mathbf{F}_E = q\mathbf{E} = q(4\hat{i} - \hat{j} - 2\hat{k}) \, \text{N/C}
\] (23)

From this chapter
\[
\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 4 & 1 \end{vmatrix} = q[(3 + 4)\hat{i} - (2 + 2)\hat{j} + (8 - 6)\hat{k}] = q(7\hat{i} - 4\hat{j} + 2\hat{k}) \, \text{N/C}
\] (24)

So the total force is given by
\[
\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = q[(4 + 7)\hat{i} + (-1 - 4)\hat{j} + (-2 + 2)\hat{k}] \, \text{N/C} = q(11\hat{i} - 5\hat{j}) \, \text{N/C} = (35.2\hat{i} - 16.0\hat{j}) \cdot 10^{-19} \, \text{N}
\] (25)

(b)
\[
\theta = \arctan \left( \frac{-5}{11} \right) = -24.4^\circ
\] (26)
**Problem 58.** Protons having a kinetic energy of \( K = 5.00 \text{ MeV} \) are moving in the \( \hat{i} \) direction and enter a magnetic field \( B = 0.050 \hat{k} \text{ T} \) directed out of the plane of the page and extending from \( x = 0 \) to \( x = 1.00 \text{ m} \) as shown in Figure P22.58. (a) Calculate the \( y \) component of the protons' momentum as they leave the magnetic field. (b) Find the angle \( \alpha \) between the initial velocity vector of the proton beam, and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that \( 1 \text{ eV} = 1.60 \cdot 10^{-19} \text{ J} \).

(b) As in our cyclotron problem (Recitation 7, Problem 12), we know

\[
F_c = m \frac{v^2}{r} = qvB 
\]

\[
mv = qrB 
\]

And

\[
K = \frac{1}{2}mv^2 
\]

\[
v = \sqrt{\frac{2K}{m}} = 30.9 \text{ Mm/s} 
\]

So the radius of the circular arc our protons make in the constant magnetic field region is

\[
r = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2K}{m}} = \frac{1}{qB} \sqrt{2Km} = 6.47 \text{ m} 
\]

Drawing out the center of the circle the beam would make and doing a bit of geometry, we see that

\[
\alpha = \arcsin \left( \frac{\Delta x}{r} \right) = 8.90^\circ 
\]

(a) Because the speed of the particles doesn’t change because of a magnetic field’s perpendicular force, we can find the protons’ speed in the \( y \) direction on exiting by

\[
v_y = v \sin(\alpha) 
\]

So the \( y \) momentum is

\[
p_y = mv_y = mv \sin(\alpha) = 0.155 \text{ kg m/s}^2 
\]