Homework 3 solutions

Problem 9. The mountain lion can jump to a height of \( h = 12.0 \) ft when leaving the ground at an angle of \( \theta = 45.0^\circ \). With what speed, in SI units, does it leave the ground to make the leap?

First, we’ll convert the height into SI units:

\[
h = 12.0 \text{ ft} \left[ \frac{1 \text{ m}}{3.28 \text{ ft}} \right] = 3.659 \text{ m}
\]  

Next, arrange the information we know in a table, calling the launch point \( P_0 \) and the peak point \( P_1 \).

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_x )</td>
<td>0 m</td>
<td></td>
</tr>
<tr>
<td>( a_y )</td>
<td>-9.8 m</td>
<td></td>
</tr>
<tr>
<td>( v_x )</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>( v_y )</td>
<td>0 m/s</td>
<td>?</td>
</tr>
<tr>
<td>( x )</td>
<td>0 m</td>
<td>?</td>
</tr>
<tr>
<td>( y )</td>
<td>3.66 m</td>
<td></td>
</tr>
<tr>
<td>( t )</td>
<td>0 s</td>
<td>?</td>
</tr>
</tbody>
</table>

Where we know \( v_1 = 0 \) m/s because \( P_1 \) is at the apex of the jump and \( x_0, y_0, \) and \( t_0 \) through our choice of coordinate frame.

We want to pick an equation to tell us something about the initial velocity (because they told us \( \theta \), either \( v_{x0} \) or \( v_{y0} \) will suffice.). Looking at our 4 constant acceleration equations (text p. 53),

\[
v_{xf} = v_{xi} + a_x t \\
x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t \\
x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2 \\
v_{x1}^2 = v_{x1}^2 + 2a_x(x_f - x_i)
\]

We see that eqn. 5 applied to the \( y \) direction is perfect, because it has no information in it that we don’t already know except \( v_{y0} \).

\[
v_{y1}^2 = (0 \text{ m/s})^2 = v_{y0}^2 + 2a_y(y_1 - y_0) \\
v_{y0}^2 = -2a_yy_1 \\
v_{y0} = \sqrt{-2a_yy_1}
\]

And we use the angle to solve for the magnitude of the inital velocity:

\[
v_{y0} = v_0 \sin \theta \\
v_0 = \frac{v_{y0}}{\sin \theta} = \frac{\sqrt{-2a_yy_1}}{\sin \theta} = \frac{\sqrt{-2 \cdot (-9.8 \text{ m/s}^2) \cdot 3.66 \text{ m}}}{\sin 45^\circ} = 12.0 \text{ m/s}
\]

Problem 19. A soccer player kicks a rock horizontally off a \( h = 40.0 \text{ m} \) high cliff into a pool of water. If the player hears the sound of the splash \( \Delta t = 3.00 \text{ s} \) later, what was the initial speed given to the rock? Assume that the speed of sound in air is \( v_s = 343 \text{ m/s} \).

Let us call the point when the player kicks the ball \( P_0 \), the point where the ball lands \( P_1 \), and the point where the sound hits the players ear \( P_2 \).

Arranging the information we know about the ball in a table,

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_x )</td>
<td>0 m</td>
<td></td>
</tr>
<tr>
<td>( a_y )</td>
<td>-9.8 m</td>
<td></td>
</tr>
<tr>
<td>( v_x )</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>( v_y )</td>
<td>0 m/s</td>
<td>?</td>
</tr>
<tr>
<td>( x )</td>
<td>0 m</td>
<td>?</td>
</tr>
<tr>
<td>( y )</td>
<td>40.0 m</td>
<td>0 m</td>
</tr>
<tr>
<td>( t )</td>
<td>0 s</td>
<td>?</td>
</tr>
</tbody>
</table>

Where we know \( v_{y0} = 0 \) m/s because the ball is kicked horizontally and \( x_0, y_0, \) and \( t_0 \) through our choice of coordinate frame.
We want to pick an equation to tell us something about the time, (because they told us \( t_2 \)). Looking at our 4 constant acceleration equations we see that eqn. (4) applied to the \( y \) direction is perfect, because it has no information in it that we don’t already know except \( t_1 \).

\[
y_1 = 0 \text{ m} = y_0 + v_{y0}t_1 + \frac{1}{2}a_y t_1^2
\]  
\[
\frac{1}{2}a_y t_1^2 = -y_0  
\]  
\[
t_1^2 = \frac{-2y_0}{a_y} 
\]  
\[
t_1 = \sqrt{-\frac{-2y_0}{a_y}} = \sqrt{\frac{-2 \cdot 40.0 \text{ m}}{-9.8 \text{ m/s}^2}} = 2.86 \text{ s} 
\]

So using eqn. (4) applied to the \( x \) direction, we have

\[
x_1 = x_0 + v_{x0}t_1 + \frac{1}{2}a_xt_1^2 = v_{x0}t_1
\]  

Now that we know \( x_1 \) and \( y_1 \), we can find the distance \( \Delta x_s \) that the sound takes returning to \( P_2 \). Because it moves in a straight line (more or less), we have

\[
\Delta x_s = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} = \sqrt{v_{x0}^2t_1^2 + y_0^2}
\]  

So the time \( (t_2 - t_1) \) taken for the sound to return is

\[
\Delta x_s = v_s(t_2 - t_1)
\]  

And they give us \( t_2 \) in the problem, so we just solve this for \( v_{x0} \).

\[
\Delta x_s = \sqrt{v_{x0}^2t_1^2 + y_0^2} = v_s(t_2 - t_1)
\]  
\[
v_{x0}^2t_1^2 + y_0^2 = [v_s(t_2 - t_1)]^2
\]  
\[
v_{x0}^2 = \left[\frac{v_s(t_2 - t_1)}{t_1}\right]^2 - y_0^2/t_1^2
\]  
\[
v_{x0} = \sqrt{\frac{v_s(t_2 - t_1)}{t_1} - y_0^2/t_1} = \sqrt{[343 \text{ m/s} \cdot (3.00 \text{ s} - 2.86 \text{ s})^2 - (40.0 \text{ m})^2]/2.86 \text{ s}} = 9.91 \text{ m/s}
\]

**Problem 24.** Suppose a copper sleeve of inner radius \( r_1 = 2.10 \text{ cm} \) and outer radius \( r_2 = 2.20 \text{ cm} \) is to be cast. To eliminate bubbles and give high structural integrity, the centripetal acceleration of each bit of metal should be at least \( a_c \text{ min} = 100g \). What rate of rotation is required? State the answer in revolutions per minute (rpm).

For circular motion

\[
a_c = \frac{v^2}{r}
\]  
\[
v = \sqrt{a_cr} 
\]

And if the circular motion is uniform, the time \( T \) taken to complete one revolution is given by

\[
\Delta x = vT
\]  
\[
T = \frac{\Delta x}{v} = \frac{2\pi r}{v}
\]

So the frequency \( f = 1/T \) of rotation is

\[
f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\sqrt{a_cr}}{2\pi r} = \frac{1}{2\pi} \sqrt{\frac{a_c}{r}}
\]

Because for a given frequency \( a_c \propto r \), we must use the inner radius \( r_1 \) to compute the \( f \) required to create an acceleration \( a_c \geq 100g \).

\[
f = \frac{1}{2\pi} \sqrt{\frac{100 \star 9.8 \text{ m/s}^2}{0.0210 \text{ m} \cdot 60 \text{ s} / 1 \text{ min}}} = 2060 \text{ rpm}
\]

To reassure ourselves that this frequency creates enough centripetal acceleration \( a_{c2} \) at \( r_2 \), we can compute \( a_{c2} \).

\[
f = \frac{1}{2\pi} \sqrt{\frac{a_c}{r}}
\]  
\[
2\pi f = \sqrt{\frac{a_c}{r}}
\]
The left hand side is constant, so applying this equation to both \( r_1 \) and \( r_2 \) we get,

\[
\sqrt{\frac{a_c \text{ min}}{r_1}} = \sqrt{\frac{a_c}{r_2}} \quad \text{(30)}
\]

\[
a_{c2} = a_c \text{ min} \frac{r_2}{r_1} = a_c \text{ min} \frac{2.20 \text{ cm}}{2.10 \text{ cm}} = 1.05 \cdot a_c \text{ min} \quad \text{(31)}
\]

So the acceleration is indeed \( \geq 100g \) throughout the cylinder.

**Problem 43.** A ball on the end of a string is whirled around in a horizontal circle of radius \( r = 0.300 \) m. The plane of the circle is \( h = 1.20 \) m above the ground. The string breaks, and the ball lands \( L = 2.00 \) m (horizontally) away from the point on the ground directly beneath the ball’s location when the string breaks. Find the radial acceleration of the ball during its circular motion.

The airbourne ball portion of the problem is a 2-D projectile motion problem. Calling the point where the string breaks \( P_0 \) and the point where the ball lands \( P_1 \), we have

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_x )</td>
<td>0 m</td>
<td></td>
</tr>
<tr>
<td>( a_y )</td>
<td>( -9.8 ) m</td>
<td></td>
</tr>
<tr>
<td>( v_x )</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>( v_y )</td>
<td>0 m/s</td>
<td>?</td>
</tr>
<tr>
<td>( x )</td>
<td>0 m</td>
<td>2.00 m</td>
</tr>
<tr>
<td>( y )</td>
<td>1.20 m</td>
<td>0 m</td>
</tr>
<tr>
<td>( t )</td>
<td>0 s</td>
<td>?</td>
</tr>
</tbody>
</table>

We use eqn. 4 applied to the \( y \) direction to find \( t_1 \)

\[
y_1 = 0 \text{ m} = y_0 + v_{y0}t_1 + \frac{1}{2}a_y t_1^2 \quad \text{(32)}
\]

\[
\frac{1}{2}a_y t_1^2 = -y_0 \quad \text{(33)}
\]

\[
t_1 = \sqrt{-\frac{-2y_0}{a_y}} \quad \text{(34)}
\]

As we saw in problem 19.

Now applying eqn. 4 to the \( x \) direction to find \( v_x \)

\[
x_1 = x_0 + v_x t_1 + \frac{1}{2}a_x t_1^2 \quad \text{(35)}
\]

\[
v_x = x_1 / t_1 \quad \text{(36)}
\]

And plugging these into our circular motion equation

\[
a_c = v^2 / r = \frac{(x_1 / t_1)^2}{r} = \frac{x_1^2 a_y}{-2y_0 r} = \frac{(2.00 \text{ m})^2 \cdot (-9.8 \text{ m/s}^2)}{-2 \cdot 1.20 \text{ m} \cdot 0.300 \text{ m}} = 54.4 \text{ m/s}^2 \quad \text{(37)}
\]