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Invited Review

The Beginning and Evolution of the Universe

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ABSTRACT. We review the current standard model for the evolution of the Universe from an early inflationary epoch to the complex hierarchy of structure seen today. We summarize and provide key references for the following topics: observations of the expanding Universe; the hot early Universe and nucleosynthesis; theory and observations of the cosmic microwave background; Big Bang cosmology; inflation; dark matter and dark energy; theory of structure formation; the cold dark matter model; galaxy formation; cosmological simulations; observations of galaxies, clusters, and quasars; statistical measures of large-scale structure; and measurement of cosmological parameters. We conclude with discussion of some open questions in cosmology. This review is designed to provide a graduate student or other new worker in the field an introduction to the cosmological literature.

1. INTRODUCTION

It is the current opinion of many physicists that the Universe is well described by what Fred Hoyle termed a Big Bang Model, in which the Universe expanded from a denser hotter childhood to its current adolescence, with a present energy budget dominated by dark energy and less so by dark matter, neither of which have been detected in the laboratory, with the stuff biological systems, planets, stars, and all visible matter are made of (called baryonic matter by cosmologists) being a very small tracer on this dark sea, and with electromagnetic radiation being an even less significant contributor. Galaxies and groups and clusters of galaxies are locally distributed inhomogeneously in space, but on large enough scales and in a statistical sense the distribution approaches isotropy. This is supported by other electromagnetic distributions such as the X-ray and cosmic microwave backgrounds, which are close to isotropic. As one looks out further into space, as a consequence of the finite speed of light, one sees objects as they were at earlier times, and there is clear observational evidence for temporal evolution in the distribution of various objects such as galaxies.

At earlier times the Universe was hotter and denser, at some stage so hot that atoms could not exist. Nuclear physics reactions between protons, neutrons, etc., in the cooling expanding Universe resulted in the (nucleo)synthesis of the lighter elements (nuclei) such as D, ^{4}He, and ^{7}Li, with abundances in good accord with what is observed, and with the photons left over forming a residual cosmic microwave background (CMB), also in good agreement with what is observed.

Given initial inhomogeneities in the mass distribution at an earlier time, processing of these by the expansion of the Universe, gravitational instability, pressure gradients, and microphysical processes, gives rise to observed anisotropies in the CMB and the current large-scale distribution of nonrelativistic matter; the situation on smaller spatial scales, where galaxies form, is murkier. Observations indicate that the needed initial inhomogeneities are most likely of the special form known as scale invariant, and that the simplest best-fitting Big Bang Model has flat spatial geometry. These facts could be the consequence of a simple inflationary infancy of the Universe—a very early period of extremely rapid expansion, which stretched zero-point quantum-mechanical fluctuations to larger length scales and transmuted them into the needed classical inhomogeneities in the mass-energy distribution. At the end of the inflationary expansion all radiation and matter is generated as the Universe moves into the usual Big Bang Model epoch. Inflation has roots in models of very high-energy physics. Because of electromagnetic charge screening, gravity is the dominant large-scale force. General relativity is the best theory of gravity.

This review attempts to elaborate on this picture. Given the Tantalus principle of cosmology (and most of astrophysics), that one can see but not "touch"—which makes this a unique field of physics—there have been many false starts and even much confusion and many missed opportunities along what most now feel is the right track. Given space constraints we cannot do justice to what are now felt to be false starts, nor will we discuss more than one or two examples of confusion and missed opportunities. We attempt here to simply describe what is now thought to be a reasonable standard model of cosmology and trace the development of what are now felt to be the important threads in this tapestry; time will tell whether our use of "reasonable

standard" is more than just youthful arrogance (or possibly middle-aged complacence).

In the following sections we review the current standard model of cosmology, with emphasis in parts on some historical roots, citing historically significant and more modern papers as well as review articles. We begin with discussion of the foundations of the Big Bang Model in § 2, which summarizes research in the half century from Einstein's foundational paper on modern cosmology until the late 1960s discovery of the CMB radiation, as well as some loose ends. Section 3 discusses inflation, which provides an explanation of the Big Bang that is widely felt to be reasonable. Dark energy and dark matter, the two (as yet not directly detected) main components of the energy budget of the present Universe are reviewed in § 4. Further topics include the growth of structure in the Universe (§ 5), observations of large-scale structure in the Universe (§ 6), and estimates of cosmological parameters (§ 7). We conclude in § 8 with a discussion of what are now thought to be relevant open questions and directions in which the field appears to be moving.

We use hardly any mathematical equations in this review. In some cases this results in disguising the true technical complexity of the issues we discuss.

We exclude from this review a number of theoretical topics: quantum cosmology, the multiverse scenario, string gas cosmology, braneworld and higher dimensional scenarios, and other modifications of the Einstein action for gravity. (We note that one motivation for modifying Einstein's action is to attempt to do away with the construct of dark matter and/or dark energy. While it is probably too early to tell whether this can get rid of dark energy, it seems unlikely that this is a viable way of getting around the idea of dark matter.)

For original papers written in languages other than English, we cite only an English translation, unless this does not exist. We only cite books that are in English. For books that have been reprinted we cite only the most recent printing of which we are aware.

As a supplement to this review, we have compiled lists of key additional reference materials and links to Web resources that will be useful for those who want to learn more about this vast topic. These materials, available on our Web site[1], include lists of more technical books (including standard textbooks on cosmology and related topics), historical and biographical references, less technical books and journal articles, and Web sites for major observatories and satellites.

2. FOUNDATIONS OF THE BIG BANG MODEL

2.1. General Relativity and the Expansion of the Universe

Modern cosmology began with Einstein (1917) when he applied his general relativity theory to cosmology. At this point in time our Galaxy, the Milky Way, was thought by most to be the Universe. To make progress Einstein assumed the Universe was spatially homogeneous and isotropic; this was enshrined as the "Copernican" cosmological principle by Milne (1933). Peebles (1993; § 3) has reviewed the strong observational evidence for large-scale statistical isotropy; observational tests of homogeneity are not as straightforward. Einstein knew that the stars in the Milky Way moved rather slowly and decided, as everyone had done before him, that the Universe should not evolve in time. He could come up with a static solution of his equations if he introduced a new form of energy, now called the cosmological constant. It turns out that Einstein's static model is unstable. In the same year de Sitter (1917) found the second cosmological solution of Einstein's general relativity equations; Lemaître (1925) and Robertson (1928) reexpressed this solution in the currently more familiar form of the exponentially expanding model used in the inflation picture. Weyl (1923) noted the importance of prescribing initial conditions such that the particle geodesics diverge from a point in the past. Friedmann (1922, 1924), not bound by the desire to have a static model, discovered the evolving homogeneous solutions of Einstein's equations; Lemaître (1927) rediscovered these "Friedmann-Lemaître" models. Robertson (1929) initiated the study of metric tensors of spatially homogeneous and isotropic spacetimes, and continuing study by him and A. G. Walker (in the mid 1930s) led to the "Robertson-Walker" form of the metric tensor for homogeneous world models. Of course, in the evolving cosmological model solutions only observers at rest with respect to the expansion/contraction see an isotropic and homogeneous Universe; cosmology thus reintroduces preferred observers!

North (1990) and Longair (2006) provide comprehensive historical reviews. See the standard cosmology textbooks for the modern formalism.

2.2. Galaxy Redshift and Distance Measurements

Meanwhile, with some first success in 1912, Slipher (1917)[2] found that most of the "white spiral nebulae" (so-called because they have a continuum spectrum; what we now term spiral galaxies) emit light that is redshifted (we now know that the few, including M31 [Andromeda] and some in the Virgo cluster, that emit blueshifted light are approaching us), and Eddington (1923) identified this with a redshift effect in the de Sitter (1917) model (not the cosmological redshift effect). Lemaître (1925) and Robertson (1928) derived Hubble's velocity-distance law $v = H_0 r$ (relating the galaxy's speed of recession v to its distance r from us, where H_0 is the Hubble constant, the present

[1] See our Web site at www.physics.drexel.edu/universe/; also see the arXiv version of this review article (arXiv:0706.1565).

[2] Although the "canals" on Mars are not really canals, they had an indirect but profound influence on cosmology. Percival Lowell built Lowell Observatory to study the Solar System, and Mars in particular, and closely directed the research of his staff. Slipher was instructed to study M31 and the other white nebulae under the hope that they were proto–solar systems.

value of the Hubble parameter) in the Friedmann-Lemaître models. The velocity-distance Hubble law is a consequence of the cosmological principle, is exact, and implies that galaxies further away than the current Hubble distance $r_H = c/H_0$ are moving away faster than the speed of light c. Hubble (1925)[3] used Leavitt's (Leavitt 1912; Johnson 2005)[4] quantitative Cepheid variable star period-luminosity relation to establish that M31 and M33 are far away (confirming the earlier somewhat tentative conclusion of Öpik 1922), and did this for more galaxies, conclusively establishing that the white nebulae are other galaxies outside our Milky Way galaxy (there was some other earlier observational evidence for this position but Hubble's work is what convinced people). Hubble got Humason (middle school dropout and one time muleskinner and janitor) to remeasure some Slipher spectra and measure more spectra, and Hubble (1929)[5] established Hubble's redshift-distance law $cz = H_0 r$, where the redshift z is the fractional change in the wavelength of the spectral line under study (although in the paper Hubble calls cz velocity and does not mention redshift). The redshift-distance Hubble law is an approximation to the velocity-distance law, valid only on short distances and at low redshifts.

North (1990) provides a comprehensive historical review; Berendzen et al. (1976) and Smith (1982) are more accessible historical summaries. See the standard cosmology textbooks for the modern formalism. Branch (1998) discusses the use of type Ia supernovae as standard candles for measuring the Hubble constant. See Fig. 1 of Leibundgut (2001) for a recent plot of the Hubble law. Harrison (1993), Davis & Lineweaver (2004), and Lineweaver & Davis (2005) provide pedagogical discussions of issues related to galaxies moving away faster than the speed of light.

2.3. The Hot Early Universe and Nucleosynthesis

As one looks out further in space (and so back in time, because light travels at finite speed) wavelengths of electromagnetic radiation we receive now have been redshifted further by the expansion, and so Wien's law tells us (from the blackbody CMB) that the temperature was higher in the past. The younger Universe was a hotter, denser place. Lemaître ("the father of the Big Bang") emphasized the importance of accounting for the rest of known physics in the general-relativistic cosmological models.

Early work on explaining the astrophysically observed abundances of elements assumed that they were a consequence of rapid thermal equilibrium reactions and that a rapidly falling temperature froze the equilibrium abundances. Tolman, Suzuki, von Weizsäcker, and others in the 1920s and 1930s argued that the observed helium-hydrogen ratio in this scenario required that at some point the temperature had been at least 10^9 K (and possibly as much as 10^{11} K). Chandrasekhar & Henrich (1942) performed the first detailed, correct equilibrium computation and concluded that no single set of temperature and density values can accommodate all the observed abundances; they suggested that it would be useful to consider a nonequilibrium process. Gamow (1946), building on his earlier work, makes the crucial point that in the Big Bang Model "the conditions necessary for rapid nuclear reactions were existing only for a very short time, so that it may be quite dangerous to speak about an equilibrium state"; i.e., the Big Bang was the place to look for this nonequilibrium process.

Gamow (1948), a student of Friedmann, and Alpher (1948), a student of Gamow, estimated the radiation (photon) temperature at nucleosynthesis, and from the Stefan-Boltzmann law for blackbody radiation noted that the energy budget of the Universe must then have been dominated by radiation. Gamow (1948) evolved the radiation to the much later epoch of matter-radiation equality (the matter and radiation energy densities evolve in different ways and this is the time at which both had the same magnitude), a concept also introduced by Gamow, while Alpher & Herman (1948) predicted a residual CMB radiation at the present time from nucleosynthesis and estimated its present temperature to be 5 K (because the zero-redshift baryon density was not reliably known then, it is somewhat of a coincidence that this temperature estimate is close to the observed modern value). Hayashi (1950) pointed out that, at temperatures about 10 times higher than during nucleosynthesis, rapid weak interactions lead to a thermal equilibrium abundance ratio of neutrons and protons determined by the neutron-proton mass difference, which becomes frozen in as the expansion decreases the temperature, thus establishing the initial conditions for nucleosynthesis. This is fortunate, in that an understanding of higher energy physics is not needed to make firm nucleosynthesis predictions; this is also unfortunate, because element abundance observations cannot be used to probe higher energy physics.

Alpher et al. (1953) concluded the early period of the standard model of nucleosynthesis. By this point it was clear that initial hopes to explain all observed abundances in this manner must fail, because of the lack of stable nuclei at mass numbers 5 and 8 and because as the temperature drops with the expansion it becomes more difficult to penetrate the Coulomb barriers. Cosmological nucleosynthesis can only generate the light elements and the heavier elements are generated from these light elements by further processing in the stars.

[3] Duncan had earlier found evidence for variable stars in M33, the spiral galaxy in Triangulum.

[4] Leavitt published a preliminary result in 1908 and Hertzsprung and Shapley helped develop the relation, but it would be another 4 decades (1952) before a reasonably accurate version became available (which led to a drastic revision of the distance scale).

[5] In the mid 1920s Lundmark and Strömberg had already noted that more distant galaxies seemed to have spectra that were more redshifted.

Zel'dovich(1963a) and Smirnov (1964) noted that the ^{4}He and D abundances are sensitive to the baryon density: the observed abundances can be used to constrain the baryon density. Hoyle & Tayler (1964) carried out a detailed computation of the ^{4}He abundance and on comparing to measurements concluded "most, if not all, of the material of our…Universe, has been 'cooked' to a temperature in excess of 10^{10} K." They were the first to note that the observed light element abundances were sensitive to the expansion rate during nucleosynthesis and that this could constrain new physics at that epoch (especially the number of light, relativistic, neutrino families).

After Penzias and Wilson measured the CMB (see below), Peebles (1966a,1966b) computed the abundances of D, ^{3}He, and ^{4}He and their dependence on, among other things, the baryon density and the expansion rate during nucleosynthesis. The monumental Wagoner et al. (1967) paper established the ground rules for future work.

For a history of these developments see pp. 125–128 and 240–241 of Peebles (1971), the articles by Alpher and Herman and Wagoner on pp. 129–157 and 159–185, respectively, of Bertotti et al. (1990), and chapter 3 and § 7.2 of Kragh (1996). See the standard cosmology textbooks for the modern formalism. Accurate abundance predictions require involved numerical analysis; on the other hand, pedagogy could benefit from approximate semianalytical models (Bernstein et al. 1989; Esmailzadeh et al. 1991).

In the simplest nucleosynthesis scenario, the baryon density estimated from the observed D abundance is consistent with that estimated from *WMAP* CMB anisotropy data, and higher than that estimated from the ^{4}He and ^{7}Li abundances. This is further discussed in § 7.2. Fields & Sarkar (2006), Cyburt (2004), and Steigman (2006) are recent reviews of nucleosynthesis.

In addition to residual CMB radiation, there is also a residual neutrino background. Above a temperature of about 10^{10} K the CMB photons have enough energy to produce a thermal equilibrium abundance of neutrinos. Below this temperature the neutrinos decouple and freely expand, resulting in about 300 neutrinos per cubic centimeter now (with three families, and this number also includes antineutrinos), at a temperature of about 2 K, lower than that of the CMB because electron-positron annihilation heats the CMB a little. See Dolgov (2002), Hannestad (2006), and the more recent textbooks cited below for more detailed discussions of the (as yet undetected) neutrino background. We touch on neutrinos again in § 5.1.

2.4. Theory and Observations of the CMB

The CMB radiation contributes of the order of 1% of the static or "snow" seen when switching between channels on a television with a conventional VHF antenna; it is therefore not surprising that it had been detected a number of times before its 1965 discovery and identification. For instance, it is now known that McKellar (1941) deduced a CMB temperature of 2.3 K at a wavelength of 2.6 mm by estimating the ratio of populations in the first excited rotational and ground states of the interstellar cyanogen (CN) molecule (determined from absorption line measurements of Adams). It is now also known that the discrepancy of 3.3 K between the measured and expected temperature of the Bell Labs horn antenna (for communicating with the *Echo I* satellite) at a wavelength of 12.5 cm found by Ohm (1961) is due to the CMB. Ohm also notes that an earlier measurement with this telescope (DeGrasse et al. 1959) ascribes a temperature of 2 ± 1 K to back and side lobe pick up, that this is " temperature not otherwise accounted for," and that "it is somewhat larger than the calculated temperature expected." Of course, McKellar had the misfortune of performing his analyses well before Gamow and collaborators had laid the nucleosynthesis foundations that would eventually explain the CN measurements (and allow the CMB interpretation), and Ohm properly did not overly stress the discrepancy beyond its weak statistical significance.

While Alpher and Herman (e.g., pp. 114–115 of Alpher & Herman 2001 and p. 130 of Weinberg 1993) privately raised the issue of searching for the CMB, and Hoyle came close to correctly explaining McKellar's CN measurements (see pp. 345–346 of Kragh 1996), Zel'dovich (1963a,1963b, 1965, p. 491, p. 89, and p. 315, respectively), Doroshkevich & Novikov (1964) and Dicke & Peebles (1965, p. 448) are the first published discussions of possible observational consequences of the (then still hypothetical) CMB in the present Universe. The relevant discussions of Zel'dovich and Doroshkevich & Novikov are motivated by the same nucleosynthesis considerations that motivated Gamow and collaborators; Dicke and Peebles favored an oscillating Universe and needed a way to destroy heavy elements from the previous cycle and so postulated an initial hotter stage in each cycle. Both Doroshkevich & Novikov (1964) and Zel'dovich (1965) referred to Ohm (1961) but neither appeared to notice Ohm's 3.3 K discrepancy; in fact Zel'dovich (1965) (incorrectly) argued that Ohm constrains the temperature to be less than 1 K and given the observed helium abundance this rules out the hot Big Bang Model.

Working with the same antenna as Ohm, using the Dicke switching technique to compare the antenna temperature to a liquid helium load at a known temperature, and paying very careful attention to possible systematic effects, Penzias & Wilson (1965) measured the excess temperature to be 3.5 ± 1 K at 7.35 cm wavelength; Dicke et al. (1965) identified this as the CMB radiation left over from the hot Big Bang.

The CMB is the dominant component of the radiation density of the Universe, with a density now of about 400 CMB photons per cubic centimeter at a temperature of about 2.7 K now. As noted in the previous subsection, observed light element abundances in conjunction with nucleosynthesis theory allows for constraints on the density of baryonic matter. Thus, there are a few billion CMB photons for every baryon; the CMB photons carry most of the cosmological entropy.

To date there is no observational indication of any deviation of the CMB spectrum from a Planckian blackbody. Partridge (1995) reviewed early measurements of the CMB spectrum. A definitive observation of the CMB spectrum was made by *COBE* (see Gush et al. 1990 for a contemporaneous rocket-based measurement), which measured a temperature of 2.725 ± 0.002 K (95% confidence) (Mather et al. 1999) and 95% confidence upper limits on possible spectral distortions: $|\mu| < 9 \times 10^{-5}$ for the chemical potential of early ($10^5 < z < 3 \times 10^6$) energy release and $|y| < 1.5 \times 10^{-5}$ for Comptonization of the spectrum at later times (Fixsen et al. 1996). Wright et al. (1994) have shown that these constraints strongly rule out many alternatives to the Big Bang Model, including the steady state model and explosive galaxy formation.

Anisotropy of the CMB temperature, first detected by *COBE* (Smoot et al. 1992), reveals important features of the formation and evolution of structure in the Universe. A small dipole anisotropy (discovered in the late 1960s and early 1970s by Conklin and Henry and confirmed by Corey and Wilkinson as well as Smoot, Gorenstein, and Muller) is caused by our peculiar motion; the CMB establishes a preferred reference frame. Higher multipole anisotropies in the CMB reflect the effect of primordial inhomogeneities on structure at the epoch of recombination and more complex astrophysical effects along the past light cone that alter this primordial anisotropy. We discuss these anisotropies, as well as the recently detected polarization anisotropy of the CMB, in § 5.2. The anisotropy signal from the recombination epoch allows precise estimation of cosmological parameters (see § 7.2).

In addition to references cited above, Dicke (1970, pp. 64–70), Wilson (1983), Wilkinson & Peebles (1990), Partridge (1995, chapter 2), and Kragh (1996, § 7.2) review the history. For the modern formalism see the more recent standard cosmology textbooks and Kamionkowski & Kosowsky (1999).

2.5. Challenges for the Big Bang Model

Since the Universe is now expanding, at earlier times it was denser and hotter. A naive extrapolation leads to a (mathematical) singularity at the beginning, with infinite density and temperature, at the initial instant of time, and over all space. This naive extrapolation is unjustified since the model used to derive it breaks down physically before the mathematical singularity is reached. Deriving the correct equations of motion for the very early Universe is an important area of current research. While there has been much work, there is as yet no predictive model that unifies gravity and quantum mechanics—and this appears essential for an understanding of the very early Universe, because as one goes back in time the gravitational expansion of the Universe implies that large cosmological length scales now correspond to tiny quantum-mechanical scales in the very very early Universe. There is a small but active group of workers who believe that only a resolution of this issue (i.e., the derivation of a full quantum theory of gravity) will allow for progress on the modeling of the very early Universe. But most others, perhaps inspired by the wonderful successes of particle physics models that have successfully described shorter and shorter distance physics, now believe that it is important to try to solve some of the "problems" of the Big Bang Model by attempting to model the cosmophysical world at an energy density higher than is probed by nucleosynthesis and other lower redshift physics but still well below the Planck energy density where quantum gravitational effects are important. This is the approach we take in the following discussion, by focusing on problems that could be resolved below the Planck density. Whether Nature has chosen this path is as yet unclear, but at least the simplest versions of the inflation scenario (discussed in the next section) are compatible with current observations and will likely be well tested by data acquired within this decade.

Assuming just nonrelativistic matter and radiation (CMB and neutrinos) in order of magnitude agreement with observations, the distance over which causal contact is possible grows with the age of the Universe. That is, if one assumes that in this model the cosmological principle is now valid because of "initial conditions" at an earlier time, then those initial conditions must be imposed over distances larger than the distance over which causal communication was possible. (And maybe this is what a quantum theory of gravitation will do for cosmology, but in the spirit of the earlier discussion we will view this as a problem of the Big Bang Model that should be resolved by physics at energies below the Planck scale.) Alpher et al. (1953, p. 1349) contains the earliest remarks (in passing) that we are aware of about this particle horizon problem. The terminology is due to Rindler (1956), which is an early discussion of horizons in general. Harrison (1968) also mentions the particle horizon problem in passing, but McCrea (1968) and Misner (1969) contain the first clear statements we are aware of, with Misner stating "These Robertson-Walker models therefore give no insight into why the observed microwave radiation from widely different angles in the sky has…very precisely…the same temperature." Other early discussions are in Dicke (1970, p. 61), Doroshkevich & Novikov (1970), and the text books of Weinberg (1972, pp. 525–526) and Misner et al. (1973, pp. 815–816), This issue was discussed in many papers and books starting in the early 1970s, but the celebrated Dicke & Peebles (1979) review is often credited with drawing prime-time attention to the particle horizon problem.

The large entropy of the Universe (as discussed above, there are now a few billion CMB photons for every baryon) poses another puzzle. When the Universe was younger and hotter there had to have been a thermal distribution of particles and antiparticles and, as the Universe expanded and cooled, particles and antiparticles annihilated into photons, resulting in the current abundance of CMB photons and baryons. Given the lack of a significant amount of antibaryons now, and the large photon to baryon ratio now, at early times there must have been a slight

(a part in a few billion) excess of baryons over antibaryons. We return to this issue in the next section.

3. INFLATION

It is possible to trace a thread in the particle horizon problem tapestry back to the singularity issue and early discussions of Einstein, Lemaître, and others who viewed the singularity as arising from the unjustified assumption of exact isotropy, and led to the intensive study of homogeneous but anisotropic cosmological models in the late 1960s and early 1970s. These attempts failed to tame the singularity but did draw attention to isotropy and the particle horizon problem of the standard Big Bang Model. It is interesting that this singularity issue also drove the development of the steady state picture, which in its earliest version was just a de Sitter model. While observations soon killed off the original steady state model (a more recent variant, the quasi-steady state model, can be adjusted to accommodate the data; see, e.g., Narlikar et al. 2003), the idea of a possible early, pre-Big-Bang, nonsingular de Sitter epoch thrived. It appears that Brout et al. (1978) were the first to note that such a cosmological model was free of a particle horizon. However, they did not seem to make the connection that this could allow for isotropy by ensuring that points well separated now shared some common events in the past and thus causal physics could, in principle, make the Universe isotropic. Zee (1980) noted that if one modifies the early Universe by speeding up the expansion rate enough over the expansion rate during the radiation-dominated epoch, the particle horizon problem is resolved (but he did not go to the exponentially expanding de Sitter solution characteristic of the early inflation scenario).

Sato(1981a,1981b), Kazanas (1980), and Guth (1981) are the ones who made the (now viewed to be crucial) point that during a phase transition at very high temperature in grand unified models it is possible for the grand unified Higgs scalar-field energy density to behave like a cosmological constant, driving a de Sitter exponential cosmological expansion, which results in a particle-horizon–free cosmological model. And the tremendous expansion during the de Sitter epoch will smooth out wrinkles in the matter distribution, by stretching them to very large scales, an effect alluded to earlier by Hoyle & Narlikar (1962) in the context of the steady state model, which could result in an isotropic Universe now, provided the initial wrinkles satisfy certain conditions. See Ellis & Stoeger (1988) and Narlikar & Padmanabhan (1991) for caveats and criticism. Of course, to get the inflationary expansion started requires a large enough, smooth enough initial patch. The contemporary explanation appeals to probability: loosely, such a patch will exist somewhere and inflation will start there. In addition, the initial conditions issue is not completely resolved by inflation, only greatly alleviated; since inflation stretches initially small length scales to length scales of contemporary cosmological interest, the cosmological principle requires that there not be very large irregularities on very small length scales in the very early Universe. This could be a clue to what might be needed from a model of very high-energy, preinflation physics. For reviews of inflation, see the more recent standard cosmology and astroparticle physics textbooks.

Building on ideas of Brout, Englert, and collaborators, Gott (1982) noted that it was possible to have inflation result in a cosmological model with open spatial hypersurfaces at the present time, in contrast to the Sato-Kazanas-Guth discussion that focused on flat spatial hypersurfaces. This open-bubble inflation model, in which the observable part of the contemporary Universe resides inside a bubble nucleated (because of a small upward "bump" in the potential-energy density function) between two distinct epochs of inflation, is a clear counterexample to the oft-repeated (but incorrect) claim that inflation explains why the Universe appears to have negligible space curvature. See Ratra & Peebles (1994, 1995) for a more detailed discussion of this model.

The open-bubble inflation model was the first consistent inflation model. Unfortunately for the Guth model, as the phase transition completes and one hopes to have a smooth transition to the more familiar radiation-dominated expansion of the hot Big Bang Model, one finds that the potential in the Guth model results in many small bubbles forming with most of energy density residing in the bubble walls. In this model the Universe at the end of inflation was very inhomogeneous because the bubble collisions were not rapid enough to thermalize the bubble wall energy density (i.e., the bubbles did not "percolate"). Linde (1982) and Albrecht & Steinhardt (1982) used a specific potential-energy density function for the Higgs field in a grand unified model and implemented Gott's scenario in the Sato-Kazanas-Guth picture, except they argued that the second epoch of inflation lasts much longer than Gott envisaged and so stretches the bubble to length scales much larger than the currently observable part of the Universe, thus resulting in flat spatial hypersurfaces now. The great advantage of the Gott scenario is that it uses the first epoch of inflation to resolve the particle horizon/homogeneity problem and so this problem does not constrain the amount of inflation after the bubble nucleates. Brout et al. (1978) and Coleman & De Luccia (1980) note that symmetry forces the nucleating bubble to have an open geometry, and this is why inflation requires open spatial hypersurfaces, but with significant inflation after bubble nucleation the radius of curvature of these hypersurfaces can be huge. Thus, the amount of space curvature in the contemporary Universe is a function of the amount of inflation after bubble nucleation, and it is now widely accepted that observational data (as discussed below in § 7.2) are consistent with an insignificant amount of space curvature and, thus, significant inflation after bubble nucleation.

It is well known that phase transitions can create topological defects. Grand unified phase transitions are no exception and often create monopoles and other topological defects. If the Universe is also inflating through this phase transition, then the

density of such topological defects can be reduced to levels consistent with the observations. This is not another argument in support of inflation, although it is often claimed to be: it is just a way of using inflation to make viable a grand unified theory that is otherwise observationally inconsistent.

One major motivation for grand unification is that it allows for a possible explanation of the observed excess of matter over antimatter (or the baryon excess) mentioned in the previous section. There are other possible explanations of how this baryon excess might have come about. One much discussed alternative is the possibility of forming it at the much lower temperature electroweak phase transition, through a nonperturbative process, but this might raise particle horizon or homogeneity issues. However, at present there is no convincing, numerically satisfying explanation of the baryon excess, from any process. Quinn & Witherell (1998), Dine & Kusenko (2004), Trodden (2004), and Cline (2006) review models now under discussion for generating the baryon excess.

At the end of inflation, as the phase transition completes and the Universe is said to reheat, one expects the generation of matter and radiation as the Universe makes the transition from rapid inflationary expansion to the more sedate radiation-dominated expansion of the hot Big Bang Model. This is an area of ongoing research, and it would be useful to have a convincing, numerically satisfying model of this epoch. The baryon excess might be generated during this reheating process.

While great effort has been devoted to inflation, resulting in a huge number of different models, at the present stage of development inflation is a very interesting general scenario desperately in need of a more precise and more convincing very high-energy particle physics–based realization. As far as large-scale cosmology is concerned, inflation in its simplest form is modeled by a scalar field (the inflaton) whose potential-energy density satisfies certain properties that result in a rapid-enough cosmological expansion at early times. It is interesting that cosmological observations within this decade might firm up this model of the very early Universe based on very high-energy physics before particle physicists do so. For reviews, see the more recent standard cosmology and astroparticle physics textbooks.

Assuming an early epoch of inflation, the cumulative effect of the expansion of the Universe from then to the present means that contemporary cosmological length scales (e.g., the length scale that characterizes the present galaxy distribution) correspond to very tiny length scales during inflation, so tiny that quantum-mechanical zero-point fluctuations must be considered in any discussion involving physics on these length scales.

As mentioned above, the idea of an early de Sitter–like expansion epoch, pre–Big Bang, was discussed in the 1970s, as a possible way of taming the initial singularity. While this de Sitter epoch was typically placed at very high energy, it differs significantly from the inflation scenario in that it was not driven by a scalar-field potential-energy density. Nevertheless, because it was at energies close to the Planck energy, there were many discussions of quantum-mechanical fluctuations in de Sitter spacetime in the 1970s.

In the inflation case quantum mechanics introduces additional fluctuations, the zero-point fluctuations in the scalar field. This was noted by Hawking (1982), Starobinsky (1982), and Guth & Pi (1982), and further studied by Bardeen et al. (1983). For a discussion of scalar-field quantum fluctuations in de Sitter spacetime and their consequences see Ratra (1985). Fischler et al. (1985) use the Dirac-Wheeler-DeWitt formalism to consistently semiclassically quantize both gravitation and the scalar field about a de Sitter background, and carry through a computation of the power spectrum of zero-point fluctuations. The simplest inflation models have a weakly coupled scalar field and so a linear perturbation theory computation suffices. The fluctuations obey Gaussian statistics and so can be completely characterized by their two-point correlation function or, equivalently, their power spectrum. Inflation models that give non-Gaussian fluctuations are possible (for a review see Bartolo et al. 2004), but the observations do not yet demand this, being almost completely consistent with Gaussianity (see discussion in § 5.2 below). The simplest models give adiabatic or curvature (scalar) fluctuations; these are what result from adiabatically compressing or decompressing parts of an exactly spatially homogeneous Universe. More complicated models of inflation can produce fluctuations that break adiabaticity, such as (tensor) gravitational waves (Rubakov et al. 1982) and (vector) magnetic fields (Turner & Widrow 1988; Ratra 1992a), which might have interesting observational consequences (see Secs. 5.1 and 5.2 below).

The power spectrum of energy density fluctuations depends on the model for inflation. If the scalar-field potential-energy density during inflation is close to flat and dominates the scalar-field energy density, the scale factor grows exponentially with time (this is the de Sitter model), and after inflation but at high redshift the power spectrum of (scalar) mass-energy density fluctuations with wavenumber magnitude k is proportional to k, or scale invariant, on all interesting length scales; i.e., curvature fluctuations diverge only as $\log k$. This was noted in the early 1980s for the inflation model (Hawking 1982; Starobinsky 1982; Guth & Pi 1982), although the virtues of a scale-invariant spectrum were emphasized in the early 1970s, well before inflation, by Harrison (1970), Peebles & Yu (1970), and Zel'dovich (1972). When the scalar-field potential-energy density is such that the scalar-field kinetic- energy density is also significant during inflation, a more general spectrum proportional to k^n can result (where the spectral index n depends on the slope of the potential-energy density during inflation); for $n \neq 1$ the spectrum is said to be tilted (Abbott & Wise 1984; Lucchin & Matarrese 1985; Ratra 1992b). Current observations appear to be reasonably well fit by $n = 1$. More complicated, non-power-law spectra are also possible.

We continue this discussion of fluctuations in § 5 below.

4. DARK MATTER AND DARK ENERGY

Most cosmologists are of the firm opinion that observations indicate the energy budget of the contemporary Universe is dominated by dark energy, with the next most significant contributor being dark matter, and with ordinary baryonic matter in a distant third place. Dark energy and dark matter are hypothetical constructs generated to explain observational data, and the current model provides a good, but not perfect, explanation of contemporary cosmological observations. However, dark energy and dark matter have not been directly detected (in the lab or elsewhere).

Hubble (1926)[6] presented the first systematic estimate of masses of the luminous part of galaxies (based on studying the motion of stars in galaxies), as well as an estimate of the mass density of the Universe (using counts of galaxies in conjunction with the estimated masses of galaxies).

Under similar assumptions (the validity of Newton's second law of motion and Newton's inverse-square law of gravitation, and that the large-scale structure under investigation is in gravitational equilibrium), Zwicky (1933), in perhaps one of the most significant discoveries of the previous century, found that galaxies in the Coma cluster of galaxies were moving with surprisingly high speeds. In modern terms, this indicates a Coma cluster mass density at least an order of magnitude greater than what would be expected from spreading the mass associated with the luminous parts of the galaxies in the Coma cluster over the volume of the cluster. Zwicky's measurements probe larger length scales than Hubble's and so might be detecting mass that lies outside the luminous parts of the galaxies, i.e., mass that does not shine, or dark matter. Ordinary baryonic matter is largely nonrelativistic in the contemporary Universe and, hence, would be pulled in by the gravitational field of the cluster. Nucleosynthesis and CMB anisotropy measurements constrain the mass density of ordinary baryonic matter, and modern data indicate that not only is the amount of gravitating mass density detected in Zwicky-like observations significantly greater than what is shining, but it is also likely a factor of 3 to 5 times the mass density of ordinary baryonic matter. (It is also known that a large fraction of the expected baryonic matter can not significantly shine.) Smith (1936) confirmed Zwicky's result, using Virgo cluster measurements, and Zwicky (1937)[7] soon followed up with a more detailed paper.

Babcock's Ph.D. thesis (Babcock 1939) was the next major (in hindsight) development in the dark matter story. He measured the rotation speed of luminous objects in or near the disk of the Andromeda (M31) galaxy, out to a distance of almost 20 kpc from the center and found that the rotation speed was still rising, not exhibiting the $1/\sqrt{r}$ Keplerian fall off with distance r from the center that would be expected if the mass distribution in M31 followed the distribution of the light. That is, Babcock found that the outer part of the luminous part of M31 was dominated by matter that did not shine. Soon thereafter Oort (1940) noted a similar result for the galaxy NGC 3115. Almost two decades later, van de Hulst et al. (1957) confirmed Babcock's result by using 21 cm wavelength observations of hydrogen gas clouds that extend beyond the luminous part of M31, finding a roughly flat rotation curve at the edge (no longer rising with distance as Babcock had found). While there was some early theoretical discussion of this issue, the much more detailed M31 flat rotation curve measured by Rubin & Ford (1970) (Rubin was a student of Gamow) forced this dark matter into the limelight.

Other early indications of dark matter came from measurements of the velocities of binary galaxies (Page 1952) and the dynamics of our Local Group of galaxies (Kahn & Woltjer 1959). Both de Vaucouleurs (1969) and Arp & Bertola (1969) found that the elliptical galaxy M87 in the Virgo cluster had a faint mass-containing halo. Ostriker & Peebles (1973) noted that one way of making the disk of a spiral galaxy stable against a barlike instability is to embed it in a massive halo, and soon thereafter Einasto et al. (1974) and Ostriker et al. (1974) showed that this suggestion was consistent with the observational evidence. These early results have been confirmed by a number of different techniques, including measuring the X-ray temperature of hot gas in galaxy clusters (which is a probe of the gravitational potential—and the mass which generates it—felt by the gas), and measurements of gravitational lensing of background sources by galaxy clusters. See § 7.2 for further discussion of this.

For reviews of dark matter see § IV of Peebles (1971) (note the fascinating comment on p. 64 on the issue of dark matter in clusters: "This quantity" M/L or the mass to luminosity ratio "is suspect because when it is used to estimate the masses of groups or clusters of galaxies the result often appears to be unreasonable," i.e., large), Faber & Gallagher (1979), Trimble (1987), Ashman (1992), Peebles (1993, § 18), and Einasto (2005).

Much as van Maanen's measurements of large (but erroneous) rotation velocities for a number of galaxies prompted Jeans (1923) to consider a modification of Newton's inverse-square law for gravity, such that the gravitational force fell off slower with distance on large distances, the large (but not erroneous) velocities measured by Zwicky and others prompted Finzi (1963), and many since then, to consider modifications of the law of gravity. The current observational indications are that this is not a very viable alternative to the dark matter hypothesis (Peebles & Ratra 2003, §§ IV.A.1 and IV.B.13). In some cases, modern high-energy physics suggests possible motivations for

[6] In this paper, among other things, Hubble also developed his galaxy classification scheme (of ellipticals, normal and barred spirals, and irregulars) and showed that the averaged large-scale galaxy distribution is spatially isotropic. Öpik (1922) had earlier estimated the mass of M31.

[7] In this paper Zwicky also proposed the remarkable idea of using gravitational lensing of background objects by foreground clusters of galaxies to estimate cluster masses.

modifications of the inverse-square law on various length scales; this is beyond the scope of our review.

Milgrom (1983, 2002) has proposed a related but alternate hypothesis: Newton's second law of motion is modified at low accelerations. This hypothesis—dubbed modified Newtonian dynamics (MOND)—does a remarkable job of fitting the flat rotation curves of spiral galaxies, but most who have cared to venture an informed opinion believe that it cannot do away completely with dark matter, especially in low-surface-brightness dwarf galaxies and rich clusters of galaxies. More importantly, the lack of a well-motivated extension of the small-length-scale phenomenological MOND hypothesis that is applicable on large cosmological length scales greatly hinders testing the hypothesis. For a recent attempt at such an extension see Bekenstein (2004). For a preliminary sketch of cosmology in this context see Diaz-Rivera et al. (2006). For a review of MOND see Sanders & McGaugh (2002).

Most cosmologists are convinced that dark matter exists. Nucleosynthesis constraints indicate that most of the dark matter is not baryonic. (Not all baryons shine; for a review of options for dark baryons, see Carr 1994). Galaxies are, in general, older than larger-scale structures (such as clusters); this indicates that the dark matter primeval velocity dispersion is small (for, if it were large, gravity would be able to overcome the corresponding pressure only on large mass—and so length—scales, first forming large-scale objects that fragment later into younger smaller-scale galaxies). Dark matter with low primeval velocity dispersion is known as cold dark matter (CDM). More precisely, the CDM model assumes that most of the nonrelativistic matter-energy of the contemporary Universe is in the form of a gas of massive, nonbaryonic, weakly interacting particles with low primeval velocity dispersion. One reason they must be weakly interacting is so they do not shine. Muñoz (2004), Bertone et al. (2005), and Baltz (2004) have reviewed particle physics dark matter candidates and prospects for experimental detection. Bond et al. (1982) and Blumenthal et al. (1982) have noted the advantages of CDM and that modern high-energy physics models provide plausible hypothetical candidates for these particles. Peebles (1982) has cast the cosmological skeleton of the CDM model, emphasizing that in this model structure forms from the gravitational growth of primordial departures from homogeneity that are Gaussian, adiabatic, and scale invariant, consistent with what is expected from the simplest inflation models. Blumenthal et al. (1984) is a first fleshing out of the CDM model. See Peebles (1993) and Liddle & Lyth (2000) for textbook discussions of the CDM model. More details about this model, including possible problems, are given in § 5 below.

To set the numerical scale for cosmological mass densities, following Einstein & de Sitter (1932), one notes that the simplest Friedmann-Lemaître model relevant to the contemporary Universe is one with vanishing space curvature and with energy budget dominated by nonrelativistic matter (and no cosmological constant). In this critical or Einstein–de Sitter case the Friedmann equation fixes the energy density of nonrelativistic matter for a given value of the Hubble constant. Cosmologists then define the mass-energy density parameter Ω for each type of mass-energy (including that of the curvature of spatial hypersurfaces Ω_K, the cosmological constant Ω_Λ, and nonrelativistic matter Ω_M) as the ratio of that mass-energy density to the critical or Einstein–de Sitter model mass-energy density. The Friedmann equation implies that the mass-energy density parameters sum to unity. (In general the Ω's are time dependent; in what follows numerical values for these parameters refer to the current epoch.)

As discussed in § 7.2 below, it has long been known that nonrelativistic matter (baryons and CDM) contributes about 25% or 30% to the critical mass-energy density. After the development of the inflation picture for the very early Universe in the 1980s there was a widespread belief that space curvature could not contribute to the mass-energy budget (this is not necessary, as discussed above), and for this and a few other reasons (among others, the timescale problem arising from the large measured values of the Hubble constant and age of the Universe), Peebles (1984) proposed that Einstein's cosmological constant contributed the remaining 70% or 75% of the mass-energy of the Universe. This picture was soon generalized to allow the possibility of a scalar-field energy density that is slowly varying in time and close to homogeneous in space—what is now called dark energy (Peebles & Ratra 1988; Ratra & Peebles 1988). As discussed in § 7.2 below, these models predict that the expansion of the Universe is now accelerating, and, indeed, it appears that this acceleration has been detected at about the magnitude predicted in these models (Riess et al. 1998; Perlmutter et al. 1999). Consistent with this, CMB anisotropy observations are consistent with flat spatial hypersurfaces, which in conjunction with the low mass-energy density parameter for nonrelativistic matter also requires a significant amount of dark energy. These issues are discussed in more detail in § 7.2 below and in reviews (Peebles & Ratra 2003; Steinhardt 2003; Carroll 2004; Padmanabhan 2005; Perivolaropoulos 2006; Copeland et al. 2006; Nobbenhuis 2006; Sahni & Starobinsky 2006).

The following sections flesh out this "standard model" of cosmology, elaborating on the model as well as describing the measurements and observations on which it is based.

5. GROWTH OF STRUCTURE

5.1. Gravitational Instability and Microphysics in the Expanding Universe

5.1.1. Gravitational Instability Theory from Newton Onward

The primary driver for the formation of large-scale structure in the Universe is gravitational instability. The detailed growth of structure depends on the nature of the initial fluctuations, the background cosmology, and the constituents of the mass-energy

density, as causal physics influences the rate at which structure may grow on different scales.

Newton, prompted by questions posed to him by Bentley, realized that a gas of randomly positioned massive particles interacting gravitationally in flat spacetime is unstable, and that as time progresses the mass density distribution grows increasingly more anisotropic and inhomogeneous. Awareness of this instability led Newton to abandon his preference for a finite and bounded Universe of stars for one that is infinite and homogeneous on average (see discussion in Harrison 2001); this was an early discussion of the cosmological principle.

Jeans (1902) studied the stability of a spherical distribution of gravitating gas particles in flat spacetime, motivated by possible relevance to the process of star formation. He discovered that gas pressure prevents gravitational collapse on small spatial scales and gives rise to acoustic oscillations in the mass density inhomogeneity, as the pressure gradient and gravitational forces compete. On large scales the gravitational force dominates and mass density inhomogeneities grow exponentially with time. The length scale on which the two forces balance has come to be known as the Jeans length or the acoustic Hubble length c_s/H_0, where c_s is the speed of sound.

On scales smaller than the Jeans length, adiabatic energy density perturbations oscillate as acoustic waves. On scales well below the Jeans length dissipative fluid effects (e.g., viscosity and radiation diffusion) must be accounted for. These effects remove energy from the acoustic waves, thus damping them. In an expanding Universe, damping is effective when the dissipation timescale is shorter than the expansion timescale, and the smallest length scale for which this is the case is called the damping length. This is discussed in more detail below.

5.1.2. Structure Growth in an Expanding Universe

Study of gravitational instability in an evolving spacetime, appropriate for the expanding Universe, began with Lemaître in the early 1930s. He pioneered two approaches, both of which are still in use: a "nonperturbative" approach based on a spherically symmetric solution of the Einstein equations (further developed by Dingle, Tolman, Bondi, and others and discussed in § 5.3 below); and a "perturbative" approach in which one studies small departures from spatial homogeneity and isotropy evolving in homogeneous and isotropic background spacetimes.

At early times, and up to the present epoch on sufficiently large scales, the growth of structure by gravitational instability is accurately described by linear perturbation theory. The growth of small density and velocity perturbations must take into account the effects of the expansion of the Universe. A fully relativistic theory must be employed to describe the growth of structure, because it is necessary to also describe the evolution of modes with wavelength larger than the Hubble length, although a Newtonian approximation is valid and used on smaller length scales.

Lifshitz (1946) laid the foundations of the general-relativistic perturbative approach to structure formation. He linearized the Einstein and stress-energy conservation equations about a spatially homogeneous and isotropic Robertson-Walker background spacetime metric and decomposed the departures from homogeneity and isotropy into independently evolving spatial harmonics (the so-called scalar, vector, and tensor modes). Lifshitz treated matter as a fluid, which is a good approximation when the underlying particle mean-free path is small. He discovered that the vector transverse peculiar velocity (the peculiar velocity is the velocity that remains after subtracting off that due to the Hubble expansion) perturbation decays with time as a consequence of angular momentum conservation and that the contemporary Universe could contain a residual tensor gravitational wave background left over from earlier times.

Unlike the exponentially growing energy density irregularity that Jeans found in flat spacetime on large scales, Lifshitz found only a much slower power-law temporal growth, leading him to the incorrect conclusion that "gravitational instability is not the source of condensation of matter into separate nebulae". It was almost two decades before Novikov (1964) (see Bonnor 1957, for an earlier hint) corrected this misunderstanding, noting that even with power-law growth there was more than enough time for inhomogeneities to grow, since they could do so even while they were on scales larger than the Hubble length $r_H = c/H_0$ at early times.

The approach to the theory of linear perturbations initiated by Lifshitz is based on a specific choice of spacetime coordinates called synchronous coordinates. This approach is discussed in detail in § V (also see § II of Peebles 1980; § III of Zel'dovich & Novikov 1983; Ratra 1988; and other standard cosmology and astroparticle physics textbooks). Bardeen (1980) (building on earlier work) recast the Lifshitz analysis in a coordinate-independent form, and this approach has also become popular. For reviews of this approach, see Mukhanov et al. (1992), as well as the standard textbooks.

A useful formalism for linear growth of density and velocity fields is given by the "Zel'dovich approximation" (Zel'dovich 1970; Shandarin & Zel'dovich 1989; Sahni & Coles 1995), based on anisotropic collapse and so "pancake" formation (a concept earlier discussed in the context of the initial singularity). This method accurately describes structure formation up to the epoch when nonlinearities become significant. Numerical simulations (see § 5.4 below) of fully nonlinear structure growth often employ the Zel'dovich approximation for setting the initial conditions of density and velocity.

5.1.3. Space Curvature

The evolution of the background spacetime influences the rate of growth of structure. An early example of this effect is seen in the Gamow & Teller (1939) approximate generalization of Jeans's analysis to the expanding Universe, in particular to a model with open spatial hypersurfaces. At late times the

dominant form of energy density in such a model is that due to the curvature of spatial hypersurfaces, because this redshifts away slower than the energy density in nonrelativistic matter. The gravitational instability growth rate is determined by the matter-energy density, but the expansion rate becomes dominated by the space curvature. As a result, the Universe expands too fast for inhomogeneities to grow and large-scale structure formation ceases. (A quarter century later, Peebles 1965 noted the importance of this effect.) This was the first example of an important and general phenomenon: a dominant spatially homogeneous contributor to the energy density budget will prevent the growth of irregularity in matter.

5.1.4. Dark Energy

Matter perturbations also cannot grow when a cosmological constant or nearly homogeneous dark energy dominates. There is strong evidence that dark energy—perhaps in the form of Einstein's cosmological constant—currently contributes $\sim 70\%$ of the mass-energy density of the universe. This dark energy was subdominant until recently, when it started slowing the rate of growth of structure (Peebles 1984); thus, its effect on dynamical evolution is milder than that of space curvature.

5.1.5. Radiation and its Interaction with Baryonic Matter

Guyot & Zel'dovich (1970) showed that a dominant homogeneous radiation background makes the Universe expand too fast to allow matter irregularities to start growing until the model becomes matter dominated (when the radiation redshifts away). Because of this effect, as discussed next, the acoustic Hubble length at the epoch when the densities of matter and radiation are equal is an important scale for structure formation in the expanding Universe. This imprints a feature in the power spectrum of matter fluctuations on the scale of the acoustic Hubble length at matter-radiation equality that can be used to measure the cosmic density of nonrelativistic matter. We return to this in § 7.2; a related CMB anisotropy effect is discussed in the next § 5.2.

Gamow (1948) noted that, at early times in the Big Bang Model, radiation (which has large relativistic pressure) dominated over baryonic matter. In addition, at high temperature radiation and baryonic matter are strongly coupled by Thomson-Compton scattering and so behave like a single fluid. As a result of the large radiation pressure during this early epoch the Jeans or acoustic Hubble length is large, and so gravitational growth of inhomogeneity occurs only on large scales, with acoustic oscillations on small scales. Peebles & Yu (1970) develop this picture.

As the Universe cooled down below a temperature $T \sim 3000$ K at a redshift $z \sim 10^3$, the radiation and baryons decoupled. Below this temperature proton nuclei can capture and retain free electrons to form electrically neutral hydrogen atoms—this process is called "recombination"—because fewer photons remained in the high-energy tail of the distribution with enough energy to disassociate the hydrogen atoms. Peebles (1968) and Zel'dovich et al. (1968) perform an analysis of cosmological recombination, finding that at the "end" of recombination there were enough charged particles left over for the Universe to remain a good conductor all the way to the present. The finite time required for recombination results in a surface of nonzero thickness within which the decoupling of now-neutral baryons and photons occurs. The mean-free path for photons quickly grew, allowing the photons to travel (almost) freely, thus this "last-scattering surface" is the "initial" source of the observed CMB; it is an electromagnetically opaque "cosmic photosphere." See the standard cosmology textbooks for discussions of recombination.

Decoupling leads to a fairly steep drop in the pressure of the baryon gas, and so a fairly steep decrease in the baryon Jeans length. Peebles (1965) was developing this picture as the CMB was being discovered. Peebles & Dicke (1968) (also see Peebles 1967) noted that the baryonic Jeans mass after decoupling is of the order of the mass of a typical globular cluster and so proposed that proto–globular clusters were the first objects to gravitationally condense out of the primordial gas. This model would seem to predict the existence of extragalactic globular clusters, objects that have not yet been observationally recognized. There are, however, dwarf galaxies of almost equal low mass, and we now also know that some globular clusters are young—and so globular clusters might form in more than one way (for a recent review, see Brodie & Strader 2006).

On scales smaller than the Jeans mass, dissipative effects become important and the ideal fluid approximation for radiation and baryonic matter is no longer accurate. As the Universe cools down toward recombination and decoupling, the photon mean-free path grows, and so photons diffuse out of more dense to less dense regions. As they diffuse the photons drag some of the baryons with them and so damp small-scale inhomogeneities in the photon-baryon fluid. This collisional damping—a consequence of Thomson-Compton scattering—is known as Silk damping in the cosmological context; it was first studied by Michie (1969)[8], Peebles (1967), and Silk (1968). The Silk damping scale is roughly that of a cluster of galaxies.

5.1.6. Possible Matter Constituents

If baryons were the only form of nonrelativistic matter the density of matter would be so low that the Universe would remain radiation dominated until after recombination. The expansion rate would be too large for gravitational instability to cause inhomogeneity growth until matter starts to dominate well after last scattering. The short time allowed for the gravitational

[8] This is a version of a manuscript submitted to the Astrophysical Journal on September 1, 1967, and only minimally revised (in response to the referee's suggestions) before the author died.

growth of inhomogeneity from the start of matter domination to today would require a large initial fluctuation amplitude to produce the observed large-scale structure. This scenario is ruled out by measurements of the anisotropy of the CMB, which indicate that fluctuations in the baryons at decoupling are too small to have grown by gravitational instability into the structures seen today in the galaxy distribution.

A solution to this puzzle is provided by dark matter, of the same type and quantity needed to explain gravitational interactions on galactic and cluster scales. Including this component of matter the Universe becomes matter dominated at a redshift comparable to, or even larger than, the redshift of last scattering. Because CDM does not directly couple to radiation, inhomogeneities in the distribution of CDM begin to grow as soon as the Universe becomes matter dominated. Growth in structure in the baryons, on scales small compared to the Hubble length, remains suppressed by Thomson-Compton scattering until recombination, after which baryons begin to gravitate toward the potential wells of dark matter, and the baryon fluctuation amplitude quickly grows. Thus, the low observed CMB anisotropy is reconciled with observed large-scale structure. (The CMB, while not directly coupled to the CDM, feels the gravitational potential fluctuations of the CDM. Consequently, measurements of the CMB anisotropy probe the CDM distribution.) This is an independent, although model-dependent and indirect, argument for the existence of CDM.

As mentioned above in Sec. 2.3, the Universe also contains low-mass neutrinos (precise masses are not yet known). These neutrinos are relativistic and weakly coupled (nearly collisionless) and so have a very long mean-free path or free-streaming length. Consequently, they must be described by a distribution function, not a fluid. Because they are relativistic, they have a large Jeans mass, and gravitational instability is effective at collecting them only on very large scales; i.e., low-mass neutrinos suppress power on small and intermediate length scales. This effect makes it possible to observationally probe these particles with cosmological measurements (Elgarøy & Lahav 2005; Lesgourgues & Pastor 2006).

5.1.7. Free Streaming

Thus, the properties of dark matter are reflected in the spectrum of density fluctuations because scales smaller than the free-streaming scale of massive particles are damped (Bond et al. 1980). For hot dark matter (HDM), e.g., neutrinos, the free-streaming scale is larger than the Hubble length at matter-radiation equality; hence, the spectrum retains only large-scale power. In such a "top-down" scenario, superclusters form first, then fragment into smaller structures including clusters of galaxies and individual galaxies, as first discussed by Zel'dovich and collaborators. The top-down model was inspired by experimental suggestions (now known to be incorrect) that massive neutrinos could comprise the nonbaryonic dark matter, and by an early (also now known to be incorrect) interpretation of observational data on superclusters and voids (see § 6.2 below) that postulated that these were the basic organizational blocks for large-scale structure. It predicts that smaller-scale structure (e.g., galaxies) is younger than larger-scale structure (e.g., superclusters), contrary to current observational indications. In fact, these observational constraints on the evolution of structure constrain the amount of HDM neutrino matter-energy density and so neutrino masses (Kahniashvili et al. 2005). Cosmological observations provide the best (model-dependent) upper limits on neutrino masses.

For CDM, e.g., weakly interacting massive particles (WIMPs), the free-streaming scale is negligible for cosmological purposes. This "bottom-up" or "hierarchical" scenario, pioneered by Peebles and collaborators, begins with the formation of bound objects on small scales that aggregate into larger structures, thus galaxies result from mergers of subgalaxies, with superclusters being the latest structures to form. This is in better agreement with the observational data. See § 4 for more details on this model.

5.1.8. Initial Density Perturbations and the Transfer Function

The current standard model for structure formation assumes that structure in the Universe arose primarily from gravitational amplification of infinitesimal scalar density perturbations in the early Universe. The processes listed in this section modify these initial inhomogeneities. Reviews are given in Peebles (1980, Sec. V), Zel'dovich & Novikov (1983, § III), Efstathiou (1990), Kolb & Turner (1990, chapter 9), Padmanabhan (1993, chapter 4), Dekel & Ostriker (1999), and Mukhanov (2005, part II).

As discussed in §§ 4 and 5.2, observations to date are consistent with primordial fluctuations that are Gaussian random phase. These are the type of fluctuations expected if the seeds for structure formation result from the superposition of quantum-mechanical zero-point fluctuations of the scalar field that drove inflation of the early Universe, in the simplest inflation models, as discussed in § 3 above. In the simplest inflation models the fluctuations are adiabatic. Furthermore, observational data are consistent with only adiabatic perturbations, so in what follows we focus on this case (see Bean et al. 2006 for a recent discussion of constraints on isocurvature models).

As discussed in § 3 above and § 7.2 below, current large-scale observational results are reasonably well fit by an $n = 1$ scale-invariant primordial spectrum of perturbations, the kind considered by Harrison (1970), Peebles & Yu (1970), and Zel'dovich (1972), and predicted in some of the simpler inflation models. The effect of causal physics on the later growth of structure, as discussed above, may then be represented by a "transfer function" that describes the relative growth of fluctuations on different wavelength scales. Observations of the anisotropy of the CMB and the clustering of galaxies and clusters at

the present epoch probe the shape of the transfer function (as well as the primordial spectrum of perturbations) and thereby constrain structure formation models. Such observations are discussed below in §§ 5.2 and 6.

5.1.9. Gravitational Waves and Magnetic Fields

As noted in § 3 above, more complicated models of inflation can generate gravitational wave or magnetic field fluctuations that break adiabaticity. A primordial magnetic field might provide a way of explaining the origin of the uniform part of contemporary galactic magnetic fields; there are enough charged particles left over after recombination to ensure that primordial magnetic field lines will be pulled in, and the field amplified, by a collapsing gas cloud. Maggiore (2000) and Buonanno (2004) have reviewed primordial gravity waves, and cosmological magnetic fields have been reviewed by Widrow (2002) and Giovannini (2004). In the next subsection we consider the effects of such fields on the CMB.

5.2. CMB Anisotropies

As a result of the gravitational growth of inhomogeneities in the matter distribution, when the photons decouple from the baryons at last scattering at a redshift $z \sim 10^3$ (see § 5.1 above), the photon temperature distribution is spatially anisotropic. In addition, in the presence of a CMB temperature quadrupole anisotropy, Thomson-Compton scattering of CMB photons off electrons prior to decoupling generates a linear polarization anisotropy of the CMB. After decoupling, the CMB photons propagate almost freely, influenced only by gravitational perturbations and late-time reionization. Measurements of the temperature anisotropy and polarization anisotropy provide important constraints on many parameters of models of structure formation. This area of research has seen spectacular growth in the last decade or so, following the *COBE* discovery of the CMB temperature anisotropy. It has been the subject of recent reviews; see White & Cohn (2002), Hu & Dodelson (2002), Peebles & Ratra (2003, § IV.B.11), Subramanian (2005), Giovannini (2005), and Challinor (2005). Here we focus only on a few recent developments.

The three-year *WMAP* observations of CMB temperature anisotropies (Hinshaw et al. 2007) are state-of-the-art data. On all but the very largest angular scales, the *WMAP* data are consistent with the assumption that the CMB temperature anisotropy is well described by a spatial Gaussian random process (Komatsu et al. 2003), consistent with earlier indications (Park et al. 2001; Wu et al. 2001). The few largest-scale angular modes exhibit a lack of power compared to what is expected in a spatially flat CDM model dominated by a cosmological constant (Bennett et al. 2003a), resulting in some debate about the assumptions of large-scale Gaussianity and spatial isotropy. This feature was also seen in the *COBE* data (Górski et al. 1998). The estimated large-angular-scale CMB temperature anisotropy power depends on the model used to remove foreground Galactic emission contamination. Much work has been devoted to understanding foreground emission on all scales (e.g., Mukherjee et al. 2003a; Bennett et al. 2003b; Tegmark et al. 2003), and the current consensus is that foregrounds are not the cause of the large-angular-scale *WMAP* effects.

The CMB temperature anisotropy is conventionally expressed as an expansion in spherical harmonic multipoles on the sky, and for a Gaussian random process the multipole (or angular) power spectrum completely characterizes the CMB temperature anisotropy. The observed CMB anisotropy is reasonably well fit by assuming only adiabatic fluctuations with a scale-invariant power spectrum. These observational results are consistent with the predictions of the simplest inflation models, where quantum-mechanical fluctuations in a weakly coupled scalar field are the adiabatic, Gaussian seeds for the observed CMB anisotropy and large-scale structure.

Smaller-scale inhomogeneities in the coupled baryon-radiation fluid oscillate (see § 5.1 above), and at decoupling some of these modes will be at a maximum or at a minimum, giving rise to acoustic peaks and valleys in the CMB anisotropy angular spectrum. The relevant length scale is the acoustic Hubble length at the epoch of recombination; this may be predicted by linear physics and so provides a standard ruler on the sky. Through the angular diameter distance relation, the multipole numbers ℓ of oscillatory features in the temperature anisotropy spectrum C_ℓ reflect space curvature ($\Omega_{\rm K}$) and the expansion history (which depends on $\Omega_{\rm M}$ and Ω_Λ) of the Universe. The angular scales of the peaks are sensitive to the value of the matter-density parameter in an open Universe, but not in a spatially flat ($\Omega_{\rm K} = 0$) Universe dominated by a cosmological constant, where the first peak is at a multipole index $\ell \sim 220$. This provides a useful way to measure the curvature of spatial hypersurfaces. Sugiyama & Gouda (1992) and Kamionkowski et al. (1994a,1994b) presented early discussions of the CMB temperature anisotropy in an open Universe, and Brax et al. (2000), Baccigalupi et al. (2002), Caldwell & Doran (2004), and Mukherjee et al. (2003b) considered the case of scalar-field dark energy in a spatially flat Universe. CMB temperature anisotropy data on the position of the first peak is consistent with flat spatial hypersurfaces (e.g., Podariu et al. 2001; Durrer et al. 2003; Page et al. 2003). Model-based CMB data analysis is used to constrain more cosmological parameters (e.g., Lewis & Bridle 2002; Mukherjee et al. 2003c; Spergel et al. 2007). For example, the relative amplitudes of peaks in this spectrum are sensitive to the mass densities of the different possible constituents of matter (e.g., CDM, baryons, and neutrinos, $\Omega_{\rm CDM}$, $\Omega_{\rm B}$, and Ω_ν).

The CMB polarization anisotropy was first detected from the ground by the DASI experiment at the South Pole (Kovac et al. 2002). The three-year *WMAP* observations are the current state of the art (Page et al. 2007). For a recent review of polarization measurements, see Balbi et al. (2006). The polarization

anisotropy peaks at a larger angular scale than the temperature anisotropy, indicating that there are inhomogeneities on scales larger than the acoustic Hubble length at recombination, consistent with what is expected in the inflation scenario. The polarization anisotropy signal is interpreted as the signature of reionization of the Universe. The ability of *WMAP* to measure polarization anisotropies allows this experiment to probe the early epochs of nonlinear structure formation, through sensitivity to the reionization optical depth τ.

Primordial gravitational waves or a primordial magnetic field can also generate CMB anisotropies. Of particular current interest are their contributions to various CMB polarization anisotropies. (Because polarization is caused by quadrupole fluctuations, these anisotropies constrain properties of the primordial fluctuations, such as the ratio of tensor-to-scalar fluctuations, r.) The effects of gravity waves on the CMB are discussed in the more recent standard cosmology and astroparticle textbooks and by Giovannini (2005). The magnetic field case has been reviewed by Giovannini (2006) and Subramanian (2006); recent topics of interest may be traced from Lewis (2004), Kahniashvili & Ratra (2005, 2007), and Brown & Crittenden (2005).

We continue discussion of the CMB anisotropies and cosmological parameters in § 7.2.

5.3. Galaxy Formation and the End of the Dark Age

The emission of the first light in the Universe, seen today as the CMB, is followed by a "dark age" before the first stars and quasars form. Bromm & Larson (2004) review formation of the first stars. Eventually, high-energy photons from stars and quasars reionize intergalactic gas throughout the Universe (for reviews, see Fan et al. 2006; Choudhury & Ferrara 2006a; Loeb 2006a,2006b). Observations of polarization of microwave background photons by *WMAP* (Page et al. 2007) suggest that reionization occurs at redshift $z \approx 11$. However, strong absorption of Lyman-α photons by intergalactic neutral hydrogen (Gunn & Peterson 1965), seen in spectra of quasars at redshift $z \approx 6$ (Becker et al. 2001; Fan et al. 2002) indicates that reionization was not complete until somewhat later. This is an area of ongoing research (see, e.g., Choudhury & Ferrara 2006b; Gnedin & Fan 2006; Alvarez et al. 2006).

Current models for galaxy formation follow the picture (Hoyle 1953; Silk 1977; Binney 1977; Rees & Ostriker 1977; White & Rees 1978) in which dark matter halos form by collisionless collapse, after which baryons fall into these potential wells, are heated to virial temperature, and then cool and condense at the centers of the halos to form galaxies as we know them. In short, baryons fall into the gravitational potentials of "halos" of dark matter at the same time that those halos grow in size, hierarchically aggregating small clumps into larger ones. The baryons cool by emitting radiation and shed angular momentum, leading to concentrations of star formation and accretion onto supermassive black holes within the dark matter halos.

In addition to the perturbative approach to structure formation discussed in § 5.1, Lemaître also pioneered a nonperturbative approach based on a spherically symmetric solution of the Einstein equations. This spherical accretion model (Gunn & Gott 1972) describes the salient features of the growth of mass concentrations. See Gott (1977), Peebles (1993, Sec. 22), and Sahni & Coles (1995) for reviews of such models.

A phenomenological prescription for the statistics of nonlinear collapse of structure, i.e., the formation of gravitationally bound objects, is given by the Press-Schechter formulae (Press & Schechter 1974; Sheth & Tormen 1999). Attempts to firm up the theoretical basis of such formulae form the "excursion set" formalism, which treats the formation of a gravitationally bound halo as the result of a random walk (Mo & White 1996; White 1996; Sheth et al. 2001). For a review, see Cooray & Sheth (2002). These methods provide probability distributions for the number of bound objects as a function of mass threshold and can be generalized to develop a complementary description of the evolution of voids (Sheth & van de Weygaert 2004). A more rigorous approach assumes structure forms at high peaks in the smoothed density field (Kaiser 1984; Bardeen et al. 1986; Sahni & Coles 1995). Recent reviews of galaxy formation include Avila-Reese (2006) and Baugh (2006). The next subsection, 5.4, describes numerical methods for studying structure formation.

Apparent confirmation of the hierarchical picture of structure formation includes the striking images of galaxies apparently in the process of assembly, obtained by the *Hubble Space Telescope* (*HST*) in the celebrated "Hubble Deep Fields" (Ferguson et al. 2000; Beckwith et al. 2006). The detailed properties of galaxies and their evolution are outside the scope of this review. Recent reviews of the observational situation are Gawiser (2006) and Ellis (2007). Texts covering this topic include Spinrad (2005) and Longair (2008).

While the current best model of structure formation, in which CDM dominates the matter-density, works quite well on large scales, current observations indicate some possible problems with the CDM model on smaller scales; see Tasitsiomi (2003), Peebles & Ratra (2003, § IV.A.2), and Primack (2005) for reviews. Simulations of structure formation indicate that CDM model halos may have cores that are cuspier (Navarro et al. 1997; Swaters et al. 2000) and central densities that are higher (Moore et al. 1999a; Firmani et al. 2001) than are observed in galaxies. Another concern is that CDM models predict a larger than observed number of low-mass satellites of massive galaxies (Moore et al. 1999b; Klypin et al. 1999). These issues have led to consideration of models with reduced small-scale power. However, it seems difficult to reconcile suppression of small-scale power with the observed small-scale clustering in the neutral hydrogen at redshifts near 3.

The relationship between the distributions of galaxies (light) and matter is commonly referred to as "biasing." The currently favored dark-energy-dominated CDM model does not require significant bias between galaxies and matter; in the best-fit model the ratio of galaxy to matter clustering is close to unity for ordinary galaxies (Tegmark et al. 2004a).

5.4. Simulations of Structure Formation

Cosmological simulations using increasingly sophisticated numerical methods provide a test bed for models of structure formation. Bertschinger (1998) has reviewed methods and results.

Computer simulations of structure formation in the Universe began with purely gravitational codes that directly compute the forces between a finite number of particles (particle-particle or PP codes) that sample the matter distribution. Early results used direct N-body calculations (Aarseth et al. 1979). Binning the particles on a grid and computing the forces using the fast Fourier transform (the particle-mesh or PM method) is computationally more efficient, allowing simulation of larger volumes of space, but has force resolution of the order of the grid spacing. A compromise is the P^3M method, which uses PM for large-scale forces supplemented by direct PP calculations on small scales, as used for the important suite of CDM simulations by Davis et al. (1985). For details on these methods, see Hockney & Eastwood (1988).

The force resolution of PM codes and the force resolution and speed of P^3M codes may be increased by employing multiple grid levels (Villumsen 1989; Couchman 1991; Bertschinger & Gelb 1991; Gnedin & Bertschinger 1996). Adaptive mesh refinement (AMR; Berger & Collela 1989) does this dynamically to increase force resolution in the PM gravity solver (Kravtsov et al. 1997; Norman & Bryan 1999).

Another approach to achieving both speed and good force resolution in gravitational N-body simulation is use of the hierarchical tree algorithm (Barnes & Hut 1986). Large cosmological simulations have used a parallelized version of this method (Zurek et al. 1994). Significant increase in speed was found with the tree particle-mesh algorithm (Bode et al. 2000). GOTPM (Dubinski et al. 2004), a parallelized hybrid PM + tree scheme, has been used for simulations involving up to 8.6×10^9 particles. PMFAST (Merz et al. 2005) is a recent parallelized multilevel PM code.

Incorporation of hydrodynamics and radiative transfer in cosmological simulations has made it possible to study not only the gravitational formation of dark matter halos but also the properties of baryonic matter, and, thus, the formation of galaxies associated with those halos. Methods for solving the fluid equations include smooth-particle hydrodynamics (SPH; see Monaghan 1992 for a review), which is an inherently Lagrangian approach, and Eulerian grid methods. Cosmological SPH simulations were pioneered by Evrard (1988) and Hernquist & Katz (1989). To date, the cosmological simulation with the largest number of particles (10^{10}) employs SPH and a tree algorithm (GADGET; Springel et al. 2001). Grid-based codes used for cosmological simulation include that described by Cen (1992) and Ryu et al. (1993).

To date, no code has sufficient dynamic range to compute both the large-scale cosmological evolution on scales of many hundreds of megaparsecs and the formation of stars from baryons, but physical heuristics have been successfully incorporated into some codes to model the conversion of baryons to stars (see, e.g., Cen 1992).

The Millenium Run simulation (Springel et al. 2005) represents the current state of the art in following the evolution of both the dark matter and baryonic components on scales from the box size, $500h^{-1}$ Mpc, down to the resolution limit of roughly $5h^{-1}$ kpc. See this article and references therein for discussion of the many pieces of uncertain physics necessary for producing the observed baryonic structures.

Another approach to modeling the properties of the galaxies associated with dark matter halos is to use the history of halo mergers together with semianalytic modeling of galaxy properties (Lacey et al. 1993; Kauffmann et al. 1993; Cole et al. 1994; Somerville & Primack 1999). When normalized to the observed luminosity function of galaxies and Tully-Fisher relation for spiral galaxies, these semianalytic models (SAMs) reproduce many of the observed features of the galaxy distribution. A common approach is to use SAMs to "paint on" the properties of galaxies that would reside in the dark matter halos found in purely gravitational simulations. See Avila-Reese (2006) and Baugh (2006) for recent reviews. Related to the SAMs approach are halo occupation models (Berlind & Weinberg 2002; Kravtsov et al. 2004) that parameterize the statistical relationship between the masses of dark matter halos and the number of galaxies resident in each halo.

6. MAPPING THE UNIVERSE

The observed features of the large-scale distribution of matter include clusters, superclusters, filaments, and voids. By mapping the distribution of galaxies in the Universe, both in two dimensions as projected on the sky and in three dimensions using spectroscopic redshifts, astronomers seek to quantify these inhomogeneities in order to test models for the formation of structure in the Universe. Not only the spatial distribution of galaxies but also the distribution of clusters of galaxies, quasars, and absorption line systems provide constraints on these models. Peculiar velocities of galaxies, which reflect inhomogeneities in the mass distribution, provide further constraints. Here we briefly review important milestones and surveys relevant for testing cosmological models.

6.1. Galaxy Photometric Surveys

Studies of the global spacetime of the Universe assume the "cosmological principle" (Milne 1933), which is the supposi-

tion that the Universe is statistically homogeneous when viewed on sufficiently large scales. The angular distribution of radio galaxies provides a good test of this approach to homogeneity, because radio-bright galaxies and quasars may be seen in flux-limited samples to nearly a Hubble distance, c/H_0. Indeed, the $\sim 31,000$ brightest radio galaxies observed at a wavelength of 6 cm (Gregory & Condon 1991) are distributed nearly isotropically, and similar results are found in the FIRST radio survey (Becker et al. 1995). (For a review of other evidence for large-scale spatial isotropy see § 3 of Peebles 1993.) In contrast, the Universe is clearly inhomogeneous on the more modest scales probed by optically selected samples of bright galaxies, For example, significant clustering is observed among the roughly 30,000 galaxies in the Zwicky et al. (1961–1968) catalog.

Maps of the distribution of nebulae revealed anisotropy in the sky before astronomers came to agree that many of these nebulae were distant galaxies (Charlier 1925). The Shapley & Ames (1932) catalog of galaxies clearly showed the nearby Virgo cluster of galaxies. Surveys of selected areas on the sky using photographic plates to detect distant galaxies clearly revealed anisotropy of the galaxy distribution and were used to quantify this anisotropy (Mowbray 1938). de Vaucouleurs (1953) recognized in this anisotropy the projected distribution of the local supercluster of galaxies.

Rubin (1954) used two-point correlations of galaxy counts from Harvard College Observatory plates to detect fluctuations on the scale of clusters of galaxies. The Shane & Wirtanen (1954) Lick Survey of galaxies used counts of galaxies found on large-format photographic plates taken at Lick Observatory to make the first large-scale map of the angular distribution of galaxies suitable for statistical analysis. Early analysis of these data included methods such as counts-in-cells analyses and the two-point correlation function (Limber 1954; Totsuji & Kihara 1969). The sky map of the Lick counts produced by Seldner et al. (1977) visually demonstrated the rich structure in the galaxy distribution. Peebles and collaborators used these data for much of their extensive work on galaxy clustering (Groth & Peebles 1977); for a review see Peebles (1980, § III).

The first Palomar Observatory Sky Survey (POSS) yielded two important catalogs: the Abell (1958) catalog of clusters and the Zwicky et al. (1961–1968) catalog of clusters and galaxies identified by eye from the photographic plates. Abell (1961) found evidence for angular "superclustering" (clustering of galaxy clusters) that was confirmed statistically by Hauser & Peebles (1973). Photographic plates taken at the United Kingdom Schmidt Telescope Unit (UKSTU) were digitized using the automatic plate measuring (APM) machine to produce the APM catalog of roughly two million galaxies. Calibration with CCD photometry made the APM catalog invaluable for accurate study of the angular correlation function of galaxies on large scales (Maddox et al. 1990). Perhaps the last large-area galaxy photometric survey to employ photographic plates was the Digitized Palomar Observatory Sky Survey (DPOSS) (Gal et al. 2004).

The largest imaging survey that employs a camera with arrays of charge-coupled devices (CCDs) is the Sloan Digital Sky Survey (SDSS; Stoughton et al. 2002). The imaging portion of this survey includes five-color digital photometry of 8000 deg^2 of sky, with 215 million detected objects. Imaging for the SDSS is obtained using a special-purpose 2.5 m telescope with a 3° field of view (Gunn et al. 2006).

Important complements to optical surveys include large-area catalogs of galaxies selected in the infrared and ultraviolet. Nearly all-sky source catalogs were produced from infrared photometry obtained with the *Infrared Astronomical Satellite* (*IRAS*; Beichman et al. 1988) and the ground-based Two Micron All Sky Survey (2MASS; Jarrett et al. 2000). The ongoing Galaxy Evolution Explorer satellite (GALEX; Martin et al. 2005) is obtaining ultraviolet imaging over the whole sky.

6.2. Galaxy Spectroscopic Surveys

Systematic surveys of galaxies using spectroscopic redshifts to infer their distances began with observations of galaxies selected from the Shapley-Ames catalog (Humason et al. 1956; Sandage 1978). Important early mapping efforts include identification of superclusters and voids in the distribution of galaxies and Abell clusters by Jôeveer et al. (1978), the Gregory & Thompson (1978) study of the Coma/Abell1367 supercluster and its environs that identified voids, and the Kirshner et al. (1981) study of the correlation function of galaxies and discovery of the giant void in Boötes. Early targeted surveys include the Giovanelli & Haynes (1985) survey of the Perseus-Pisces supercluster.

Redshift surveys of large areas of the sky began with the first Center for Astrophysics redshift survey (CfA1; Huchra et al. 1983), which includes redshifts for 2401 galaxies brighter than apparent magnitude $m_B = 14.5$ over a wide area toward the North Galactic Pole. CfA2 (Falco et al. 1999) followed over roughly the same area, extending to apparent magnitude $m_B = 15.5$. At this depth, the rich pattern of voids, clusters, and superclusters were strikingly obvious (de Lapparent et al. 1986). Giovanelli & Haynes (1991) review the status of galaxy redshift surveys ca. 1991.

Both CfA redshift surveys used the Zwicky catalog of galaxies to select targets for spectroscopy. The Southern Sky Redshift Survey (SSRS; da Costa et al. 1998) covers a large area of the southern hemisphere (contiguous with CfA2 in the northern galactic cap) to similar depth, using the ESO/Uppsala Survey to select galaxy targets and a spectrograph similar to that employed for the CfA surveys. The Optical Redshift Survey (ORS) supplemented existing redshift catalogs with 1300 new spectroscopic redshifts to create a nearly all-sky survey (Santiago et al. 1995).

Deep "pencil-beam" surveys of narrow patches on the sky revealed apparently periodic structure in the galaxy distribution (Broadhurst et al. 1990).

The Las Campanas Redshift Survey (LCRS; Shectman et al. 1996), the first large-area survey to use fiber optics, covered over 700 deg^2 near the South Galactic Pole. This survey was important because it showed that structures such as voids and superclusters found in shallower surveys are ubiquitous, but the structures seen by LCRS were no larger than those found earlier. The Century Survey (Geller et al. 1997) and the ESO Deep Slice survey (Vettolani et al. 1998) were likewise useful for statistically confirming this emerging picture of large-scale structure.

Sparse surveys of galaxies to efficiently study statistical properties of the galaxy distribution include the Stromlo-APM redshift survey (Loveday et al. 1996) based on $1/20$ sampling of the APM galaxy catalog and the Durham/UKSTU redshift survey (Ratcliffe et al. 1998).

While optically selected surveys are relatively blind to structure behind the Milky Way, redshift surveys based on objects detected in the infrared provide nearly all-sky coverage. A sequence of surveys of objects detected by *IRAS* were carried out, flux-limited to 2 Jy (Strauss et al. 1992), 1.2 Jy (Fisher et al. 1995), and 0.6 Jy (Saunders et al. 2000). The 6dF Galaxy Survey (Jones et al. 2004) will measure redshifts of 150,000 galaxies photometrically identified by 2MASS (Jarrett et al. 2000).

The Two Degree Field Galaxy Redshift Survey (2dFGRS) of 250,000 galaxies (Colless et al. 2001) was selected from the APM galaxy catalog and observed using the Two Degree Field multifiber spectrograph at the Anglo-Australian 4 m telescope. The survey is complete to apparent magnitude $m_J = 19.45$ and covers about 1500 deg^2.

The spectroscopic component of the SDSS (Stoughton et al. 2002) includes medium-resolution spectroscopy of 675,000 galaxies and 96,000 quasars identified from SDSS photometry. These spectra are obtained with dual fiber-optic CCD spectrographs on the same 2.5 m telescope. The main galaxy redshift survey is complete to $m_r = 17.77$ and is complemented by a deeper survey of luminous red galaxies. The ongoing extension of this survey (SDSS-II) will expand the spectroscopic samples to more than 900,000 galaxies and 128,000 quasars.

Spectroscopic surveys that trace structure in the galaxy distribution at much larger redshift include the DEEP2 survey (Coil et al. 2004) and others (Steidel et al. 2004) employing the Keck Observatory, and the VIMOS VLT Deep survey (Le Fèvre et al. 2005).

6.3. Cluster Surveys

Mapping of the Universe using galaxy clusters as tracers began with study of the Abell catalog (Abell 1958; Abell et al. 1989). Studies of the angular clustering of Abell clusters includes Hauser & Peebles (1973). Several redshift surveys of Abell clusters have been conducted, including those described by Postman et al. (1992) and Katgert (1996). Important cluster samples have also been identified from digitized photographic plates from the UKSTU, followed up by redshift surveys of cluster galaxies (Lumsden et al. 1992; Dalton et al. 1992). More distant samples of clusters have been identified using deep CCD photometry (see, e.g., Postman et al. 1996; Gladders & Yee 2005). In X-ray bandpasses, cluster samples useful for studying large-scale structure have been identified using data from *ROSAT* (Romer et al. 1994; Böhringer et al. 2004). The SDSS is producing large catalogs of galaxy clusters using a variety of selection methods (Bahcall et al. 2003). Use of the Sunyaev-Zel'dovich effect (the microwave decrement caused by Thomson-Compton scattering of the CMB photons by the intracluster gas) holds great promise to identify new deep samples of galaxy clusters (Carlstrom et al. 2002). General reviews of clusters of galaxies include Rosati et al. (2002), Voit (2005), and Borgani (2006).

6.4. Quasar Surveys

The advent of multiobject wide-field spectrographs has made possible the collection of very large samples of spectroscopically confirmed quasars, as observed by the 2dF QSO Redshift Survey (Croom et al. 2004) and the SDSS (Schneider et al. 2005). For a ca. 1990 review of the field, see Hartwick & Shade (1990). While quasars themselves are too sparsely distributed to provide good maps of the large-scale distribution of matter, their clustering in redshift space has been measured (Osmer 1981) and generally found to be similar to that of galaxies (Outram et al. 2003). Similar results have been obtained from clustering analyses of active galactic nuclei in the nearby universe (Wake et al. 2004), although this clustering depends in detail on the type of active galactic nucleus (AGN; Constantin & Vogeley 2006).

The distribution of absorption lines from gas, particularly from the Lyman-α "forest" of neutral hydrogen clouds along the line of sight toward bright quasars (Lynds 1971; Rauch 1998) provides an important statistical probe of the distribution of matter (see, e.g., McDonald et al. 2005) on small scales and at large redshift.

6.5. Peculiar Velocity Surveys

When measured over sufficiently large scales, the peculiar motions of galaxies or clusters simply depend on the potential field generated by the mass distribution (see Peebles 1980; 1993; Davis & Peebles 1983). Techniques for measuring distances to other galaxies are critically reviewed in Rowan-Robinson (1985), Jacoby et al. (1992), Strauss & Willick (1995), and Webb (1999). Together with the galaxy or cluster redshifts, these measurements yield maps of the line-of-sight component of the peculiar velocity. From such data it is possible to reconstruct a map of the matter-density field (e.g., Bertschinger & Dekel

1989; Dekel 1994) or to trace the galaxy orbits back in time (e.g., Peebles 1990; Goldberg & Spergel 2000). Analyses of correlations of the density and velocity fields also yield constraints on the cosmic matter-density (e.g., Willick et al. 1997).

Rubin et al. (1976) were the first to find evidence for bulk flows from galaxy peculiar velocities. Dressler et al. (1987) found evidence for a bulk flow toward a large mass concentration, dubbed the "Great Attractor." Lauer & Postman (1994) found surprising evidence for motion of the Local Group on a larger scale. However, analysis of subsequent peculiar velocity surveys indicates that the inferred bulk flow results, including those of Lauer and Postman, are consistent within the uncertainties (Hudson et al. 2000). The status of this field ca. 1999 is surveyed by Courteau & Willick (2000); recent results include Hudson et al. (2004), and Dekel (1994) and Strauss & Willick (1995) review this topic. Comparison of peculiar velocity surveys with the peculiar velocity of our Galaxy implied by the CMB dipole indicates that a significant component of our motion must arise from density inhomogeneities that lie at rather large distance, beyond $60h^{-1}$ Mpc (Hudson et al. 2004).

6.6. Statistics of Large-Scale Structure

The clustering pattern of extragalactic objects reflects both the initial conditions for structure formation and the complex astrophysics of formation and evolution of these objects. In the standard picture described above, linear perturbation theory accurately describes the early evolution of structure; thus measurement of clustering on very large scales, where the clustering remains weak, closely reflects the initial conditions. On these scales the density field is very nearly Gaussian random phase, therefore the two-point correlation function of the galaxy number density field (also called the autocorrelation or covariance function) or its Fourier transform, the power spectrum, provides a complete statistical description. (Temperature anisotropies of the CMB discussed in § 5.2 arise from density fluctuations at redshift $z \sim 10^3$ that evolve in the fully linear regime.) On the scales of galaxies and clusters of galaxies, gravitational evolution is highly nonlinear, and the apparent clustering depends strongly on the detailed relationship between mass and light in galaxies. In between the linear and nonlinear regimes lies the "quasi-linear" regime in which clustering growth proceeds most rapidly. A wide range of statistical methods have been developed to quantify this complex behavior. Statistical properties of the galaxy distribution and details of estimating most of the relevant statistics are described in depth by Peebles (1980), Martínez & Saar (2002), and Bernardeau et al. (2002). Methods of using galaxy redshift surveys to constrain cosmology are reviewed by Lahav & Suto (2004) and Percival (2006). Constraints on cosmological parameters from such measurements are discussed below in § 7.2.

The simplest set of statistical measures are the n-point correlation functions, which describe the joint probability in excess of random of finding n galaxies at specified relative separation. Early applications of correlation functions to galaxy data include Limber (1954), Totsuji & Kihara (1969), and Groth & Peebles (1977). The n-point functions may be estimated by directly examining the positions of n-tuples of galaxies or by using moments of galaxy counts in cells of varying size. Tests of scaling relations among the n-point functions are discussed in detail by Bernardeau et al. (2002).

Power spectrum analyses of large galaxy redshift surveys (Vogeley et al. 1992; Fisher et al. 1993; Tegmark et al. 2004b) yield useful constraints on cosmological models. Closely related to power spectrum analyses are estimates of cosmological parameters using orthogonal functions (Vogeley & Szalay 1996; Pope et al. 2004). Tegmark et al. (1998) discuss the merits of different methods of power spectrum estimation. Verde et al. (2002) describe a measurement of the galaxy bispectrum.

A number of statistics have been developed to quantify the geometry and topology of large-scale structure. The topological genus of isodensity contours characterizes the connectivity of large-scale structure (Gott et al. 1987). Measurements of the genus are consistent with random phase initial conditions (as predicted by inflation) on large scales (Gott et al. 1989), with departures from Gaussianity on smaller scales where nonlinear gravitational evolution and biasing of galaxies are evident (Vogeley et al. 1994; Gott et al. 2006). Similar techniques are used to check on the Gaussianity of the CMB anisotropy (Park et al. 2001; Wu et al. 2001; Komatsu et al. 2003), as well as identify foreground emission signals in CMB anisotropy data (Park et al. 2002).

The void probability function, which characterizes the frequency of completely empty regions of space (White 1979), has been estimated from galaxy redshift surveys (Maurogordato & Lachièze-Rey 1987; Hoyle & Vogeley 2004). Catalogs of voids have been constructed with objective void-finding algorithms (El-Ad et al. 1996; Hoyle & Vogeley 2002).

Early investigations of the pattern of galaxy clustering dating back to Charlier (1925) suggested a clustering hierarchy. The fractal model of clustering introduced by Mandelbrot (1982, and references therein) further motivated investigation of the possibility of scale-invariant clustering of galaxies. Results of such analyses of galaxy survey data were controversial (compare, e.g., Sylos Labini et al. 1998 with Hatton 1999 and Martínez et al. 2001 and references therein). While fractal behavior is seen on small scales, there is fairly strong evidence for an approach to homogeneity in galaxy redshift and photometric surveys on very large scales. Thus, a simple scale-invariant fractal description seems to be ruled out. A multifractal description of clustering continues to provide a useful complement to other statistical descriptors (Jones et al. 2005). Consideration of modified forms of the fractal picture are of interest for providing slight non-Gaussianity on very large scales that might be needed to explain the very largest structures in the Universe.

Anisotropy of galaxy clustering in redshift space results from bulk flows on large scales that amplify clustering along the line of sight to the observer and from motions of galaxies in virialized systems such as clusters that elongate those structures along the line of sight (Kaiser 1987). Hamilton (1998) provides an extensive review and Tinker et al. (2006) describe recent methods for estimating cosmological parameters from redshift-space distortions of the correlation function or power spectrum.

The dependence of clustering statistics on properties of galaxies provides important clues to their history of formation and reflects the complex relationship between the distributions of mass and luminous matter. The amplitude of galaxy clustering is seen to vary with galaxy morphology (e.g., Davis & Geller 1976; Guzzo et al. 1997) and with luminosity (e.g., Hamilton 1988; Park et al. 1994). In recent analyses of the SDSS and 2dFGRS, these and similar trends with color, surface brightness, and spectral type are seen (Norberg et al. 2002; Zehavi et al. 2005).

Spectroscopy obtained with 8–10 m class telescopes has recently made it possible to accurately study structure in the galaxy distribution at higher redshift (Coil et al. 2004; Adelberger et al. 2005; Le Fèvre et al. 2005).

7. MEASURING COSMOLOGICAL PARAMETERS

7.1. The Case for a Flat, Accelerating Universe

As mentioned in § 4, observations of Type Ia supernovae (SNeIa) provide strong evidence that the expansion of the Universe is accelerating. Type Ia supernovae have the useful property that their peak intrinsic luminosities are correlated with how fast they dim, which allows them to be turned into standard candles. At redshifts approaching unity, observations indicate that they are dimmer (and so farther away) than would be predicted in an unaccelerating Universe (Riess et al. 1998; Perlmutter et al. 1999). In the context of general relativity this acceleration is attributed to dark energy that varies slowly with time and space, if at all. A mass-energy component that maintains constant (or nearly constant) density has negative pressure. Because pressure contributes to the active gravitational mass density, negative pressure, if large enough, can overwhelm the attraction caused by the usual (including dark) matter mass density and result in accelerated expansion. For a careful review of the early supernova tests see Leibundgut (2001). For discussions of the cosmological implications of this test see Peebles & Ratra (2003) and Perivolaropoulos (2006). Current supernova data show that models with vanishing cosmological constant are more than four standard deviations away from the best fit.

The supernova test assumes general relativity and probes the geometry of spacetime. The result is confirmed by a test using the CMB anisotropy that must, in addition, assume the CDM model for structure formation discussed in § 4 (see § 5.3 for apparent problems with this model). As discussed in § 5.2, CMB anisotropy data on the position of the first peak in the angular power spectrum are consistent with the curvature of spatial hypersurfaces being small. Many independent lines of evidence indicate that the mass density of nonrelativistic matter (CDM and baryons)—a number also based on the CDM structure formation model—is about 25% or 30% of the critical Einstein–de Sitter density (see §§ 4 and 7.2). Because the contemporary mass density of radiation and other relativistic matter is small, a cosmological constant or dark energy must contribute 70% or 75% of the current mass budget of the Universe. For reviews of the CMB data constraints, see Peebles & Ratra (2003), Copeland et al. (2006), and Spergel et al. (2007).

7.2. Observational Constraints on Cosmological Parameters

The model suggested by the SNeIa and CMB data, spatially flat and with contemporary mass-energy budget split between a cosmological constant or dark energy ($\sim$70%), dark matter ($\sim$25%), and baryonic matter ($\sim$5%), is broadly consistent with the results of a large number of other cosmological tests. In this subsection we present a very brief discussion of some of these tests and the constraints they impose on the parameters of this "standard" cosmological model. Two nice reviews of the cosmological tests are § 13 of Peebles (1993) and Sandage (1995). Hogg (1999) provides a concise summary of various geometrical measures used in these tests. Section IV of Peebles & Ratra (2003) reviews more recent developments and observational constraints. Here we summarize some of these as well as the significant progress of the last four years. Numerical values for cosmological parameters are listed in Lahav & Liddle (2006), although in some cases there is still significant ongoing debate.

There have been many—around 500—measurements of the Hubble constant H_0, (Huchra 2007), the current expansion rate. Since there is debate about the error estimates of some of these measurements, a median statistics meta-analysis estimate of H_0 is probably the most robust estimate (Gott et al. 2001). At two standard deviations this gives $H_0 = 100h$ km s^{-1} Mpc$^{-1} = 68 \pm 7$ km s^{-1} Mpc$^{-1} = (14 \pm 1$ Gyr$)^{-1}$ (Chen et al. 2003), where the first equation defines h. It is significant that this result agrees with the estimate from the *HST* Key Project (Freedman et al. 2001), the *HST* estimate of Sandage and collaborators (Sandage et al. 2006), and the *WMAP* three-year data estimate (which assumes the CDM structure formation model; Spergel et al. 2007).

A measurement of the redshift dependence of the Hubble parameter can be used to constrain cosmological parameters (Jimenez & Loeb 2002; Simon et al. 2005). For applications of this test using preliminary data, see Samushia & Ratra (2006) and Sen & Scherrer (2007).

Expansion time tests are reviewed in Peebles & Ratra (2003, § IV.B.3). A recent development is the *WMAP* CMB anisotropy data estimate of the age of the Universe, $t_0 = 13.7 \pm 0.3$ Gyr at

two standard deviations (Spergel et al. 2007), which assumes the CDM structure formation model. This *WMAP* t_0 estimate is consistent with t_0 estimated from globular cluster observations (Krauss & Chaboyer 2003; Gratton et al. 2003; Imbriani et al. 2004) and from white dwarf star measurements (Hansen et al. 2004). The above values of H_0 and t_0 are consistent with a spatially flat, dark-energy dominated Universe.

As discussed in § 2.3, Peebles & Ratra (2003), Fields & Sarkar (2006), and Steigman (2006), ^{4}He and ^{7}Li abundance measurements favor a higher baryon density than the D abundance measurements and the *WMAP* CMB anisotropy data. (This difference is under active debate.) However, it is remarkable that high-redshift ($z \sim 10^3$) CMB data and low-redshift ($z \lesssim$ few) abundance measurements indicate a very similar baryon density. A summary range of the baryonic density parameter from nucleosynthesis is $\Omega_B = (0.0205 \pm 0.0035)h^{-2}$ at two standard deviations (Fields & Sarkar 2006).

As mentioned above, Type Ia supernovae apparent magnitudes as a function of redshift may be used to constrain the cosmological model. See Peebles & Ratra (2003, § IV.B.4) for a summary of this test. Riess et al. (1998) and Perlmutter et al. (1999) provided initial constraints on a cosmological constant from this test, and Podariu & Ratra (2000) and Waga & Frieman (2000) generalized the method to constrain scalar-field dark energy. Developments may be traced back from Wang & Tegmark (2005), Clocchiatti et al. (2006), Astier et al. (2006), Riess et al. (2007), Nesseris & Perivolaropoulos (2005), Jassal et al. (2006), and Barger et al. (2007). Proposed satellite experiments are under active discussion and should result in tight constraints on dark energy and its evolution. See Podariu et al. (2001), Perlmutter et al. (2006), and Réfrégier et al. (2006) for developments in this area.

The angular size of objects (e.g., quasars, compact radio sources, radio galaxies) as a function of redshift provides another cosmological test. These observations are not as numerous as the supernovae, so this test is much less constraining, but the results are consistent with those from the SNeIa apparent magnitude test. Developments may be traced back through Chen & Ratra(2003a) and Podariu et al. (2003). Daly & Djorgovski (2006) describe a way of combining the apparent magnitude and angular size data to more tightly constrain cosmological parameters.

"Strong" gravitational lensing, by a foreground galaxy or cluster of galaxies, produces multiple images of a background radio source. The statistics of strong lensing may be used to constrain the cosmological model. Fukugita et al. (1990) and Turner (1990) have noted that for low nonrelativistic matter-density the predicted lensing rate is significantly larger in a cosmological-constant dominated spatially flat model than in an open model. The scalar-field dark-energy case is discussed in Ratra & Quillen (1992) and lies between these two limits. For reviews of the test see Peebles & Ratra (2003, § IV.B.6) and Kochanek (2006). Recent developments may be traced back from Fedeli & Bartelmann (2007). Cosmological constraints from the CLASS gravitational lens statistics data are discussed in Chae et al. (2002, 2004), and Alcaniz et al. (2005). These constraints are consistent with those derived from the supernova apparent magnitude data, but are not as restrictive.

Galaxy motions respond to fluctuations in the gravitational potential, and, thus, peculiar velocities of galaxies may be used to estimate the nonrelativistic matter-density parameter $\Omega_{\rm M}$ (as discussed in §§ 4 and 6.5 above and in Peebles 1999 and Peebles & Ratra 2003, § IV.B.7) by comparing the pattern of flows with maps of the galaxy distribution. Note that peculiar velocities are not sensitive to a homogeneously distributed mass-energy component. For a summary of recent results from the literature, see Pike & Hudson (2005). Measurements of the anisotropy of the redshift-space galaxy distribution that is produced by peculiar velocities also yield estimates of the matter-density (Kaiser 1987; Hamilton 1998). Most methods measure this anisotropy in the galaxy autocorrelation or power spectrum (see, e.g., Tinker et al. 2006). Recent analyses include Hawkins et al. (2003) and da Ângela et al. (2005) from the 2dFGRS and 2QZ surveys. Also of interest are clustering analyses of the SDSS that explicitly take into account this redshift-space anisotropy either by using the predicted distortions when constructing eigenmodes (Pope et al. 2004) or by constructing modes that are sensitive to radial vs. angular fluctuations (Tegmark et al. 2004b).

A median statistics analysis of density estimates from peculiar velocity measurements and a variety of other data indicates that the nonrelativistic matter-density parameter lies in the range $0.2 \lesssim \Omega_0 \lesssim 0.35$ at two standard deviations (Chen & Ratra 2003b). This is consistent with estimates based on other data, e.g., the *WMAP* CMB data result in a very similar range (Spergel et al. 2007).

"Weak" gravitational lensing (which mildly distorts the images of background objects), in combination with other data, should soon provide tight constraints on the nonrelativistic matter-density parameter. For reviews of weak lensing see Réfrégier (2003), Schneider (2006), and Munshi et al. (2006). See Schimd et al. (2007), Hetterscheidt et al. (2007), and Kitching et al. (2007) for recent developments. Weak gravitational lensing also provides evidence for dark matter (see, e.g., Clowe et al. 2006; Massey et al. 2007).

Rich clusters of galaxies are thought to have originated from volumes large enough to have fairly sampled both the baryons and the dark matter. In conjunction with the nucleosynthesis estimate of the baryonic mass density parameter, the rich cluster estimate of the ratio of baryonic and nonrelativistic (including baryonic) matter—the cluster baryon fraction—provides an estimate of the nonrelativistic matter-density parameter (White et al. 1993; Fukugita et al. 1998). Estimates of $\Omega_{\rm M}$ from this test are in the range listed above. A promising method for measuring the cluster baryonic gas mass fraction uses the Sunyaev-Zel'dovich effect (Carlstrom et al. 2002).

An extension of this cluster test makes use of measurements of the rich cluster baryon mass fraction as a function of redshift. For relaxed rich clusters (not those in the process of collapsing) the baryon fraction should be independent of redshift. The cluster baryon fraction depends on the angular diameter distance (Sasaki 1996; Pen 1997), so the correct cosmological model places clusters at the right angular diameter distances to ensure that the cluster baryon mass fraction is independent of redshift. This test provides a fairly restrictive constraint on $\Omega_{\rm M}$, consistent with the range above; developments may be traced back through Allen et al. (2004), Chen & Ratra (2004), Kravtsov et al. (2005), and Chang et al. (2006). When combined with complementary cosmological data, especially the restrictive SNeIa data, the cluster baryon mass fraction versus redshift data provide tight constraints on the cosmological model, favoring a cosmological constant but not yet ruling out slowly varying dark energy (Rapetti et al. 2005; Alcaniz & Zhu 2005; Wilson et al. 2006).

The number density of rich clusters of galaxies as a function of cluster mass, both at the present epoch and as a function of redshift, constrains the amplitude of mass fluctuations and the nonrelativistic matter-density parameter (see § IV.B.9 of Peebles & Ratra 2003 and references therein). Current cluster data favor a matter-density parameter in the range discussed above (Rosati et al. 2002; Voit 2005; Younger et al. 2005; Borgani 2006).

The rate at which large-scale structure forms could eventually provide another direct test of the cosmological model. The cosmological constant model is discussed in Peebles (1984) and some of the more recent textbooks listed below. The scalar-field dark-energy model is not as tractable; developments may be traced from Mainini et al. (2003), Mota & van de Bruck (2004), and Maio et al. (2006).

Measurements of CMB temperature and polarization anisotropies (see § 5.2 above and § IV.B.11 of Peebles & Ratra 2003) provide some of the strongest constraints on several cosmological model parameters. These constraints depend on the assumed structure formation model. Current constraints are usually based on the CDM model (or some variant of it). As discussed in § 5.2, the three-year *WMAP* data (Hinshaw et al. 2007) provide state-of-the-art constraints (Spergel et al. 2007).

Data on the large-scale power spectrum (or correlation function) of galaxies complement the CMB measurements by connecting the inhomogeneities observed at redshift $z \sim 10^3$ in the CMB to fluctuations in galaxy density close to $z = 0$, and by relating fluctuations in gravitating matter to fluctuations in luminous matter (which is an additional complication). For a recent discussion of the galaxy power spectrum, see Percival et al. (2007), from which earlier developments may be traced. It is a remarkable success of the current cosmological model that it succeeds in providing a reasonable fit to both sets of data. The combination of *WMAP* data with clustering measurements from SDSS or the 2dFGRS reduces several of the parameter uncertainties. For recent examples of such analyses, see Tegmark et al. (2004a) and Doran et al. (2007a).

The peak of the galaxy power spectrum reflects the Hubble length at matter-radiation equality and so constrains $\Omega_{\rm M}h$. The overall shape of the spectrum is sensitive to the densities of the different matter components (e.g., neutrinos would cause damping on small scales) and the density of dark energy. The same physics that leads to acoustic peaks in the CMB anisotropy causes oscillations in the galaxy power spectrum—or a single peak in the correlation function. Eisenstein et al. (2005) report a three standard deviation detection of this "baryon acoustic oscillation" peak at $\sim 100h^{-1}$ Mpc in the correlation function of luminous red galaxies (LRGs) measured in the SDSS. The resulting measurement of $\Omega_{\rm M}$ is independent of and consistent with other low-redshift measurements and with the high-redshift *WMAP* result. This is remarkable given the widely different redshifts probed (LRGs probe $z = 0.35$) and notable because possible systematics are different. For discussions of the efficacy of future measurements of the baryon acoustic oscillation peak to constrain dark energy, see Wang (2006), McDonald & Eisenstein (2006), and Doran et al. (2007b). For constraints from a joint analysis of these data with supernovae and CMB anisotropy data, see Wang & Mukherjee (2006).

Tegmark et al. (2006) include a nice description of how the large-scale galaxy power spectrum provides independent measurement of $\Omega_{\rm M}$ and $\Omega_{\rm B}$, which breaks several parameter degeneracies and thereby decreases uncertainties on $\Omega_{\rm M}, h$ and t_0. A combined $WMAP+$ SDSS analysis reduces uncertainties on the matter-density, neutrino density, and tensor-to-scalar ratio by roughly a factor of 2. See Sánchez et al. (2006) for an analysis of the 2dFGRS large-scale structure data in conjunction with CMB measurements.

Measurements of the clustering of Lyman-α forest clouds complement larger-scale constraints, such as those from the CMB and large-scale structure, by probing the power spectrum of fluctuations on smaller scales (McDonald et al. 2005). Combining observations of 3000 SDSS Lyman-α forest cloud spectra with other data, Seljak et al. (2006) constrain possible variation with the length scale of the spectral index of the primordial power spectrum and find that Lyman-α cloud clustering may indicate a slightly higher power spectrum normalization, σ_8 (the fractional mass density inhomogeneity smoothed over $8h^{-1}$ Mpc), than do the *WMAP* data alone, or the *WMAP* data combined with large-scale structure measurements.

The presence of dark energy or nonzero spatial curvature causes time evolution of gravitational potentials as CMB photons traverse the Universe from their "emission" at $z \sim 10^3$ to today. The resulting net redshifts or blueshifts of photons cause extra CMB anisotropy, known as the Integrated Sachs-Wolfe (ISW) contribution. This contribution has been detected by cross-correlation of CMB anisotropy and large-scale structure data. The resulting constraints on dark energy are consistent with the model discussed above (and references cited in Boughn

& Crittenden 2005; Gaztañaga et al. 2006). In principle, measurements of the ISW effect at different redshifts can constrain the dark-energy model. Pogosian (2006) discusses recent developments and the potential of future ISW measurements.

7.3. Cosmic Complementarity: Combining Measurements

The plethora of observational constraints on cosmological parameters has spawned interest in statistical methods for combining these constraints. Lewis & Bridle (2002), Verde et al. (2003), and Tegmark et al. (2004a) have discussed statistical methods employed in some of the recent analyses described above. Use of such advanced statistical techniques is important because of the growing number of parameters in current models and possible degeneracies between them in fitting the observational data. Developments may be traced back through Alam et al. (2007), Zhang et al. (2007), Zhao et al. (2007), Davis et al. (2007), Wright (2007), and Kurek & Szydłowski (2007).

To describe large-scale features of the Universe (including CMB anisotropy measured by *WMAP* and some smaller-angular-scale experiments, large-scale structure in the galaxy distribution, and the SNeIa luminosity-distance relation) the simplest version of the "power-law-spectrum spatially flat ΛCDM model" requires fitting no fewer than six parameters (Spergel et al. 2007): nonrelativistic matter-density parameter $\Omega_{\rm M}$, baryon density parameter $\Omega_{\rm B}$, Hubble constant H_0, amplitude of fluctuations σ_8, optical depth to reionization τ, and scalar perturbation index n. This model assumes that the primordial fluctuations are Gaussian random phase and adiabatic. As suggested by its name, this model further assumes that the primordial fluctuation spectrum is a power law (running power-spectral index independent of scale $dn/d\ln k = 0$), the Universe is flat ($\Omega_{\rm K} = 0$), the bulk of the matter-density is CDM ($\Omega_{\rm CDM} = \Omega_{\rm M} - \Omega_{\rm B}$) with no contribution from hot dark matter (neutrino density $\Omega_\nu = 0$), and that dark energy in the form of a cosmological constant comprises the balance of the mass-energy density ($\Omega_\Lambda = 1 - \Omega_{\rm M}$). Of course, constraints on this model assume the validity of the CDM structure formation model.

Combinations of observations provide improved parameter constraints, typically by breaking parameter degeneracies. For example, the constraints from *WMAP* data alone are relatively weak for H_0, Ω_Λ, and $\Omega_{\rm K}$. Other measurements such as from SNeIa or galaxy clusters are needed to break the degeneracy between $\Omega_{\rm K}$ and Ω_Λ, which lies approximately along $\Omega_{\rm K} \approx -0.3 + 0.4\Omega_\Lambda$. The degeneracy between $\Omega_{\rm M}$ and σ_8 is broken by including weak lensing and cluster measurements. The degeneracy between $\Omega_{\rm M}$ and H_0 can be removed, of course, by including a constraint on H_0. As a result, including H_0 data restricts the geometry to be very close to flat. A caveat regarding this last conclusion is that it assumes that the dark-energy density does not evolve.

CDM-model-dependent clustering limits on baryon density ($\Omega_{\rm B} = (0.0222 \pm 0.0014)h^{-2}$ from *WMAP* and SDSS data combined at 95% confidence; Tegmark et al. 2006) are now better than those from light element abundance data (because of the tension between the ^{4}He and ^{7}Li data and the D data). It is important that the galaxy observations complement the CMB data in such a way as to lessen reliance on the assumptions stated above for the power-law flat ΛCDM model. If the SDSS LRG $P(k)$ measurement is combined with *WMAP* data, then several of the prior assumptions used in the *WMAP*-alone analysis ($\Omega_{\rm K} = 0$, $\Omega_\nu = 0$, no running of the spectral index n of scalar fluctuations, no inflationary gravity waves, no dark-energy temporal evolution) are not important. A major reason for this is the sensitivity of the SDSS LRG $P(k)$ to the baryon acoustic scale, which sets a "standard ruler" at low redshift.

The SNeIa observations are a powerful complement to CMB anisotropy measurements because the degeneracy in $\Omega_{\rm M}$ versus Ω_Λ for SNeIa measurements is almost orthogonal to that of the CMB. Without any assumption about the value of the Hubble constant but assuming that the dark energy does not evolve, combining SNeIa and CMB anisotropy data clearly favors nearly flat cosmologies. On the other hand, assuming the Universe is spatially flat, combined SNeIa and cluster baryon fraction data favors dark energy that does not evolve—a cosmological constant—see Rapetti et al. (2005), Alcaniz & Zhu (2005), and Wilson et al. (2006).

The bottom line is that statistical analyses of these complementary observations strongly support the flat ΛCDM cosmological model. It is remarkable that many of the key parameters are now known to better than 10%. However, several weaknesses remain, as discussed in the following, and final, section of this review. Time and lots of hard work will tell if these weaknesses are simply details to be cleaned up, or if they reveal genuine failings of the model, the pursuit of which will lead to a deeper understanding of physics and/or astronomy. It is worth recalling that, at the beginning of the previous century just before Einstein's burst of 1905 papers, it was thought by most physicists that classical physics fit the data pretty well.

8. OPEN QUESTIONS AND MISSING LINKS

We conclude this review by emphasizing that cosmology is by no means "solved." Here we list some outstanding questions, which we do not prioritize, although the first two questions are certainly paramount (What is most of the Universe made of?). It may interest the reader to compare this discussion of outstanding problems in cosmology to those discussed in 1996 (Turok 1997). Recent discussions of key questions, with regard to funding for answering such questions, may be found in reports of the National Research Council (2001, 2003).

8.1. What is "Dark Energy"?

As discussed in §§ 4 and 7, there is strong evidence that the dominant component of mass-energy is in the form of something like Einstein's cosmological constant. In detail, does the dark energy vary with space or time? Data so far are consistent

with a cosmological constant with no spatial or temporal evolution, but the constraints do not strongly exclude other possibilities. This uncertainty is complemented by the relatively weak direct evidence for a spatially flat universe; as Wright (2006), Tegmark et al. (2006), and Wang & Mukherjee (2007) point out, it is incorrect to assume $\Omega_K = 0$ when constraining the dark-energy time dependence, because observational evidence for spatial flatness assumes that the dark energy does not evolve.

More precisely, dark energy is often described by the XCDM parameterization, where it is assumed to be a fluid with pressure $p_X = \omega_X \rho_X$, where ρ_X is the energy density and ω_X is time independent and negative but not necessarily -1 as in the Λ CDM model. This is an inaccurate parameterization of dark energy; see Ratra (1991) for a discussion of the scalar-field case. In addition, dark energy and dark matter are coupled in some models now under discussion, so this also needs to be accounted for when comparing data and models; see Amendola et al. (2007), Bonometto et al. (2007), Guo et al. (2007), and Balbi et al. (2007) for recent discussions.

On the astronomy side, the evidence is not ironclad; for example, inference of the presence of dark energy from CMB anisotropy data relies on the CDM structure formation model and the SNeIa redshift-magnitude results require extraordinary nearly standard candle-like behavior of the objects. Thus, work remains to be done to measure (or reject) dark-energy spatial or temporal variation and to shore up the observational methods already in use.

With tighter observational constraints on "dark energy," one might hope to be guided to a more fundamental model for this construct. At present, dark energy (as well as dark matter) appears to be somewhat disconnected from the rest of physics.

8.2. What Is Dark Matter?

Astronomical observations currently constrain most of the gravitating matter to be cold (small primeval free-streaming velocity) and weakly interacting. Direct detection would be more satisfying and this probably falls to laboratory physicists to pursue. The Large Hadron Collider (LHC) may produce evidence for the supersymmetric sector that provides some of the most-discussed current options for the culprit. As mentioned in the previous question, some models allow for coupling between the dark matter and dark energy. On the astronomy side, observations may provide further clues and, perhaps already do; there are suggestions of problems with "pure" CDM from the properties of dwarf galaxies and galactic nuclear density profiles. Better understanding of the complex astrophysics that connect luminous (or, at least, directly detectable) matter to dark matter will improve such constraints.

8.3. What Are the Masses of the Neutrinos?

In contrast to various proposed candidates for the more dominant "cold" component of dark matter, we know that neutrinos exist. While there are indications from underground experiments of nonzero neutrino mass (Eguchi et al. 2003) and the cosmological tests discussed above yield upper bounds on the sum of masses of all light neutrino species, there has yet to be a detection of the effect of neutrinos on structure formation. A highly model-dependent analysis of Lyman-α forest clustering (Seljak et al. 2006) results in an upper bound of $\sum m_\nu < 0.17$ eV (95% confidence; the sum is over light neutrino species).

8.4. Are Constraints on Baryon Density Consistent?

Using the standard theory for nucleosynthesis to constrain the baryon density from observations of light element abundances, measurements of ^{4}He and ^{7}Li imply a higher baryon density than do D measurements, see §§ 2.3 and 7.2 and Fields & Sarkar (2006) and Steigman (2006). Constraints on the baryon density from *WMAP* CMB anisotropy data are consistent with that from the D abundance measurements. It is possible that more and better data will resolve this discrepancy. On the other hand, this might be an indication of new physics beyond the standard model.

8.5. When and How Was the Baryon Excess Generated?

Matter is far more common than antimatter. It is not yet clear how this came to be. One much-discussed option is that grand unification at a relatively high temperature is responsible for the excess. An alternate possibility is that the matter excess was generated at much lower temperature during the electroweak phase transition.

8.6. What is the Topology of Space?

The observational constraints we have reviewed are local; they do not constrain the global topology of space. On the largest observable scales, CMB anisotropy data may be used to constrain models for the topology of space (see, e.g., Key et al. 2007 and references cited therein). Current data do not indicate a real need for going beyond the simplest spatially flat Euclidean space with trivial topology.

8.7. What Are the Initial Seeds for Structure Formation?

The exact nature of the primordial fluctuations is still uncertain. The currently favored explanation posits an inflationary epoch that precedes the conventional Big Bang era (see § 3). The simplest inflation models produce nearly scale-invariant adiabatic perturbations. A key constraint on inflation models is the slope of the primordial spectrum; *WMAP* data (Spergel et al. 2007) suggest a deviation from the scale-invariant $n = 1$ value, but this is not yet well measured. At present, the most promising method for observationally probing this early epoch is through detection of (the scale-invariant spectrum of) inflationary gravity waves predicted in a number of inflation

models. Detection of these waves or their effects (e.g., measuring the ratio of tensor-to-scalar fluctuations via CMB anisotropy data), would constrain models for inflation; however, nondetection would not rule out inflation because there are simple inflation models without significant gravity waves.

Another critical area for studying the initial fluctuations regards the possibility of non-Gaussian perturbations or isocurvature (rather than adiabatic) perturbations. The evidence indicates that these are subdominant, but that does not exclude a nonvanishing and interesting contribution.

Some models of inflation also predict primordial magnetic field fluctuations. These can have effects in the low-redshift Universe, including on the CMB anisotropy. Observational detection of some of these effects will place interesting constraints on inflation.

8.8. Did the Early Universe Inflate and Reheat?

Probably (although we would not be astonished if the answer turned out to be no). With tighter observational constraints on the fossil fluctuations generated by quantum mechanics during inflation one might hope to be guided to a more fundamental model of inflation. At present, inflation is more of a phenomenological construct; an observationally consistent, more fundamental model of inflation could guide the development of very high-energy physics. This would be a major development. Another pressing need is to have a more precise model for the end of inflation, when the Universe reheats and matter and radiation are generated. It is possible that the matter excess is generated during this reheating transition.

8.9. When, How, and What Were the First Structures Formed?

Discovery of evidence for the epoch of reionization, from observations of absorption line systems toward high-redshift quasars and the polarization anisotropy of the CMB, has prompted intense interest, both theoretical and observational, in studying formation of the first objects. See § 5.3 above.

8.10. How Do Baryons Light Up Galaxies and What Is Their Connection to Mass?

Carrying on from the previous question, the details of the process of turning this most familiar component of mass-energy into stars and related parts of galaxies remains poorly understood. Or so it seems when compared with the much easier task of predicting how collisionless dark matter clusters in a Universe dominated by dark matter and dark energy. Important problems include the effects of "feedback" from star formation and active galactic nuclei, cosmic reionization, radiative transfer, and the effect of baryons on halo profiles. High-resolution hydrodynamic simulations are getting better, but even Moore's law will not help much in the very near future (see comment in Gott et al. 2006). Solving these problems is critical, not only for understanding galaxy formation, but also for using galaxies—the "atoms of cosmology"—as a probe of the properties of dark matter and dark energy.

Clues to the relationship between mass and light and, therefore, strong constraints on models of galaxy formation include the detailed dependence of galaxy properties on environment. Outstanding puzzles include the observation that, while galaxy morphology and luminosity strongly vary with environment, the properties of early-type (elliptical and S0) galaxies (particularly their colors) are remarkably insensitive to environment (Park et al. 2007).

8.11. How Do Galaxies and Black Holes Coevolve?

It is now clear that nearly every sufficiently massive galaxy harbors a supermassive black hole in its core. The masses of the central supermassive black holes are found to correlate strongly with properties of the host galaxy, including bulge velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000). Thus, galaxy formation and the formation and feeding of black holes are intimately related (see, e.g., Silk & Rees 1998; Kauffmann & Haehnelt 2000; Begelman & Nath 2005).

8.12. Does the Gaussian, Adiabatic CDM Structure Formation Model Have a Real Flaw?

This model works quite well on large scales. However, on small scales it appears to have too much power at low redshift (excessively cuspy halo cores, excessively large galactic central densities, and too many low-mass satellites of massive galaxies). Modifications of the power spectrum to alleviate this excess small-scale power cause too little power at high redshift and, thus, delay formation of clusters, galaxies, and Lyman-α clouds. Definitive resolution of this issue will require more and better observational data as well as improved theoretical modeling. If the CDM structure formation model is found to be inadequate, this might have significant implications for a number of cosmological tests that assume the validity of this model.

8.13. Is the Low Quadrupole Moment of the CMB Anisotropy a Problem for Flat ΛCDM?

The small amplitude of the quadrupole moment observed by *COBE* persists in the *WMAP* observations even after many rounds of reanalysis of possible foreground contributions (see Park et al. 2007 and references cited therein). Although one cannot, by definition, rule out the possibility that it is simply a statistical fluke (with significance of about 95% in flat ΛCDM), this anomaly inspires searches for alternative models, including multiply connected Universes (see above).

8.14. Are the Largest Observed Structures a Problem for Flat ΛCDM?

The largest superclusters, e.g., the "Sloan Great Wall" (Gott et al. 2005), seen in galaxy redshift surveys are not reproduced

by simulations of the concordance flat ΛCDM cosmology (Einasto et al. 2006). Perhaps we need larger simulations (see discussion in Gott et al. 2006) or better understanding of how galaxies trace mass.

8.15. Why Do We Live Just Now?

Because we see the Universe from only one place, at only one time, we must wrestle with questions related to whether or not we (or at least our location) is special.

Peebles (2005) notes the remarkable coincidences that we observe the Universe when (1) it has just begun making a transition from being dominated by matter to being dominated by dark energy, (2) the Milky Way is just running out of gas for forming stars and planetary systems, and (3) galaxies have just become useful tracers of mass. While anthropic arguments have been put forward to answer the question of why we appear to live at a special time in the history of the Universe, a physically motivated answer might be more productive and satisfying. Understanding of the details of structure formation, including conversion of baryons to stars (mentioned above), and constraints on possible evolution of the components of mass-energy in the Universe may provide clues.

Progress in cosmology is likely to come from more and higher-quality observational and simulation data as well as from new ideas. A number of ground-based, space-based, and numerical experiments continue to collect data and new near-future particle physics, cosmology, astronomy, and numerical experiments are eagerly anticipated. It is less straightforward to predict when a significant new idea might emerge.

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