PHYS 305 - Assignment #7

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Develop further our physical intuition about the driven non-linear Duffing model. Start thinking about random numbers. In particular, explore sums of random variables as the limiting process to Brownian motion.

Submission: You will not email this assignment to me, due to the large file sizes. Instead you will copy your data over to the local directory `/home/newton8/thoppe/share` in a directory with your name on it. When complete, email me the name of this directory.

Duffing model

The chaotic one-well and two-well attractors produced by the Duffing oscillator are amazing objects.

- The "movie" showing the buildup of the two-well chaotic attractor showed how the attractor is build "all at once", i.e., the dots appeared everywhere all along. The same is true of the one-well chaotic attractor. Illustrate this fact by making a movie of the chaotic attractor formation. Follow the same approach as done in class for the 2 wells case, except that the 10,000 initial conditions (100x100 grid) in x and v should cover a rectangle $x = [3.0, 3.3]$ and $v = [-4.5, -4.2]$ and and that the GNUPLOT viewing window should be $x = [0.5, 4.5]$ and $v = [-8.0, 6.0]$.

- The building of the chaotic oscillator "all at once" happens as well when the attractor is built from a single trajectory. To illustrate this, make a movie from the graphs of the trajectory (starting at $x = 1.0$, $v = 0.0$ – default in duffing.c) for increasing time spans (i.e., $100T$, $200T$, ... $2500T$).

- The application of the stroboscopic projection to coincide with the period T of the external forcing term is arbitrary. A fraction of the period could be used instead. Generate the chaotic attractor (2000 dots) that would result from choosing the half-period mark for the projection, i.e., recording the x and v values at $t_n = (n + 1/2)T$.

Hint: Save time!!! Use the RK4 Duffing oscillator solution, *duffing.c*, from the WEB as is. Use the stroboscopic projection code, *stroboscopic.c*, as is (in parts 1 and 2) or with minimum changes in part 3..

Follow exactly the same steps as explained on the web pages (and in class) in making the movies of the one-well and two-well chaotic attractor. Use the BASH script *script_movie* as is, or with minimum changes.

Uniform random numbers

Write the code *uniform.cpp* that does the following things:

- Generate a list of 10^5 random integers over the integral $x \in [0...9]$
- Output the bin or histogram counts for each number
- Include a plot of the histogram with a labeled axis. Normalize the plot so that its integral sums to one (make it a proper probability function). Have the plot display the percent deviation for each item from the expected value.

2D random walk

Write the code *2Dwalk_traj.cpp* that does the following things:

- Generate a trajectory of length $N = 10^5$ of a random walk over a 2D grid starting at $(0, 0)$.
- Output and plot this trajectory.

Write the code *2Dwalk_displacement.cpp* that does the following things:

- Generate a trajectory of length N of a random walk over a 2D grid where N is input from the command line.
- Output total displacement from the initial position $\langle r \rangle$.
- Plot $\langle r \rangle$ vs. N over the interval $N = [10^1, .., 10^5]$ using at least 1000 different values of N. Find the best fit line to your data (quadratic, exponential, etc...).

Diffusion Limited Aggeration

Write the code *DLA.cpp* that does the following things:

- Start with a single 'seed' particle at $(0, 0)$.
- New particles enter the system on the boundary of the unit circle one at a time and follow a random walk. Each step the particle takes is in a random direction (ie. the direction is choosen uniformly at random $\theta \in [0...2\pi]$) and a magnitude dr. If at any time the particle exits the unit circle, start over with a new particle. If the particle gets within a distance dr of another seed particle the motion is over and the particle is added to the list of seeds.
- Stop the simulation when 300 seeds are found using a step size $dr = .05$.
- Plot the location of all seeds on a single graph.