PHYS 305 - Assignment #4

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Study a Phase Space Portrait. Write an analysis tool.

Non-Linear Oscillators

Non-linear oscillators derive from the harmonic oscillators, mass-on-a-spring problem, by introducing terms in the force that are non-linear (power $\neq 1$) at large distance. Consider the 1-D motion of a mass subjected to the potential

$$V(x) = \frac{k\ x^2}{2}\ \left(1 - \frac{2}{3}\alpha x\right)$$

The non-linearity is introduced by the α term.

Potential and force (using Maple)

- Define $V(x)$
- Plot $V(x)$ (use $k = 4.0$ and $\alpha = 0.7$) over the range $x = -1.5..3$
- Derive the force field, $F(x) = -\frac{dV}{dx}$
- Plot the force field over the same range using the command `fieldplot`.

Solving Newton equation (C or C++)

Solve for the motion of a particle of mass $m = 1.0$ in the potential above using the ODE solver RK4.

- Start with the code solving the mass-on-the-spring problem
- Adapt the code to the current problem
- Use execution line arguments to specify the initial conditions
- Print t, x, v, E (time, position, velocity and total energy) to $< stdout >$
- Plot the energy as a function of time, check that it is adequately conserved

Phase Space Portrait

Draw a phase space portrait that illustrates the different types of trajectories supported by this potential. Do this by specifying five trajectories of your choice with initial conditions $x(0) = 0.0$ and *negative velocities* $v(0) < 0.0$ of various magnitudes.

- Comment on your choice of trajectories
- Write a shell (bash, tcsh, python etc..) script to generate the various trajectories that appear in the phase space portrait at once, and pipe the data into a file *traj.dat*

Oscillatory Motion – Analysis tool

This potential supports non-linear oscillations. These are characterized by the fact that, contrary to the mass-on-the-spring case, the period of the oscillating motion is not a constant for the various trajectories.

- Plot in one graph $x(t)$ vs t for all the trajectories you generated in the phase space portrait
- Do the same for the velocities versus time
- Comment on these 2 graphs
- Write a program *period.c* or *period.cpp* to read in the data in the file *traj.dat* by piping it in:

  cat traj.dat | ./period
  or

  period traj.dat

  and calculate the periods of the different oscillatory trajectories. Devise the algorithm, describe it in words and implement it in this program.
- Plot the period of these trajectory versus $v(0)$. As a point of comparison, add to your graph the (constant) period of the harmonic oscillator ($\alpha = 0$).

  What is the period of the harmonic oscillator?