

# PHYS 305 - Assignment #5

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*Make sure your name is listed as a comment at the beginning of all your work. Please make sure that every graph and chart you turn in is well-labeled with axes and a title (you will lose points if I have to guess!).*

*Purpose:* Develop a physical intuition about Chaotic scattering. Complete **all** exercises on chaotic scattering in the course web page (reproduced below):

`../Physics-305/Chaotic_Scattering/chaotic_scattering_story/node3.html`

## Chaotic Scattering

Adapt the general Runge-Kutta 4th order program (or write your own) to solve this scattering problem. Use a scaled mass  $m = 1.0$ . Use a time step parameter  $\eta = 0.01$ .

- Have the code compute each trajectory until the distance of the mass from the origin exceeds a distance of 4.1 in scaled units. Choose  $v_0 = 0.15, k = 1.0, b = 0.25$ , and compute the trajectory ( $y(t)$  versus  $x(t)$ ) for  $y_0 = 0.0$  and  $y_0 = 0.2$ . Plot the two trajectories. Plot  $E(t)$  as a check. Use the code `potential.c` as a guide to compute the latter.
- Add to the code a calculation of the scattering angle  $\Phi$  and the scattering time  $T$ . Compute the trajectories for  $y_0 = 0, \dots, 0.8$  in 1000 equal increments. Plot separately the scattering angle and the scattering time for these trajectories versus  $y_0$ . You should find that, for some ranges in  $y_0$ , both  $\phi$  and  $T$  vary smoothly with  $y_0$ . However, you should also find certain irregular regions where both quantities change greatly from one trajectory to the next. This sensitive dependence of the outcome on small changes in the initial conditions is the hallmark of chaos. The behavior of the system in the irregular regions is known as chaotic scattering.
- Choose one of the irregular regions in the previous exercise and “zoom in” on it by successively narrowing the range in  $y_0$  and covering the new range in the same number (1000) of steps. Do you see regular regions within the chaotic band? Continue to zoom in on the new chaotic regions until you have decreased the range of  $y_0$  by a factor of 1000 below that used initially. You should find self-similar structure—the same pattern of regularity and irregularity keeps recurring on smaller and smaller scales.
- Identify some of the impact parameter  $y_0$  values that yield the longest scattering times and plot the corresponding trajectories. You may want to draw small circles centered on the three repeller positions to guide the eyes. See for instance: `draw_circles.c` Better yet is to draw the interception of a plane at the appropriate energy  $E$  with the potential surface. See for instance: `potential_contour.c`

- Can you identify any critical trajectories that seem to separate regular from irregular motion?

## Energy Dependence

The scattering functions, scattering angle and escape time, in reality depend on both the impact parameter and the energy of the incoming particle.

Form two color images ( 1024 x 1024 ) of the scattering angle and escape time, having the incoming kinetic energy and the impact parameter as labels. Use an impact parameter range  $b = 0.0 - 0.8$  and an energy range corresponding to 0.1 to 1.2 the hill height. Modify the code *setup\_scatter.c* to generate initial conditions evenly distributed within these ranges. These are to be fed in the differential equation solver code.