

PHYS 305 - Assignment #2

Due: Friday, January 23rd

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Illustrate fractals. Practice programming and plotting.

Mandelbrot Set

The best known fractal and one of the most complex mathematical structure was discovered by Benoit Mandelrot in 1980. It is known as the Mandelbrot Set. The set is produced by the simple map

$$z_{n+1} = z_n^2 + c$$

where the variable z and parameter c are complex numbers and $z_0 = 0$. This relation can be broken in a *real* and *imaginary* notation, $z = Re(z) + iIm(z)$ and $c = Re(c) + iIm(c)$, to yield

$$Re(z_{n+1}) = Re(z_n)^2 - Im(z_n)^2 + Re(c)$$

$$Im(z_{n+1}) = 2 Re(z_n) Im(z_n) + Im(c)$$

The Mandelbrot Set is the set of points in the complex parameter space c for which the iterated map variable z remains finite for an infinite number of iterations. In practice, the maximum number of iterations is taken to be finite and the set is described on a 2D numerical lattice for C centered onto the origin. The beautiful images of the set that you might have already seen are produced based on the number of iterations of the map that lead to an escape or that reach the maximum allowed.

Google will give you thousands of hits on the Mandelbrot Set. Visit some.

Part A

Write a C (C++) code to generate the Mandelbrot Set

- Set a 2D grid in c . Default the range of c to be $Re(c) = [-1.0, 0.8]$, $Im(c) = [-1.0, 1.0]$ and $N_{real} = 200$, $N_{img} = 180$.
- For each c values on the c grid iterate the map in z starting at $z = (Re(z), Im(z)) = (0, 0)$ for a depth (maximum number) of iterations of 80 or until z escapes to infinity
- The z escape criteria is based on the fact that z will necessarily escape if it reaches the region outside of a circle of radius 2 centered onto the origin $z = 0$
- Output the number of iterations for each value of c in $\langle stdout \rangle$

Part B

- Produce a graph of the Mandelbrot Set as a surface using GNUPLOT
- Produce color map images of the Mandelbrot Set using the the Python based image tool *plot_image.py*

Part C

- Modify your code to specify the image size (c grid size) and the c domain. Produce a few large images – 600 x 500 – of the Mandelbrot Set, over spots you find interesting. Take enlargements of the set in regions that you think are worth visiting, save a few of them and submit them with your assignment.