PHYS 160 - Homework #8

Finite Differences

You are to help your friend in Biology. She is using a finite-difference model for population dynamics. Namely she wants to model the competition between two species, a predator and a prey (or two firms on Wall Street!). The model is specified by

$$\frac{\Delta u1}{\Delta t} = u1\ (a - b\ u2) \tag{1}$$

$$\frac{\Delta u2}{\Delta t} = u2\ (-c + p\ u1) \tag{2}$$

The functions $u1(t)$ and $u2(t)$ correspond to the populations of species #1 and species #2 respectively and t is the time. The constants a, b, c and p are chosen to best model specific species. Note that these equations are similar to the one encountered in solving the *Radioactive Decay* problem.

Using Maple

- Start from the Mid-point solution to the *Radioactive Decay* problem we did in class. You need to modify this solution to solve the model above.
- Specify the model parameters: $a = 0.1$, $b = 0.01$, $c = 0.05$ and $p = 0.004$.
- Define the time grid: $dt = 0.1$, $Nt = 1235$ (# of grid points, use a small number while debugging!) and initial time 0.0
- Use initial values: $u1(0.0) = 20.5$ and $u2(0.0) = 18.0$

Adapt the notation in the worksheet to this problem. For instance use uu1[] and uu2[] to denote the populations. Use rhs_u1 and rhs_u2 for the right hand sides of the equations.

- Plot $u1(t)$ versus time. Label the plot with a title.
- Plot $u2(t)$ versus time. Label the plot with a title.
- Plot $u2(t)$ versus $u1(t)$. This plot is the trajectory. Label the plot with a title.
- Write some comments on these plots and the behavior of the solution.
- What are the time and the values of the populations $u1(t)$ and $u2(t)$ on the last point of the time grid.

Do a Google search on the Lotka-Volterra model. Your friend in Biology certainly did such a search!