Finite Differences

This assignment is to practice the use of finite difference formulae to compute the derivative of a function given numerically on an equally spaced lattice.

Let the numerical lattice be defined by the domain, \(x_{\text{min}}\) and \(x_{\text{max}}\), and the number of equally spaced points, \(N_{\text{grid}}\). The constant spacing between the points on the numerical lattice is given by \(dx = \frac{x_{\text{max}} - x_{\text{min}}}{N_{\text{grid}} - 1}\). The coordinate of the \(i^{\text{th}}\) point is then given by \(x_i = x_{\text{min}} + (i - 1)dx\) with the convention that the first point at \(x = x_{\text{min}}\) is labeled by \(i = 1\). A convenient notation for a function \(f(x)\) evaluated at the \(i^{\text{th}}\) point on the lattice is \(f_i = f(x_i)\).

We saw in class the forward, backward and symmetric forms to compute the first derivative based on the tabulated function values \(f_i\).

\[
\text{slope}_f^i = \frac{f_{i+1} - f_i}{dx} \quad (1)
\]
\[
\text{slope}_b^i = \frac{f_i - f_{i-1}}{dx} \quad (2)
\]
\[
\text{slope}_s^i = \frac{f_{i+1} - f_{i-1}}{2dx} \quad (3)
\]

In this assignment you will use the sample function

\[
f(x) = \frac{1}{2}x^3 + 2x^2 + x - \frac{1}{5} \quad (4)
\]

\[
g(x) = f(x)e^{-x^2} \quad (5)
\]

Using Excel

- Define the x-grid using \(x_{\text{min}} = -3\), \(x_{\text{max}} = 3\) and \(N_{\text{grid}} = 35\)
- Calculate the x-grid
- Calculate \(g(x)\) on this grid
- Plot \(g(x)\)
- Calculate the first derivative of \(g(x)\) via the forward, backward and symmetric forms respectively
- Plot these three approximate derivatives
- Guess (read off the graph) the location of the maxima and minima of \(g(x)\) and the values of \(g(x)\) at those points. Write your answers in a small table in the Excel worksheet.
Using Maple

- Define \( g(x) \) and then \( f(x) \)
- Plot \( g(x) \) in the domain \( x = [-3, 3] \)
- Calculate \( \text{slope}(x) \), the exact (analytical) derivative of \( g(x) \)
- Plot \( \text{slope}(x) \) in the domain \( x = [-3, 3] \)
- Find the location and the function \( g(x) \) values at the maxima and minima of \( g(x) \) in the interval \( x = [-3, 3] \)
- How good (percent error) were your previous estimate of these maxima and minima using Excel?

Back to Excel

- Calculate the (numerically) exact derivative of \( g(x) \) in Excel by typing in by hand the analytical formula for it as generated in Maple
- Plot the first derivative of \( g(x) \) as obtained by the \textit{symmetric} formula and the (numerically) exact one above
- Plot the difference between the first derivative of \( g(x) \) as obtained by the \textit{symmetric} formula and the (numerically) exact one above

Oops – we forgot an extremum! Back to Maple

In reality there is another maximum or minimum somewhere outside of the interval \( x = [-3, 3] \). Use Maple to find the location of this extremum and the \( g(x) \) function value at this particular point.

\textit{Hint}: Find the zeros of \( f(x) \).

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