PHYS 160 - Homework #6

Finite Differences

This assignment is to practice the use of *finite difference* formulae to compute the derivative of a function given numerically on an equally spaced lattice.

Let the numerical lattice be defined by the domain, x_{min} and x_{max}, and the number of equally spaced points, N_{grid}. The constant spacing between the points on the numerical lattice is given by $dx = \frac{x_{max}-x_{min}}{N_{grid}-1}$. The coordinate of the i^{th} point is then given by $x_i = x_{min} + (i-1)dx$ with the convention that the first point at $x = x_{min}$ is labeled by $i = 1$. A convenient notation for a function $f(x)$ evaluated at the i^{th} point on the lattice is $f_i = f(x_i)$.

We saw in class the *forward*, *backward* and *symmetric* forms to compute the first derivative based on the tabulated function values f_i.

$$slope_i^f = \frac{f_{i+1} - f_i}{dx} \tag{1}$$

$$slope_i^b = \frac{f_i - f_{i-1}}{dx} \tag{2}$$

$$slope_i^s = \frac{f_{i+1} - f_{i-1}}{2dx} \tag{3}$$

In this assignment you will use the sample function

$$f(x) = \frac{1}{2}x^3 + 2x^2 + x - \frac{1}{5} \tag{4}$$

$$g(x) = f(x)e^{-x^2} \tag{5}$$

Using Excel

- Define the x-grid using $x_{min} = -3$, $x_{max} = 3$ and $N_{grid} = 35$
- Calculate the x-grid
- Calculate $g(x)$ on this grid
- Plot $g(x)$
- Calculate the first derivative of $g(x)$ via the *forward*, *backward* and *symmetric* forms respectively
- Plot these three approximate derivatives
- Guess (read off the graph) the location of the maxima and minima of $g(x)$ and the values of $g(x)$ at those points. Write your answers in a small table in the Excel worksheet.

Using Maple

- Define $g(x)$ and then $f(x)$
- Plot $g(x)$ in the domain $x = [-3, 3]$
- Calculate $slope(x)$, the exact (analytical) derivative of $g(x)$
- Plot $slope(x)$ in the domain $x = [-3, 3]$
- Find the location and the function $g(x)$ values at the maxima and minima of $g(x)$ in the interval $x = [-3, 3]$
- How good (percent error) were your previous estimate of these maxima and minima using Excel?

Back to Excel

- Calculate the (numerically) exact derivative of $g(x)$ in Excel by typing in by hand the analytical formula for it as generated in Maple
- Plot the first derivative of $g(x)$ as obtained by the *symmetric* formula and the (numerically) exact one above
- Plot the difference between the first derivative of $g(x)$ as obtained by the *symmetric* formula and the (numerically) exact one above

Oops – we forgot an extremum! Back to Maple

In reality there is another maximum or minimum somewhere outside of the interval $x = [-3, 3]$. Use *Maple* to find the location of this extremum and the $g(x)$ function value at this particular point.

Hint: Find the zeros of $f(x)$.