PHYS 160 - Homework #5

Due: Wednesday November 5^{th}

Model Signal

You are to help your scientific friend with her model of an electronic signal. She describes this signal via a *Fourier Series* as

$$signal(N1, N2, t) = \sum_{n=N1}^{N2} term(2n-1, t) \tag{1}$$

where n is the sum index which ranges over the values from $N1$ to $N2$. The variable t is the time. The $term(n, t)$ function is defined as

$$term(n, t) = a(n) sin(2\pi nt/TP) \tag{2}$$

where TP is the *period of the signal* and the amplitude $a(n)$ is

$$a(n) = \frac{4}{\pi}\frac{1}{n} \tag{3}$$

Using Maple

- Define $signal(N1, N2, t)$, $term(n, t)$ and $a(n)$ (*in this order, i.e., eq. 1, 2, and 3*).
- Assign the constant: $TP = 1.125$.
- As a check, plot the signal function, $signal(N1, N2, t)$, with the first 50 terms included in the series starting from the fundamental frequency, $N1 = 1$, over the time domain $t = [0, 2TP]$. Label this plot with a title.

Minima and Maxima in Truncated Signals

A signal can be electronically modified, either on purpose or by accident (poor design or malfunction). In the language of the Fourier Series this corresponds to applying a filter to the signal to cut or modify either the *low* or *high* frequency terms, or both. A filter applies a modification to the signal by multiplying the amplitude $a(n)$ of each term in the Fourier Series by a filtering function that depends on n.

For example, consider the following high frequency filtering function

$$filter(n) = \frac{1}{1 + \exp(n - 9)}$$

It is applied to the Fourier Series via

$$filtered\_signal(N1, N2, t) = \sum_{n=N1}^{N2} filter(2n-1) term(2n-1, t)$$

- Define $filter(n)$
- Define the filtered signal $filtered\_signal(N1, N2, t)$.

- Plot this filtered signal for $N1 = 1$ and $N2 = 15$ over the time domain $t = [0, TP]$. Label this plot with a title.
- Calculate the *slope* function of this filtered signal.
- Plot the *slope* function over the time domain $t = [0, TP]$.
- Find the maxima and minima of the filtered signal over the first half of the cycle of the signal (first half of the period). Compute the times at which these occur as well as the values of the signal.
- Can you find any symmetry in the times and signal values you just found?
- How would you get the minima and maxima of the filtered signal in the *second half* of the signal without having to *fsolve()* for them?

Note: The next exercise, Exercise #3, will be administered on Wednesday, November 5^{th}.