PHYS 160 - Homework #4

Model Signal

You are to help your scientific friend with her model of an electronic signal. She describes this signal via a *Fourier Series* as

$$signal(N1, N2, t) = \sum_{n=N1}^{N2} term(2n-1, t) \tag{1}$$

where n is the sum index which ranges over the values from $N1$ to $N2$. The variable t is the time. The $term(n, t)$ function is defined as

$$term(n, t) = a(n) sin(\pi n t/T0) \tag{2}$$

where $T0$ is a constant and the amplitude $a(n)$ is

$$a(n) = \frac{4}{\pi}\frac{1}{n} \tag{3}$$

Using Maple

- Define $signal(N1, N2, t)$, $term(n, t)$ and $a(n)$ (*in this order, i.e., eq. 1, 2, and 3*)
- Assign the constant: $T0 = 1.125$
- Plot the signal function, $signal(N1, N2, t)$, with the first 150 terms included in the series starting from the fundamental frequency, $N1 = 1$, over the time domain $t = [0, 5]$. Label this plot with a title.

Power Spectrum

The power (energy/time) per frequency intervals carried by a signal modeled via a Fourier Series can be demonstrated to be proportional to the square of the amplitude, $a(n)^2$.

- Plot *point style* $a(n)^2$ as a function of n for the first 15 terms appearing in the Fourier Series of the signal. Label this plot with a title.
  Hint: Form a sequence of points coordinates $[n, a(n)^2]$ using the Maple *seq*() command and then plot this sequence.
- Repeat the plot using a semi-log plot (log scale vertically – linear scale horizontally) to better illustrate the range in values. Label this plot with a title.
  Hint: Look in the *plots* library for a suitable plot command.
- Comment on the advantage of the semi-log plot in this case.

Threshold Values in Truncated Signals

A signal can be electronically modified, either on purpose or by accident (poor design or malfunction). In the language of the Fourier Series this corresponds to applying a filter to the signal to cut or modify either the *low* or *high* frequency terms, or both.

- High frequency filter. Plot the filtered signal function, $signal(N1, N2, t)$, with the first 5 terms included in the series starting from the fundamental frequency, $N1 = 1$, over the time domain $t = [0, 5]$. Label this plot with a title.
- Repeat the plot above with an added horizontal line at 1.0 AND over the time domain $t = [0, 2T0]$
- Find the time interval(s) during which the 5 terms filtered signal is larger than 1.0 within the time domain $t = [0, 2T0]$
- Consider a threshold function: $threshold(t) = -0.5 + 0.15t - 0.05t^2$
  Find the time interval during which the 5 terms filtered signal is smaller than the threshold function within the time domain $t = [0, 2T0]$
- Plot a composite graph of the 5 terms filtered signal function over the time domain $t = [0, 2T0]$ AND the threshold function over the time interval found above.

More on Filters

A filter applies a modification to the signal by multiplying the amplitude $a(n)$ of each term in the Fourier Series by a filtering function that depends on n. For example, consider the following filtering function

$$filter(n) = \frac{1}{1 + \exp(n - 9)}$$

It will be applied to the Fourier Series via

$$filtered\_signal(N1, N2, t) = \sum_{n=N1}^{N2} filter(2n - 1) term(2n - 1, t)$$

- Define $filter(n)$
- Plot $filter(n)$ over a domain $n = [0, 15]$. Label this plot with a title. Note that the function is very close to 1 for small n and dies off quickly for large n.
- Is this a low frequency or a high frequency filter?
- Define the filtered signal $filtered\_signal(N1, N2, t)$
- Plot this filtered signal for $N1 = 1$ and $N2 = 50$ over the time domain $t = [0, 5]$. Label this plot with a title.
- Comment on the differences between this filtered signal and the one in the previous section when the filter resulted in simply cutting off all terms beyond the first 5 terms.