PHYS 115

Contemporary Physics - Spring '07

Rec. Assignment #5

Magnetic induction

Consider a very small magnet near a loop of wire. The wire has closely spaced N=20 turns and a radius of 0.01m. It is oriented such that its normal vector points in the $\hat{x}$ direction. If we can approximate the B field of the small magnet as:

$$B_x \propto \frac{1}{x^3} \tag{1}$$

We have enough information to compute the emf induced onto the wire from Faradays law:

$$emf = -N\frac{d\Phi_M}{dt} \tag{2}$$

Assume that the wire starts off 5m away from the center of the circle and moves with a velocity v(t) in the $\hat{x}$ direction. You will plot the induced emf as a function of time for the interval from x = [-5, -1]m. For the first plot assume that the velocity is a constant. For the next plot, find the velocity, v(t) such that the emf induced is a constant. Plot this and indicate on the graph the equation for v(t).

Now let the magnet be fixed at 5m, but allow it to rotate perpendicular to the normal plane of the wire. We can consider the magnet as a dipole whose magnetic field is proportional to:

$$\vec{B} \propto \frac{1}{r^3}(3(\hat{m} \bullet \hat{r})\hat{r} - \hat{m}) \tag{3}$$

Where $\vec{m}$ is a vector that gives the dipole moment of the magnet:

$$\vec{m} = m(\cos\theta\hat{x} + \sin\theta\hat{y}) \tag{4}$$

Since we are only interested in a qualitative solution, we leave out the strength of the magnet and the messy $\frac{\mu_0}{4\pi}$ constants. You can assume that the magnetic field is constant over the area of the loop. Plot the emf induction for a constant $\omega(t)$. Is there any angular speed for which you have a constant emf? If so, plot the graph and give an equation for $\omega(t)$. If not, please explain why.

* * * For bonus points do not make this assumption that the magnetic field is constant over the area of the loop. Break up the loop into small area elements to calculate the total magnetic flux. If you do this, plot the result for a constant $\omega(t)$ versus the graph obtained above. Is it a reasonable approximation?