PHYS 114

Rec. Assignment #6

February 28, 2007

Gauss's Law - The hard way

You already know the solution to the $\vec{E}$ field of a uniformly charged spherical shell, both inside and out (if not, read through your book again). We want to calculate the $\vec{E}$ around this sphere using numerical integration.

How do you uniformly sample a sphere? In general, I don't know the answer for a discrete number of points, but we can use the physicists favorite trick, the Monte Carlo method. While we can not sample the sphere uniformly, we can generate random points on the surface of the sphere. In the limit of a large number of points, the sum becomes an integration.

- Use this sample code to generate random points along the sphere:

  ```
  def S2_rand(R):
      x=0;y=0;s = 2
      while s > 1:
  ->      x = uniform(-1,1)
  ->      y = uniform(-1,1)
  ->      s = x**2+y**2
      z = 2*s - 1
      s = sqrt( (1-z**2)/s )
      x = x*s; y = y*s
      return [R*x,R*y,R*z]
  ```

  It's a function that takes in a radius and returns out a posistion vector (x,y,z).

- For a given point $\vec{r} = (x, y, z)$ at $\vec{r'} = (x', y', z')$ calculate the electric field, remember that it is a vector quantity!

$$\vec{E}_{\vec{r},\vec{r'}} = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r} - \vec{r'}|^3}(\vec{r} - \vec{r'}) \tag{1}$$

- Now you have to 'integrate' over the whole sphere. Choosing a different point on the sphere each time add up the contribution for N number of points (where A is the surface area of your sphere):

$$\vec{E}_{\vec{r}} \approx \frac{A}{N}\sum_{i=1}^{N} E_{\vec{r},\vec{r}'_i} \tag{2}$$

Use $N = 5000$, $R = 1(m)$, $\sigma = 1(\frac{C}{m^2})$, and like the previous assignment use 'units' of $\frac{1}{\epsilon_0}$, i.e. set the value of $\epsilon_0 = 1$. You may wonder what the value of 'q' is. Remember that if you have a uniform object,

$$\sigma = \frac{Q_{total}}{A_{total}} \tag{3}$$

Now use your program to plot $|\vec{E}|$ along the line y=0, z=0, x=[0,3]. Include this graph along with your code when you turn your assignment in.