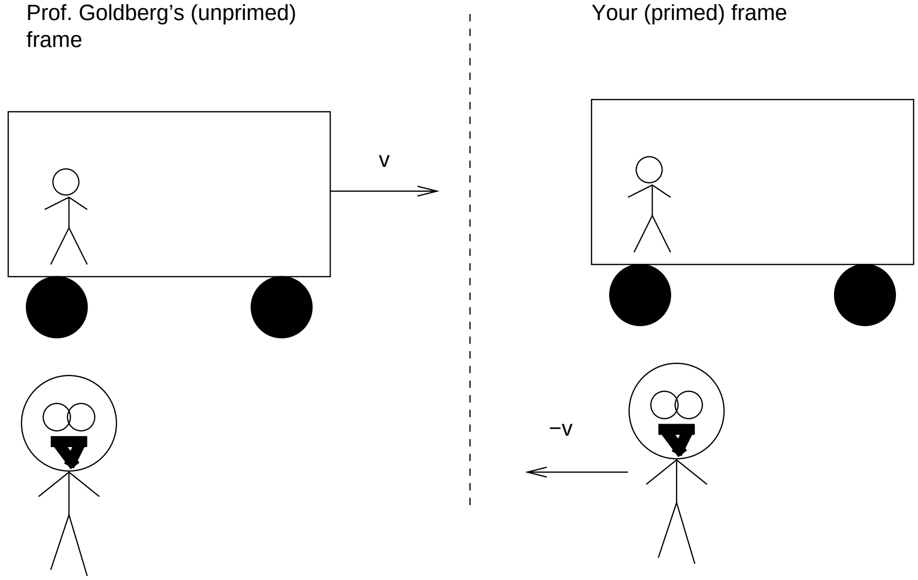
Spacetime Diagrams

We keep seeing the word "relativity" appear in our discussion. To the person on the street, relativity (normally associated with Einstein) elicits the statement "everything is relative," implying that we cannot know anything objectively. In fact, both Einstein and Galileo (who we will also discuss) would have emphatically disagreed with this assessment. We *can* say things objectively – it's just that part of our objective statement needs to include information about how to translate what *I* see, to what you see.

Our standard piece of equipment for this experiment will be a train moving to the right at some known speed, v. You will be on that train, while I will be on the platform.



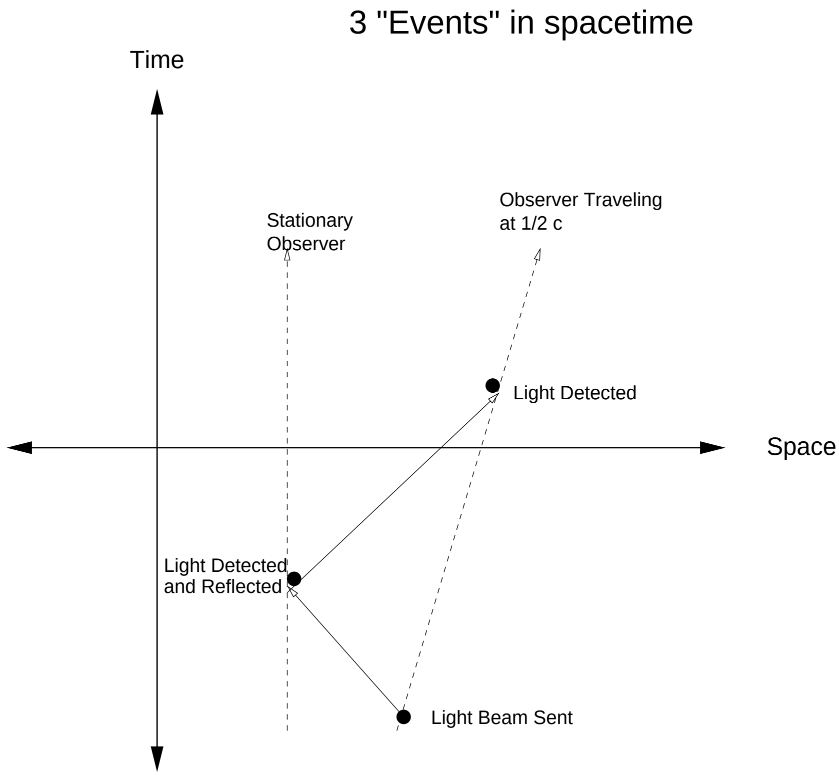
Note that the train is moving at all times. The "Special" in special relativity comes from the fact that there are no accelerations.

Of course, you also know from your own experience that from your vantage, it appears that I am moving backwards at speed, v.

In general, when we talk about different perspectives, we'll refer to the "unprimed" (meaning, in this case, my) or "primed" (meaning your) frame. For example, if I saw a puppy roll over 10 meters in front of me, I'd say, $x = 10m$. However, if you also observed the same "event," but it was *behind* you 5m, we'd say, $x' = -5m$, which is just a shorthand for saying that the position of the puppy rolling over is -5m from the perspective of the person in the train.

I used the word "event" in the previous paragraph. For us, there are only events, and events have a time, and a position. Thus, "The party begins at 9:00pm at the funky disco" indicates an event.

We can imagine a particularly simple 1-d universe full of events, and plot it up on a "space-time" diagram.



In general we tend to scale these things so that 1 unit in the x-direction corresponds to, say, 1 light-second, and 1 unit in the y-direction corresponds to a second, and thus, light travels at 45 degree angles (or -45 deg. if it's traveling to the left). Someone sitting still will move vertically through time (but not through space). And, of course, since no-one can travel faster than light, your "world line" can't ever be shallower than 45 degrees.

On the space-time diagram, I've drawn a light-beam fired from a ship moving half the speed of light toward an observer at rest (Event 1). The beam is detected by the stationary observer (event 2), and re-deflected. Finally, the the lightbeam is detected by the moving observer (event 3).

For each observer, each event has a definite time and position. I have drawn things from the perspective of the stationary observer, since from your own perspective it always *seems* as though you are sitting still.

Galilean Relativity

All of this is just notation leading us up to relativity. And when you think relativity, you no doubt think of Galileo. Well, probably not. But in reality, Galileo's relativity is the one in our everyday experience.

Time: For Galileo, time was an absolute, and thus, all clocks ran at the same rate. So, even though it is legitimate (say if you and I are in different time-zones) for us to measure an event at different "times" – say $t_1 = 0$, and $t'_1 = 3600s$ (if you measure a particular event as being one hour later than may because you never reset your watch from daylight savings), it is true that our watches run at the same *rate*. Thus, if we measure a second event, and I measure it at $t_2 = 10s$, you'll measure it as $t' = 3610s$.

According to Galileo, For both of us:

$$Galileo: \quad \Delta t = t_2 - t_1 = \Delta t' = t_2' - t_1'$$

Space: With space it's a different matter. Consider that for a moving observer, objects, and hence events, will look like they're in different positions at different times. Thus, if a particular event appears to me at some position, x, you'll observe it as

$$x' = x - vt$$

which should accord with your intuition. If you don't believe me, think about how a stationary observer appears as you pull away from the station. As more time passes, his position looks more and more negative.

Finally, we can relate the velocity of an observed object. Consider that velocity is simply defined as:

$$u \equiv \frac{\Delta x}{\Delta t} \tag{1}$$

where we simply observe a moving object at two different times. Using the relations:

$$\Delta x' = \Delta x - v\Delta t \tag{2}$$
$$\Delta t' = \Delta t \tag{3}$$

simple algebra yields:

$$u' = u - v \tag{4}$$

In other words, if you were on a train going 100mph, and I fired a bullet parallel to the train at 150mph, you would observe the bullet to be moving at 50mph.

Finally, note that these equations are completely symmetric. We could just as easily say:

$$\begin{aligned} \Delta x &= \Delta x' + v\Delta t' \\ \Delta t &= \Delta t' \\ u &= u' + v \end{aligned}$$

And thus, there's no way from Newton physics to tell who is moving and who is staying still.

According to Newton, the only thing that matters are accelerations, and consider that a system which obeys Newton's laws in one frame will also satisfy them in any other inertial frame.

No problem, right?

Wrong. Wait until next time.