

Solid State Physics
Homework 3

Physics 715

Fall 2006 Due: **THURSDAY, Oct. 12**

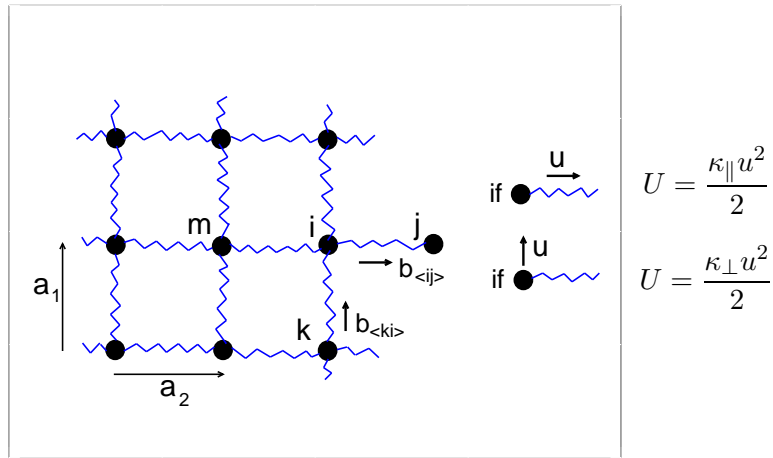
1. Consider a two dimensional square lattice with the lattice constant a . Nearest neighbor atoms of mass m are coupled by harmonic forces with the spring constants κ_{\parallel} and κ_{\perp} :

$$U = U_{\parallel} + U_{\perp}$$

$$U_{\parallel} = (\kappa_{\parallel}/2) \sum_{\langle ij \rangle} \left[\mathbf{b}_{\langle ij \rangle} \cdot (\mathbf{u}_j - \mathbf{u}_i) \right]^2$$

$$U_{\perp} = (\kappa_{\perp}/2) \sum_{\langle ij \rangle} \left[\mathbf{b}_{\langle ij \rangle} \times (\mathbf{u}_j - \mathbf{u}_i) \right]^2$$

where $\mathbf{b}_{\langle ij \rangle}$ is a unit vector in the direction of the bond connecting nearest neighbor points i and j , i.e. it is either $\pm \mathbf{a}_1/a$ or $\pm \mathbf{a}_2/a$. (see Figure).



- (a) Derive phonon dispersion laws for this lattice and verify that $\omega_j(\mathbf{k}) \rightarrow 0$ when $\mathbf{k} \rightarrow 0$.
- (b) Draw the first Brillouin zone (BZ) for this lattice, and verify that the derivative $\partial\omega_j(\mathbf{k})/\partial\mathbf{k}$ is zero in the direction perpendicular to the

BZ boundary (by symmetry, it is sufficient to consider only one segment of the boundary).

(c) In which directions the classification of waves in terms of transverse and perpendicular is exact (**Hint** : do not miss directions along which the two phonon branches have the same energy - one is free to rotate the eigenvector basis at points of degeneracy).

(d) What relation between κ_{\parallel} and κ_{\perp} has to be satisfied for *all* modes to be transverse or longitudinal?

2. Consider a solid consisting of atoms with mass $m = m_{\text{He}} \approx 6.7 \cdot 10^{-24} \text{ g}$. The pair-wise potential holding atoms together is rather weak and a rough estimate is that moving an atom 1 \AA away from equilibrium would increase the energy by 100 K . Estimate the typical vibration frequency of this solid (in degrees Kelvin).

3. **Problem 5.1 in Kittel.**

(a) From the dispersion relation for a monoatomic linear lattice of N atoms with nearest neighbor interactions, $\omega(k) = 2|\sin(ka/2)|\sqrt{\kappa/m}$, show that the density of phonon states is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{\omega_{max}^2 - \omega^2}$$

where ω_{max} is the maximum frequency.

(b) Suppose that an optical phonon branch has the form $\omega(\mathbf{k}) = \omega_0 - A\mathbf{k}^2$, near $\mathbf{k} = 0$ in three dimensions. Show that $D(\omega) = (L/2\pi)^3 (2\pi/A^{3/2})\sqrt{\omega_0 - \omega}$ for $\omega_0 > \omega$ and $D(\omega) = 0$ for $\omega_0 < \omega$.

4. **The first problem is long and will be counted as two.**