Chapter 2b

More on the Momentum Principle

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In this chapter we continue to apply the Momentum Principle to a variety of systems.

The major topics in this chapter are:

- The derivative form of the Momentum Principle, with applications
- Applying the Momentum Principle to multiparticle systems
- Conservation of momentum

2b.1 Derivative form of the Momentum Principle

The forms of the Momentum Principle given so far, $\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t$ and $\vec{p}_f = \vec{p}_i + \vec{F}_{\text{net}}\Delta t$, are particularly useful when we know the momentum at a particular time and want to predict what the momentum will be at a later time. We use these forms in repetitive computer calculations to predict future motion.

Another important form is obtained by dividing by the time interval Δt:

$$\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{\text{net}}$$

Just as we did in finding the instantaneous velocity of an object (see Section 1.7.6), we can find the instantaneous rate of change of momentum by letting the time interval Δt approach zero (an infinitesimal time interval). The ratio of the infinitesimal momentum change (written as $d\vec{p}$) to the infinitesimal time interval (written as dt) is the instantaneous rate of change of the momentum. As the time interval gets very small, $\Delta t \to 0$, the ratio of the momentum change to the time interval approaches the ratio of the infinitesimal quantities

$$\lim_{\Delta t \to 0} \frac{\Delta\vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}$$

and we obtain the following form:

THE MOMENTUM PRINCIPLE (DERIVATIVE FORM)

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$$

In words, "the instantaneous time rate of change of the momentum of an object is equal to the net force acting on the object." Or in calculus terms, since the limit of a ratio of infinitesimal quantities is called a derivative, we can say that "the derivative of the momentum with respect to time is equal to the net force acting on the object."

This form of the Momentum Principle is useful when we know something about the rate of change of the momentum at a particular instant. Knowing the rate of change of momentum, we can use this form of the Momentum Principle to deduce the net force acting on the object, which is numerically equal to the rate of change of momentum. Knowing the net force, we may be able to figure out particular contributions to the net force.

Ex. 2b.1 At a certain instant the z component of the momentum of an object is changing at a rate of 4 kg·m/s per second. At that instant, what is the z component of the net force on the object?

Ex. 2b.2 If an object is sitting motionless, what is the rate of change of its momentum? What is the net force acting on the object?

Ex. 2b.3 If an object is moving with constant momentum $\langle 10, -12, -8 \rangle$ kg·m/s, what is the rate of change of momentum $d\vec{p}/dt$? What is the net force acting on the object?

2b.1.1 An approximate result: *F = ma*

In simple cases the derivative form of the Momentum Principle reduces to a form that may be familiar to you from a previous course. If the mass is constant (the usual situation), and the speed is small enough compared to the speed of light that the momentum is well approximated by $\vec{p} \approx m\vec{v}$, we have this:

$$\frac{d\vec{p}}{dt} \approx \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = \vec{F}_{net} \text{ (nonrelativistic form; constant mass)}$$

The quantity $\frac{d\vec{v}}{dt}$ is called *acceleration*, and is often given the symbol $\vec{a}$.

This form of the Momentum Principle (Newton's second law) says that mass times acceleration (time rate of change of velocity, $d\vec{v}/dt$) is equal to the net force, or in simplified, scalar form "*ma=F*" or "*F=ma*".

The approximate form is not valid in situations where an object's mass isn't constant. One example is a rocket with exhaust gases ejecting out the back; as a result, the rocket has decreasing mass. In such cases the momentum-based formula $d\vec{p}/dt = \vec{F}_{net}$ gives the correct results, whereas the constant-mass formula $md\vec{v}/dt = \vec{F}_{net}$ cannot be used.

The approximate form is also not valid for objects moving at speeds close to the speed of light, because in these circumstances the approximation $\vec{p} \approx m\vec{v}$ is not valid.

When the principle is written in terms of momentum, it is valid even for objects whose mass changes or which are moving at speeds close to the speed of light, as long as we use the relativistically correct definition of momentum. So $d\vec{p}/dt = \vec{F}_{net}$ is the more general and more powerful form.

The extra symbols on $d\vec{p}/dt = \vec{F}_{net}$ compared to the simplified version *F=ma* are important:

- The Momentum Principle is a vector principle, so the arrows over the symbols are extremely important; they remind us that there are really three separate component equations, for *x*, *y*, and *z*.
- It is also important to remember that we have to add up all vector forces to give the "net" force, so we write $\vec{F}_{net}$, not just *F*.

Ex. 2b.4 The velocity of a 80 gram ball changes from $\langle 5, 0, -3 \rangle$ m/s to $\langle 5.02, 0, -3.04 \rangle$ m/s in 0.01 s, due to the gravitational attraction of the Earth and to air resistance. What is the acceleration of the ball? What is the rate of change of momentum of the ball? What is the net force acting on the ball?

2b.2 Momentum not changing

We saw in Chapter 1 that if a system is in uniform motion its momentum is constant, and does not change with time. A special case of uniform motion is a situation in which object is at rest and remains at rest; this situation is called "static equilibrium."

2b.2.1 Static equilibrium

? If the system never moves, what is $\frac{d\vec{p}}{dt}$?

The momentum of the system isn't changing, so the rate of change of the momentum is zero:

$$\frac{d\vec{p}}{dt} = \vec{0} = \langle 0, 0, 0\rangle \text{ (static equilibrium)}$$

? What does this imply about the net force acting on the system?

From the derivative form of the Momentum Principle, $d\vec{p}/dt = \vec{F}_{\text{net}}$, we deduce that the net force acting on a system that never moves must be zero:

$$\vec{F}_{\text{net}} = \vec{0} = \langle 0, 0, 0\rangle \text{ (static equilibrium)}$$

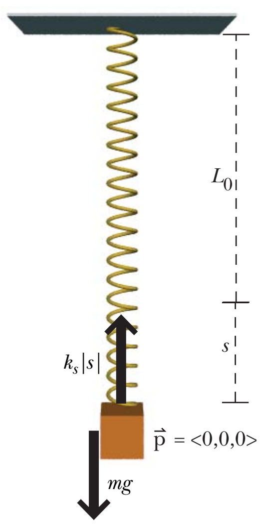


Figure 2b.1 A block hangs motionless from a spring (static equilibrium).

For example, consider a spring whose stiffness is $k_s = 80$ N/m. You suspend the spring vertically and hang a block of mass 0.5 kg from the bottom end. At first the block oscillates up and down but eventually air resistance brings it to a stop, and the block hangs motionless, never moving (Figure 2b.1).

? How much has the spring stretched? Use the Momentum Principle to prove your result rigorously.

Choose the block as the system. If the block no longer moves at all, the rate of change of its momentum $d\vec{p}/dt$ is zero. From the derivative form of the Momentum Principle we conclude that the net force acting on the block must be zero.

? What objects in the surroundings exert forces on the block?

The spring pulls up and the Earth pulls down.

? Apply the derivative form of the Momentum Principle to find the stretch.

The vector sum of these two forces must equal zero:

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}} = \vec{F}_{\text{spring}} + \vec{F}_{\text{Earth}}$$

$$\langle 0, 0, 0\rangle = \langle 0, k_s s, 0\rangle + \langle 0, -mg, 0\rangle = \langle 0, k_s s - mg, 0\rangle$$

$$0 = k_s s - mg \text{ which gives } s = mg/k_s$$

$$s = \frac{(0.5\text{ kg})(9.8\text{ N/kg})}{80\text{ N/m}} = 0.061\text{ m}$$

2b.2.2 Uniform motion

Suppose the spring and hanging mass are on an elevator that goes up at a constant speed of 4 m/s. How does putting the apparatus into uniform motion affect our analysis?

? What is $\frac{d\vec{p}}{dt}$ for the block, which moves upward at 4 m/s?

Now the momentum of the block is not zero, but constant, and $d\vec{p}/dt$ is still zero, which means that the net force $\vec{F}_{net}$ must also be zero. The spring will be stretched 0.061 m, just as when there was no motion. This is an example of the principle of relativity, that physical laws (in this case, the Momentum Principle) work in the same in all reference frames that are in uniform motion (that is, constant velocity) with respect to the cosmic microwave background, which is the stage on which phenomena play themselves out.

2b.2.3 Momentarily at rest *vs.*static equilibrium

Pull the block down a ways and release it, and it oscillates up and down at the end of the spring. During this oscillating motion, whenever the block reaches the bottom of its travel its speed is momentarily zero. At first glance, this situation may seem similar to the case of block hanging at rest which we considered in Section 2b.2.1, but there are important differences.

Figure 2b.2 Just before the block comes to a momentary stop, its momentum is downward.

? At this instant, when the speed is momentarily zero, is the rate of change of the block's momentum zero?

No! The momentum just before hitting bottom points downward (Figure 2b.2), and just after hitting bottom the momentum points upward (Figure 2b.4). The momentum is changing all during this time, and the momentum is zero only momentarily (Figure 2b.3).

Figure 2b.3 The block's momentum is zero only at the instant it reaches its lowest position.

? Apply the derivative form of the Momentum Principle. What can we deduce about the magnitude and direction of the net force on the block at the instant the block is at rest at its lowest position?

Since $d\vec{p}/dt$ is nonzero, the net force must be nonzero as well. Since the initial momentum was downward, and the final momentum is upward, the direction of $d\vec{p}/dt$ must be upward. This implies that the net force is also upward at the instant the block is at rest.

? Does an upward net force make sense physically?

If the net force at the bottom of the motion were zero, the momentum couldn't change from its current value (zero), and the block would stay forever at the bottom point of the oscillation. But that's not what happens. The block slows to a stop but then starts back upwards. In order for the y component of momentum to change from zero to a positive quantity, there must be an upward net force to cause this change.

? At the bottom of the oscillating motion, which force is larger, the force exerted upward by the spring or the force exerted downward by the Earth? How do you know?

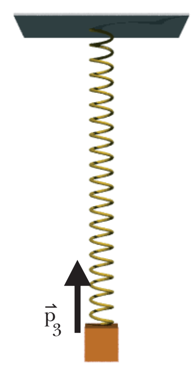


Figure 2b.4 As the block starts back upward, its momentum is upward.

Since the net force must be upward in order to produce the observed change in the momentum, it must be that the spring force is larger than the gravitational force exerted by the Earth. This conclusion is also consistent with the fact that the block when headed downward slows to a stop, which means that the net force must point upward to oppose the downward motion.

? How does the stretch of the spring when the block is momentarily at rest at its lowest point compare to the stretch of the spring in the previous situation, where the block hung motionless in equilibrium?

The stretch of the spring is longer in this case (block momentarily at rest) than it is in the previous case (static equilibrium).

Let's look at this numerically. When a block of 0.5 kg hangs motionless from a spring whose stiffness is 80 N/m, we found that in static equilibrium the spring is stretched 0.061 m (6.1 cm). Now suppose the block is oscillating up and down in such a way that at the bottom of its motion the spring is stretched 0.1 m (10 cm).

> **?** What is the net force when the block's momentum is (momentarily) zero?

The y component of the net force is $F_{\text{net},y} = k_s s - mg = 0$, which gives

$$F_{\text{net},y} = (80 \text{ N/m})(0.1 \text{ m}) - (0.5 \text{ kg})(9.8 \text{ N/kg}) = +3.1 \text{ N}$$

As expected, the net force points upward (positive y component), consistent with the rate of change of momentum being upward.

Avoiding confusion

One of the hardest things to keep straight when first learning about motion and the causes of motion is that being momentarily at rest is not at all like being permanently at rest (static equilibrium). When you encounter a situation where the momentum is zero, always ask yourself whether it is staying zero permanently. If not, the momentum is changing, and the net force is nonzero.

There are many other situations in which the net force on an object is nonzero at the instant when the object's momentum is (momentarily) zero. This happens whenever an object reverses direction. Throw a ball up in the air. When the ball reaches its maximum height, p_y is momentarily zero, but the gravitational force still points downward and makes p_y decrease. If the net force dropped to zero when p_y dropped to zero, the ball would stay motionless at the top of its trajectory!

When a tennis ball hits a wall and rebounds, the momentum of the ball momentarily goes to zero, but a force due to the wall acts on the ball, changing the momentum of the ball to go in the opposite direction. If the force exerted by the wall went to zero when the speed of the ball went to zero, the ball would just remain motionless, squashed against the wall.

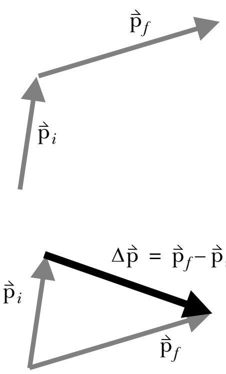


Figure 2b.5 Calculation of $\Delta\vec{p}$.

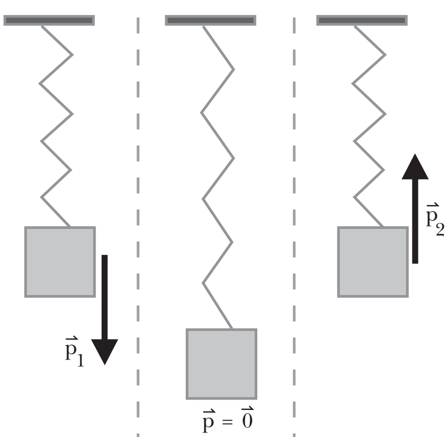


Figure 2b.6 The momentum of a block oscillating on a spring, at an instant before it comes to a momentary rest at the bottom of its path (1) and an instant after it starts back upward (2).

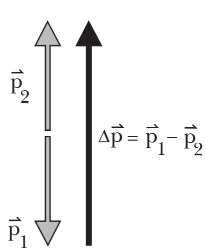


Figure 2b.7 The direction of $d\vec{p}/dt$ is the direction of $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$.

2b.2.4 Finding the rate of change of momentum

In order to apply the derivative form of the Momentum Principle, it is important to be able to decide if $d\vec{p}/dt$, the instantaneous rate of change of momentum, is zero or nonzero, and if it is nonzero, what its direction is. The procedure for finding $d\vec{p}/dt$ is based on the procedure for finding $\Delta\vec{p}$ which you practiced in Chapter 1, and requires that you draw a simple diagram, as shown in Figure 2b.5.

1. Draw two arrows:

   one representing $\vec{p}_i$, the momentum of the object a short time before the instant of interest, and

   a second arrow representing $\vec{p}_f$, the momentum of the object a short time after the instant of interest

2. Graphically find $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ by placing the arrows tail to tail, and drawing the resultant arrow starting at the tail of $\vec{p}_i$ and going to the tip of $\vec{p}_f$

3. This arrow indicates the direction of $\Delta\vec{p}$, which is the same as the direction of $d\vec{p}/dt$, provided the time interval involved is sufficiently small

The diagrams in Figure 2b.6 and Figure 2b.7 show the result of this procedure when applied to the oscillating block on the end of a spring, at the instant it is at rest at the bottom. The direction of $d\vec{p}/dt$ at this instant is upward.

Ex. 2b.5 A block oscillating up and down on a spring comes to the top of its path, where it is momentarily at rest. Follow the procedure outlined above to answer the following questions: At the instant it is at rest, is $d\vec{p}/dt$ zero or nonzero? If nonzero, what is its direction?

Ex. 2b.6 You throw a ball straight up into the air. Follow the procedure outlined above to answer the following questions: At the instant the ball reaches its highest location, is $d\vec{p}/dt$ zero or nonzero? If nonzero, what is its direction?

Ex. 2b.7 A ping pong ball hits a paddle and bounces straight back. While the ball is in contact with the paddle, is $d\vec{p}/dt$ zero or nonzero? If nonzero, what is its direction?

2b.2.5 Example: Hanging block (static equilibrium)

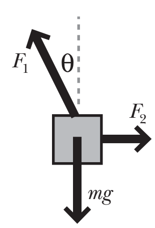
A block of mass m hangs from a string (Figure 2b.8). Then you pull the block to the side by pulling horizontally with a second string. What is the magnitude F_2 of the force you must apply to the second string in order for the first string to be at an angle of θ to the vertical? What is the tension F_1 in the first string? (The “tension” in a string is the force it exerts on another object. Because a string is flexible, the force a string exerts is always in the direction of the string; a string cannot exert a force perpendicular to the string.)

1. Choose a system

System: the block

2. List external objects that interact with the system, with diagram

string 1, string 2, the Earth



3. Apply the Momentum Principle

$$d\vec{p}/dt = \vec{F}_{net}$$
$$\langle 0, 0, 0\rangle = \langle F_2 - F_1\cos\theta, F_1\sin\theta - mg, 0\rangle$$

4. No need to apply the position update formula (no motion)

5. Solve for the unknowns (algebraic manipulation)

$$0 = F_2 - F_1\cos\theta \text{ so } F_1\cos\theta = F_2$$

$$0 = F_1\sin\theta - mg \text{ so } F_1\sin\theta = mg \text{ which gives } F_1 = \frac{mg}{\sin\theta}$$

$$F_2 = F_2\cos\theta = \frac{mg}{\sin\theta}\cos\theta$$

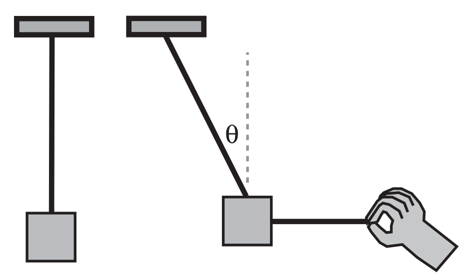


Figure 2b.8 A block hangs from a string. Then you pull the block to the side with another string.

6. Check

We can make some checks on the reasonableness of the results.

If $\theta = 90°$, the first string is vertical, and we expect the tension in this string to be *mg*. Since $\sin(90°) = 1$, we do indeed find $F_1 = mg$. Also, $F_2 = 0$, as it should in this case.

On the other hand, if $\theta = 0$, $\sin(0) = 0$, the first string is horizontal, and $F_2 = mg/0$, which is infinitely large and impossible. This makes sense: if both strings are horizontal, there is no *y* component to support the block.

We could also choose the system of the ball alone, and show that the tension in the vertical portion of the first string must be equal to *mg*, since the net force must be zero (since the ball is constantly at rest).

2b.2.6 Preview of the Angular Momentum Principle

In many static equilibrium situations, the Momentum Principle is not sufficient for carrying out a full analysis and we also need the Angular Momentum Principle, which we will discuss in a later chapter.

For example, consider two children sitting motionless on a seesaw (Figure 2b.9). Choose as the system the two children and the (lightweight) board. For this choice of system, the objects in the surroundings that exert significant (external) forces on the system are the Earth (pulling down) and the support pivot (pushing up). The momentum of the system isn't changing, so the Momentum Principle correctly tells us that the net force must be zero, and the support pivot pushes up with a force equal to the weight of the children (we are neglecting the low-mass board).

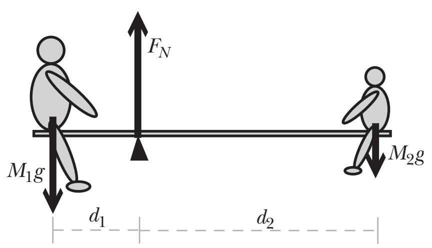


Figure 2b.9 Two children sit on a seesaw in static equilibrium.

However, the Momentum Principle alone doesn't tell us where the children have to sit in order to achieve balance and static equilibrium. The force on a child exerts a twist (the technical term is "torque") about the pivot which tends to make the board turn. The torque associated with the forces on the two children have to add up to zero (one tends to twist the board clockwise, the other counterclockwise). Torque is defined as force times lever arm, and a nonzero net *torque* causes changes in a quantity called angular momentum, just as a nonzero net *force* causes changes in the ordinary momentum we have been studying.

2b.3 Curving motion

Recall from Chapter 1 that the rate of change of momentum along a curving path has a component toward the center of the kissing circle:

PERPENDICULAR COMPONENT OF $d\vec{p}/dt$
FOR MOTION ALONG A CURVING PATH

Perpendicular component of $\frac{d\vec{p}}{dt}$ is given by $\left|\left(\frac{d\vec{p}}{dt}\right)_\perp\right| = \frac{|\vec{v}|}{R}|\vec{p}|$

The direction is toward the center of the kissing circle of radius *R*.

If an object is moving along a curved path, and you know the speed and the radius of curvature of the path, you know $|(d\vec{p}/dt)_\perp|$ and therefore you also know the component of the net force acting toward the center of the kissing circle, since $(d\vec{p}/dt)_\perp = \vec{F}_{\text{net}_\perp}$.

2b.3.1 Example: The Moon around the Earth (curving motion)

The first part of this example was discussed in Chapter 1. The Moon, which has a mass of 7×10^{22} kg, orbits the Earth once every 28 days (a lunar month), following a path which is nearly circular (Figure 2b.10). The distance from the Earth to the Moon is 4×10^{8} m. What is the magnitude of the rate of change of the Moon's momentum? Calculate the gravitational force exerted on the Moon by the Earth, whose mass is 6×10^{24} kg. Compare these two results.

$$|\vec{v}| = \frac{2\pi(4\times10^{8}\ \text{m})}{(28\ \text{days})(24\ \text{hr/day})(60\ \text{min/hr})(60\ \text{s/min})} = 1\times10^{3}\text{m/s}$$

$$|\vec{p}| \approx m|\vec{v}| = (7\times10^{22}\ \text{kg})(1\times10^{3}\text{m/s}) = 7.3\times10^{25}\ \text{kg}\cdot\text{m/s}$$

$$\left|\frac{d\vec{p}}{dt}\right| = \frac{|\vec{v}|}{R}|\vec{p}| = \frac{(1\times10^{3}\text{m/s})}{(4\times10^{8}\ \text{m})}(7.3\times10^{25}\ \text{kg}\cdot\text{m/s}) = 1.8\times10^{20}\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

The Moon's speed is not changing, so $d\vec{p}/dt$ is perpendicular to $\vec{p}$, as shown in Figure 2b.10.

The magnitude of the gravitational force of the Earth on the Moon is

$$\left|\vec{F}_{\text{net}}\right| = G\frac{Mm}{R^2} = \left(6.7\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(6\times10^{24}\ \text{kg})(7\times10^{22}\ \text{kg})}{(4\times10^{8}\ \text{m})^2}$$

$$\left|\vec{F}_{\text{net}}\right| = 1.8\times10^{20}\ \text{N}$$

The net force is equal in magnitude to the observed magnitude of the rate of change of the Moon's momentum. The directions are also the same: toward the center of the kissing circle.

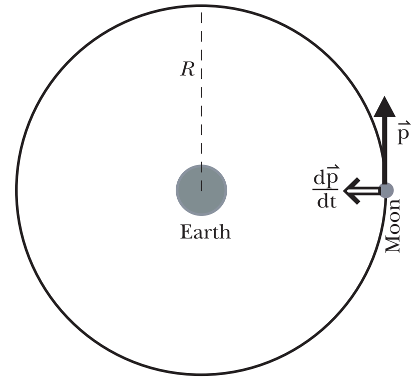


Figure 2b.10 The Moon's orbit around the Earth is nearly circular. Not to scale: the sizes of the Earth and Moon are exaggerated.

This comparison was first made by Newton, who realized that the fall of an apple and the orbit of the Moon were due to the same fundamental cause, the gravitational attraction of the Earth for the apple and for the Moon. Until Newton had this insight no one had made this connection.

Ex. 2b.8 A child of mass 40 kg sits on a wooden horse on a carousel. The wooden horse is 5 meters from the center of the carousel, which completes one revolution every 90 seconds. What is the rate of change of the momentum of the child, both magnitude and direction? What is the net force acting on the child?

Ex. 2b.9 A person rides on a Ferris wheel at constant speed (Figure 2b.11). When the person is at the location shown, what is the direction of the person's rate of change of momentum, $d\vec{p}/dt$? What must be the direction of the net force acting on the person? (The net force is due to the Earth's gravitational pull plus the force of the seat on the person.)

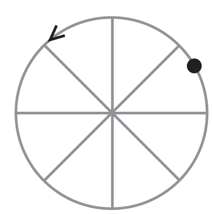


Figure 2b.11 A person rides on a Ferris wheel at constant speed (Exercise 2b.9). When the person is at the location shown, what is the direction of the net force acting on the person?

Ex. 2b.10 The orbit of the Earth around the Sun is approximately circular, and takes one year to complete. The Earth's mass is 6×10^{24} kg, and the distance from the Earth to the Sun is 1.5×10^{11} m. What is the magnitude of the rate of change of the Earth's momentum? What is the direction of the rate of change of the Earth's momentum? What is the magnitude of the gravitational force the Sun (mass 2×10^{30} kg) exerts on the Earth? What is the direction of this force?

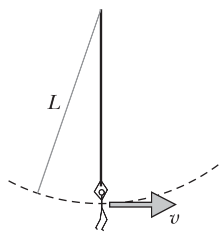


Figure 2b.12 Tarzan swings from a vine. The kissing circle has radius L, the length of the vine.

2b.3.2 Example: Tarzan swings from a vine (curving motion)

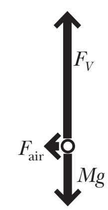
Tarzan swings from a vine, and the radius of his circular motion is L, the length of the vine (Figure 2b.12). We will calculate how strong the vine must be so that at the bottom of the swing (with the vine momentarily vertical) the vine doesn't break. Tarzan's mass is M and his speed at the bottom of the swing is v. Because Tarzan's motion is circular, we know that the component of the rate of change of his momentum that is perpendicular to his motion has magnitude $(|\vec{v}|/L)|\vec{p}|$ and is directed toward the center of the kissing circle (the top of the vine).

1. Choose a system

System: Tarzan

2. List external objects that interact with the system, with diagram

the vine (tension F_V), the Earth, and the air (represent the system by a circle)



3. Apply the Momentum Principle

$$d\vec{p}/dt = \vec{F}_{net}$$

$$\langle 0, \frac{|\vec{v}|}{L}|\vec{p}|, 0\rangle = \langle 0, F_V - Mg, 0\rangle$$

We'll neglect air resistance.

4. No need to apply the position update formula

5. Solve for the unknowns

$$\frac{|\vec{v}|}{L}|\vec{p}| = F_V - Mg \text{ so } F_V = \left(\frac{v}{L}\right)Mv + Mg$$

6. Check

Mg has units of N, and $(v/L)Mv$ has units of (1/s)(kg)(m/s) or kg·m/s^2, which are the units of the rate of change of momentum, which are equivalent to N. So the units check.

Is the result reasonable? The result shows that the vine has to exert an upward force that is greater than Tarzan's weight Mg. That makes sense. If Tarzan were hanging motionless (static equilibrium), the vine would only have to exert an upward force of Mg (net force would be zero). But to make Tarzan's momentum turn from horizontal to upward requires that the net force be upward.

As a numerical example, Tarzan's mass might be 100 kg (about 220 pounds), and the vine might be 10 m long. Suppose his speed at the bottom of the motion is 15 m/s. His weight Mg is 980 N, but the vine has to provide a much larger force:

$$F_V = \frac{(100\ \text{kg})(15\ \text{m/s})^2}{10\ \text{m}} + 980\ \text{N}$$

$$F_V = (2250\ \text{N}) + (980\ \text{N}) = 3230\ \text{N}$$

If the maximum tension this vine can support without breaking is less than 3230 N, Tarzan is in trouble. Note that the vine has to support a force about three times as big as Tarzan's weight.

2b.3.3 The Momentum Principle relates different things

The equation $d\vec{p}/dt = \vec{F}_{net}$ does *not* mean that $d\vec{p}/dt$ and $\vec{F}_{net}$ are the same thing! They are not.

In the analysis of Tarzan and the vine, note carefully that we knew the rate of change of the momentum, $d\vec{p}/dt$, because we knew what kind of curving path Tarzan was following. This result, $d\vec{p}/dt = \langle 0, (v/L)Mv, 0\rangle$, does not involve the strength of the vine nor g, the magnitude of the gravitational field; it just depends on the way Tarzan's motion is curving.

On the other hand, we know that the net force is $\langle 0, F_V - Mg, 0\rangle$ (the vector sum of all the external forces). This expression for the net force doesn't contain anything about how Tarzan is moving.

It is the derivative form of the Momentum Principle which links these two kinds of information together, telling us that the (vector) rate of change of the momentum is numerically equal to the (vector) net force. We set the two vectors equal to each other and solve for the quantity of interest.

The result $|(d\vec{p}/dt)_\perp| = (|\vec{v}|/R)|\vec{p}|$ is essentially a geometrical result, which we proved in Chapter 1 using a geometrical argument. It is important to understand that it doesn't involve anything at all concerning the particular force that is causing the change of direction. For example, the gravitational force law constant G does not appear in this result, nor is there anything about electric charges or electric forces in the result.

Yet the Momentum Principle says that $d\vec{p}/dt$ is equal to $\vec{F}_{net}$. These two quantities are very different in kind. They even have different units: $(\mathrm{kg}\cdot\mathrm{m/s})/\mathrm{s}$ and newtons (although the Momentum Principle indicates that these two physical units must be equivalent in some sense).

The Momentum Principle, $d\vec{p}/dt = \vec{F}_{net}$, is powerful *precisely because it relates two very different things to each other.* If you know something about the net force, you know how rapidly the momentum must be changing. Or if you know how rapidly the momentum is changing, you know what the net force must be. The Momentum Principle relates an *effect* (rate of change of momentum) to its *cause* (net force due to interactions with other objects).

This is very different from a mere definition such as the definition of momentum, $\vec{p} \equiv m\vec{v}/\sqrt{1-(|\vec{v}|/c)^2}$. We introduce the symbol $\vec{p}$ and let it stand for the quantity $m\vec{v}/\sqrt{1-(|\vec{v}|/c)^2}$ as a convenience, but there is no *physics* in this, because there is no difference between what we mean by $\vec{p}$ and what we mean by $m\vec{v}/\sqrt{1-(|\vec{v}|/c)^2}$.

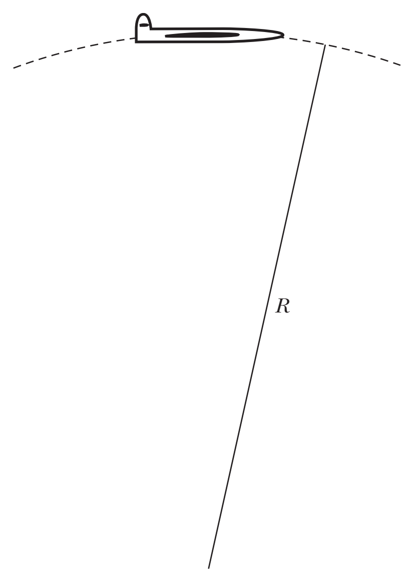


Figure 2b.13 An airplane goes over the top of its path with speed v. The kissing circle has radius R at the top of the path.

2b.3.4 Example: Sitting in an airplane (curving motion)

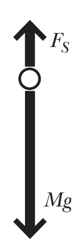
An airplane makes a curving motion at constant speed v as shown in Figure 2b.13, where the radius of the kissing circle at the top of the path is R. Choose a passenger as the system of interest and determine the force that the seat exerts on the passenger's bottom.

1. Choose a system

System: passenger in airplane

2. List external objects that interact with the system, with diagram

the seat and the Earth
(represent the system by a circle)



3. Apply the Momentum Principle

$$d\vec{p}/dt = \vec{F}_{net}$$

$$\langle 0, -\left(\frac{|\vec{v}|}{R}\right)|\vec{p}|, 0\rangle = \langle 0, F_S - Mg, 0\rangle$$

We'll neglect air resistance.

The rate of change of the momentum $d\vec{p}/dt$ as usual points toward the center of the kissing circle, so the y component of $d\vec{p}/dt$ is negative.

4. No need to apply the position update formula

5. Solve for the unknowns

$$-\left(\frac{|\vec{v}|}{R}\right)|\vec{p}| = F_S - Mg \text{ so } F_S = Mg - \left(\frac{v}{R}\right)Mv$$

6. Check

The units check; see previous example on these units.

Let's explore the physical significance of this result. There is an interesting and important minus sign in our result. The upward force of the seat F_S on the passenger's bottom is *less* than the passenger's weight Mg.

? That means that the net force points downward. Does that make sense?

Yes, because the net force should point toward the center of the kissing circle, which is below the airplane.

? But since $F_S = Mg - Mv^2/R$, if the airplane's speed v is big enough, the force exerted by the seat could be zero. Does that make any sense?

Yes, that's the case where the airplane changes direction so quickly that the seat is essentially jerked out from under the passenger, and there is no longer contact between the seat and the passenger. No contact, no force of the seat on the passenger.

In this case, what does the situation look like to the passenger? From the passenger's point of view it is natural to think that the passenger has been thrown upward, away from the seat. But actually the seat has been yanked out from under the passenger, in which case the passenger is falling toward the Earth (being no longer supported by the seat), but changing velocity less rapidly than the airplane and so the ceiling comes closer to the passenger.

In the extreme case, if the airplane's maneuver is very fast, it may seem as though the passenger is "thrown up against the cabin ceiling." In actual fact, however, it is the cabin ceiling that is yanked down and hits the passenger!

You can see the value of wearing a seat belt, to prevent losing contact with the seat. When the airplane yanks the seat downward, the seat belt yanks the passenger downward. This may be uncomfortable, but it is a lot better than having the cabin ceiling hit you hard in the head.

? How would this analysis change if the airplane goes through the bottom of a curve rather than the top (Figure 2b.14)?

Now the center of the kissing circle is above the airplane, so $d\vec{p}/dt$ points upward. That means that the net force must also point upward.

? Which is larger, the force of the seat on the passenger, or the gravitational force of the Earth on the passenger?

The force of the seat is greater than Mg, since the net force is upward. The passenger sinks deeper into the seat. Or to put it another way, the airplane yanks the seat up harder against the passenger's bottom, squeezing the seat against the passenger.

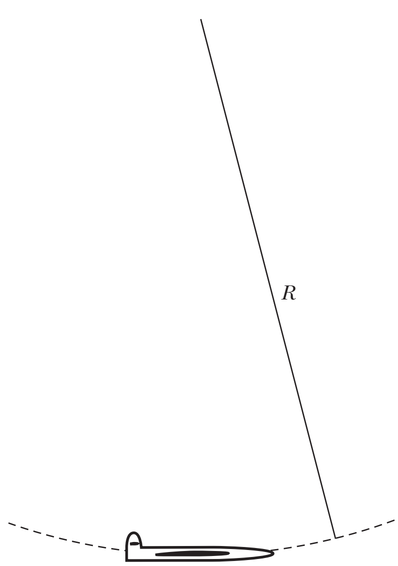


Figure 2b.14 An airplane goes through the bottom of its path.

The feeling of weight, and of weightlessness

As you sit reading this text, you feel a sensation that you associate with weight. If someone suddenly yanks the chair out from under you, you feel "weightless," which feels funny and odd. It also feels scary, because you know from experience that whenever you lose the support of objects under you (chair, airplane seat, floor, mountain path), something bad is about to happen.

In reality, what we perceive as "weight" is actually not the gravitational force at all but rather the forces of atoms in the chair (or airplane seat or floor or mountain path) on atoms in your skin. You have nerve endings that sense the compression of interatomic bonds in your skin, and you interpret this as evidence for gravity acting on you. But if you were placed in a spaceship that was accelerating, going faster and faster, its floor would squeeze against you in a way that would fool your nerve endings and brain into thinking you were subjected to gravity, even if there are no stars or planets nearby.

If you lose contact with the seat in an airplane that is going rapidly over the top of a curving path, your nerve endings no longer feel any contact forces, and you feel "weightless." Yet this is a moment when the only force acting on you is in fact the Earth's gravitational force. Weightlessness near the Earth paradoxically is associated with being subject only to your weight Mg.

Nor is it just nerve endings in your skin that give you the illusion of "weight." As you sit here reading, your internal organs press upward on other organs above them, making the net force zero. You feel these internal contact forces. If you are suddenly "weightless" a main reason for feeling funny is that the forces one organ exerts on another inside your body are suddenly gone.

To train astronauts, NASA has a cargo plane that is deliberately flown in a curving motion over the top, so that people in the padded cargo bay lose contact with the floor and seem to float freely (in actual fact, they are accelerating toward the Earth due to the Mg forces acting on them, but the plane is also deliberately accelerating toward the Earth rather than flying level, so the people don't touch the walls and appear to be floating). All the nerve endings are crying out for the usual comforting signals and not getting them. This airplane is sometimes called the "vomit comet" in honor of the effects it can have on trainees.

2b.3.5 Example: A turning car (curving motion)

Suppose you're riding as the passenger in a convertible with left hand drive (American or continental European), so you are sitting on the right (Figure 2b.15). Assume you have foolishly failed to fasten your seat belt. The driver makes a sudden right turn. Before studying physics you would probably have said that you are "thrown to the left," and in fact you do end up closer to the driver.

? There's something wrong with this "thrown to the left" idea. What object in the surroundings exerted a force to the left to make you move to the left?

There is no such object! To understand what's happening, watch the turning car from a fixed vantage point above the convertible, and observe carefully what happens to the passenger. In the absence of forces to change the passenger's direction of motion, the passenger keeps moving ahead, in a straight line at the same speed as before (Figure 2b.16). It is the car that is yanked to the right, out from under the passenger. The driver moves closer to the passenger; it isn't that the passenger moves toward the driver.

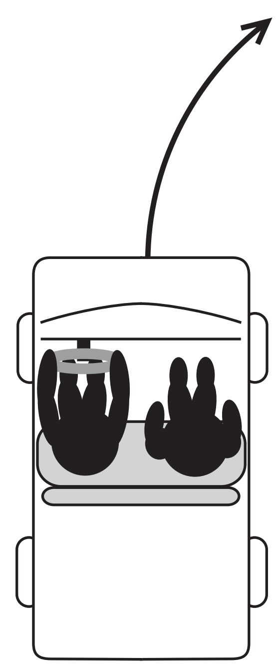


Figure 2b.15 You are a passenger in a car that makes a sudden right turn.

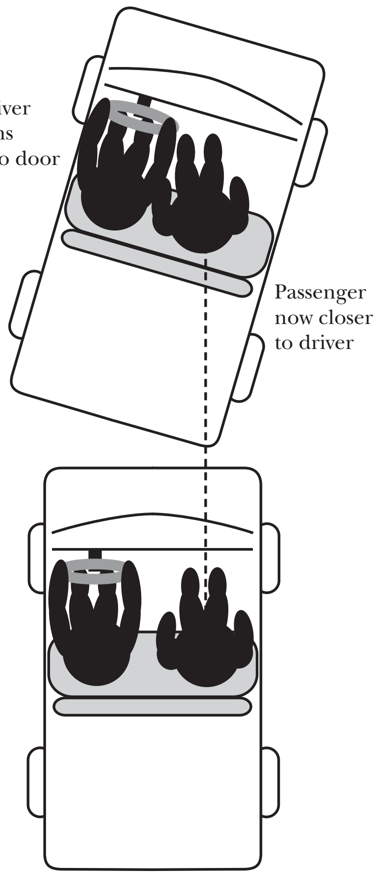


Figure 2b.16 The passenger moves straight ahead and is now closer to the driver, who was thrown to the right. The driver was thrown to right by the door pushing the driver to the right.

The driver may also feel “thrown to the left” against the door. But it is the door that runs into the driver, forcing the driver to the *right*, not to the left.

Sometimes people invent fictitious forces such as the so-called “centrifugal” force to account for the passenger and driver being “thrown to the left.” But this just adds a second confusion to the first. The passenger and driver are thrown to the right in a right turn, not to the left, and there are real forces to the right that make them go to the right (for example, the force of the door on the driver’s left shoulder).

Moreover, forces are associated with interactions between objects in a chosen system and objects in the surroundings. The “centrifugal” force is just made up and is not associated with any real object.

2b.4 A special case: Circular motion at constant speed

Motion in a circle at constant speed is a common kind of motion. This is approximately the situation for the planets in our Solar System.

Although the orbits are actually ellipses, except for Pluto these elliptical orbits are very close to circular. In the case of the Earth, the ellipse is so close to a circle that on a drawing the size of this page you couldn’t tell the difference. The orbit of our Moon around the Earth is also nearly circular at constant speed. Also, each day you are carried around in a circle at constant speed around the Earth’s axis by the rotation of the Earth.

A Ferris wheel or a merry-go-round moves its passengers in a circle at constant speed. Some physical properties of the hydrogen atom can be understood by a simple model of an electron going around in a circle around the proton at constant speed (though this model is too simple to explain many properties of hydrogen atoms, and quantum mechanics is needed for a full analysis).

For a planet of mass m moving around a star of mass M in a circular orbit at constant speed, we can write

$$\left|\frac{d\vec{p}}{dt}\right| = \left|\vec{F}_{net}\right|$$

$$\frac{|\vec{v}|}{R}|\vec{p}| = \left|\vec{F}_{grav}\right| = G\frac{Mm}{R^2}$$

Planets move at very high speeds compared to baseballs, but at speeds that are very small compared to the speed of light. So approximately, $\vec{p} \approx m\vec{v}$, and we can write

$$\frac{|\vec{v}|}{R}m|\vec{v}| \approx G\frac{Mm}{R^2}$$

Canceling the common factors m and R, we have the following result:

$$|\vec{v}|^2 \approx G\frac{M}{R} \quad \text{(special case; see following paragraph)}$$

This is such a special-case formula that it is not a good idea to memorize it. If you do, you are likely to misuse it, using it in situations where it does not apply. It applies only to

- circular motion at constant speed
- with a gravitational force
- between a massive nearly motionless star and
- a comparatively low-mass planet.

You should be able to derive this result from the Momentum Principle and the gravitational force law, as done above. Only then can you say that you really understand the result. Note how short the derivation is!

The gravitational force is unusual because the force is proportional to the mass m of the object, so when the gravitational force appears in the Momentum Principle, the mass m cancels. This is not the case when the force is an electric force, which depends on electric charge, not on mass.

In Einstein's general theory of relativity, which describes gravity in a very different way than Newton did, the star's mass alters the space around the star, and all small masses move in that space in the same way, no matter what their (small) mass. We see an echo of this Einstein view in the cancellation of m in the result above.

2b.4.1 The initial conditions required for a circular orbit

What can we do with our result, $|\vec{v}|^2 \approx GM/R$ for a circular orbit at constant speed? We know the mass M of the Sun, and we know the gravitational constant G, so for a circular orbit whose radius of curvature is R we can calculate the speed that a planet must have in such a circular orbit. Conversely, for a particular speed, we can calculate the radius that will give a circular orbit.

? What if the planet starts out in a direction perpendicular to the line between Sun and planet, but the planet has less than the calculated speed for a circular orbit?

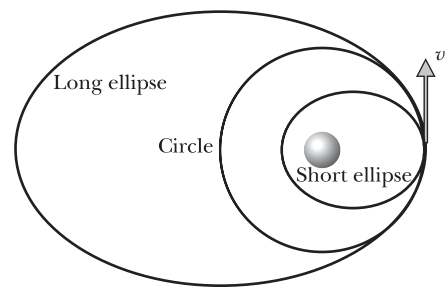


Figure 2b.17 The initial speed v has to be just right to initiate a circular orbit. If the speed is more or less than this value, you get an ellipse (or for very high speeds, a parabola or hyperbola).

In that case the planet will not go in a circle; its orbit will be a short ellipse (Figure 2b.17). If the planet has more than this calculated speed, the orbit will be a long ellipse (or a parabola or a hyperbola), not a circle. The circular orbit is very special and requires just the right combination of radius of curvature and speed.

For a given radius, only one value of the speed gives a circular orbit. Likewise, if you know the speed of an object in a circular orbit, you can figure out what the radius of its orbit has to be.

Newton himself published a diagram that shows very clearly how the initial speed determines the type of orbit. He imagined standing on a very high mountain on the Earth, so high as to be above the atmosphere, and throwing a rock horizontally at various speeds. In Figure 2b.18 you see that if Newton throws the rock with a low speed the rock simply falls toward the Earth and hits the ground near the base of the mountain. If he throws a bit faster, the rock goes further as it falls, hitting the ground a bit farther from the base of the mountain. With a somewhat higher initial speed the rock goes a significant distance around the Earth, falling all the time, before hitting the ground. But if Newton throws the rock just right, the rock falls, and falls, and falls, and falls in a circle, and comes around and hits him in the back of the head!

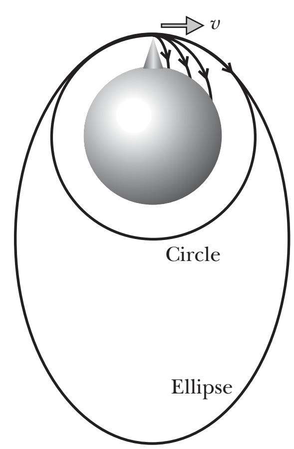


Figure 2b.18 Newton throws a rock horizontally from a high mountain. Only one particular speed makes the rock move in a circular orbit.

This is the magic, special speed for circular motion, and there is only one speed that will produce a circle. It is a speed which you can calculate (as Newton did) from what you now know, and it is the speed of those satellites that are in circular orbits just above our atmosphere. The speed is thousands of meters per second, so this is a "thought experiment" as far as an unaided human thrower is concerned.

If Newton throws the rock even faster than the magic speed for circular motion, the rock moves in a long elliptical orbit. And for even higher speeds (still small compared to the speed of light), the orbit is a parabola or hyperbola, and the rock never comes back.

Example: The Earth's elliptical orbit can be well approximated for many purposes by a circular orbit of radius 1.5×10^{11} m. What would we predict for the orbital speed of the Earth? Start from the

Momentum Principle as applied to circular motion. Solar system data are given in the inside back cover of the textbook.

Solution: $\left|\frac{d\vec{p}}{dt}\right| = |\vec{F}_{net}|$ (Momentum Principle)

$$\left(\frac{|\vec{v}|}{R}\right)|\vec{p}| = G\frac{Mm}{R^2} \text{ (circular motion; gravitational force)}$$

$$\left(\frac{|\vec{v}|}{R}\right)m|\vec{v}| \approx G\frac{Mm}{R^2} \quad (v << c)$$

$$\frac{|\vec{v}|^2}{R} = G\frac{M}{R^2} \text{ (mass } m \text{ of the Earth cancels)}$$

$$|\vec{v}|^2 = G\frac{M}{R} \text{ (solve for speed squared)}$$

$$|\vec{v}| = \sqrt{G\frac{M}{R}} = \sqrt{(6.7\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{2\times10^{30}\ \text{kg}}{1.5\times10^{11}\ \text{m}}} = 3\times10^4\ \text{m/s}$$

Ex. 2b.11 What is the speed of a satellite in a circular orbit near the Earth? You can use the radius of the Earth as the approximate radius of the orbit, because the atmosphere is only about 50 km thick, whereas the radius of the Earth is 6.4×10^6 m . The mass of the Earth is 6×10^{24} kg . Follow the analysis in the example above, but think about the meaning of M and R for this situation.

2b.4.2 Nongravitational situations

Our analysis is not limited to gravitational forces. Any time an object moves at constant speed in a circular orbit when subjected to a net force of constant magnitude, directed radially inward, we have from the Momentum Principle

$$\left|\frac{d\vec{p}}{dt}\right| = \left(\frac{|\vec{v}|}{R}\right)|\vec{p}| = |\vec{F}_{net}| \text{ (circular motion at constant speed)}$$

For example, if you swing a rock at the end of a string in a horizontal circular orbit at constant speed, the net force is the horizontal component of the tension in the string.

For this reason, rather than memorizing the very special result $|\vec{v}|^2/R \approx GM/R^2$ for the case of circular motion due to a gravitational force, it is better to memorize the more general result for the time rate of change of the perpendicular component of momentum, $|(d\vec{p}/dt)_\perp| = (|\vec{v}|/R)|\vec{p}|$, and then equate this perpendicular component of the rate of change of momentum to whatever the perpendicular component of the net force happens to be.

Ex. 2b.12 In outer space, a ball of mass 5 kg at the end of a spring moves in a circle of radius 2 m at a constant speed of 3 m/s. What is the magnitude of the force exerted by the spring on the rock? In words, in what direction does the force act?

2b.4.3 Period

Often we know or would like to calculate the time it takes a planet to go around its star once, called the "period" T. For example, the period of the Earth's orbit around the Sun is one calendar year.

? If a particle goes all the way around a circle of radius R in a time T at constant speed, what is the speed?

The distance around once is the circumference $2\pi R$, so the speed is this distance divided by the time it takes:

$$|\vec{v}| = \frac{2\pi R}{T}$$

This makes it possible to predict a relationship between the period T of a planet and its distance R from the Sun. Just put this value for the speed into the result we obtained above.

This relationship was first discovered empirically by Johannes Kepler, who lived before Newton. Kepler produced his formula by fitting mathematical curves to accurate observations of planetary orbits made by Tycho Brahe (without telescopes, which hadn't been invented yet). The later explanation of Kepler's equation in terms of the Momentum Principle and the gravitational force law was a triumph of the Newtonian approach.

Ex. 2b.13 On the previous page we predicted that the speed of the Earth around the Sun should be 3×10^4 m/s. Do you get the same result for the speed from the simple geometrical formula $|\vec{v}| = (2\pi R)/T$? (Hint: You need the period in seconds. How many seconds in a year?) Compare with our prediction from the Momentum Principle.

2b.4.4 Example: Circular pendulum (curving motion)

As another, rather complex example of an analysis using the derivative form of the Momentum Principle, we will analyze a ball hanging at the end of a string, but with the ball in motion. There are many possible motions of this system. The ball could swing back and forth in a plane; this is the motion of a "simple pendulum." The ball could go around in a path something like an ellipse, in which case the ball's height would vary, just as with simple pendulum motion. The ball could move so violently that the string goes slack for part of the time, with abrupt changes of momentum every time the string suddenly goes taut. If the string can stretch noticeably, there can be a sizable oscillation superimposed on the swinging.

All of these motions are difficult to analyze. In principle, we could use a computer to predict the general motion of the ball hanging from the string if we had a formula for the tension force in the string, as a function of its length. Essentially, we would need the effective spring stiffness for this stiff "spring." However, the string may be so stiff that even a tiny stretch implies a huge tension (a nearly "inextensible" string). The observable length of the string is nearly constant, but the tension force that the string applies to the ball can vary a great deal. This makes it very difficult to do a computer numerical integration, because a tiny error in position makes a huge change in the force.

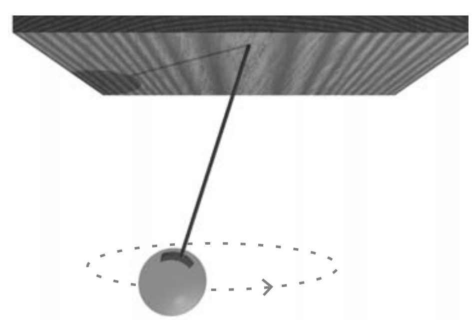


Figure 2b.19 A circular pendulum.

There is however one particular motion of the ball that is simple enough to model and to analyze without a lot of mathematics—circular motion with a fixed stretch of the string. This is a "circular pendulum" (Figure 2b.19). As you discovered when studying planetary orbits, circular motion doesn't just happen. To get the ball moving in a circle you have to start it moving with just the right initial conditions. We tackle a problem that we can solve, by imposing the requirement that we only consider the case of circular motion.

A large circular pendulum was used by Newton to measure g, the magnitude of the gravitational field, with considerable accuracy. You could observe and measure the motion of a circular pendulum yourself, and your analysis of the motion would let you deduce a value for g, which you could compare with the accepted value.

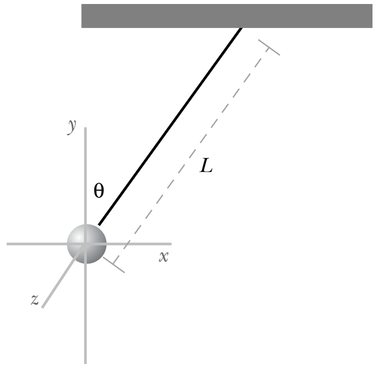


Figure 2b.20 A diagram of the circular pendulum, showing the main geometrical aspects of the situation.

A physics diagram (Figure 2b.20) shows a snapshot of a side view of the circular pendulum, at an instant when the string (of length L to the center of the ball) lies in the xy plane (at an angle θ to the vertical), and the ball is at the leftmost point in its circular path of radius $r = L\sin\theta$. Let's assume that the ball is moving counter-clockwise as seen from above. Here is the analysis:

1. Choose a system

System: the ball

2. List external objects that interact with the system, with diagram

the string, the Earth, and the air (but neglect air resistance) (see Figure 2b.21)

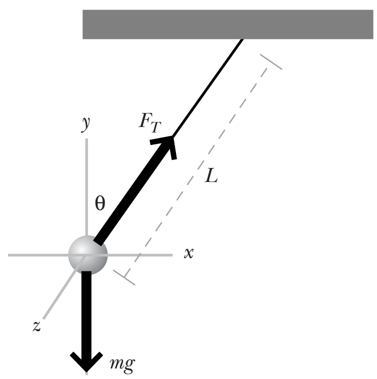


Figure 2b.21 We show known and unknown forces to create a useful force diagram.

3. Apply the Momentum Principle

$$d\vec{p}/dt = \vec{F}_{net}$$

$$\left\langle \frac{|\vec{v}|}{r}|\vec{p}|, 0, 0\right\rangle = \langle F_T\sin\theta, F_T\cos\theta - mg, 0\rangle$$

4. No need to apply the position update formula

5. Solve for the unknowns

$$\frac{|\vec{v}|}{r}|\vec{p}| = F_T\sin\theta \text{ so } F_T = \frac{mv^2}{r\sin\theta} \text{ where } r = L\sin\theta$$

$$0 = F_T\cos\theta - mg \text{ so } F_T = \frac{mg}{\cos\theta}$$

From the two expressions for the tension in the string,

$$F_T = \frac{mv^2}{L(\sin\theta)^2} = \frac{mg}{\cos\theta} \text{ so } g = \frac{v^2\cos\theta}{L(\sin\theta)^2}$$

Measuring the speed v, length L and angle θ determines g.

If the time to go around once is T, a distance $2\pi r = 2\pi L\sin\theta$, the speed is $v = (2\pi L\sin\theta)/T$, so you can measure L, θ, and T to get g.

6. Check

The units can be shown to be correct. Also, note that if $\theta = 0$ the string hangs vertically and there is no motion, so the tension in the string ought to be mg, which is consistent with $F_T = \dfrac{mg}{\cos 0} = mg$.

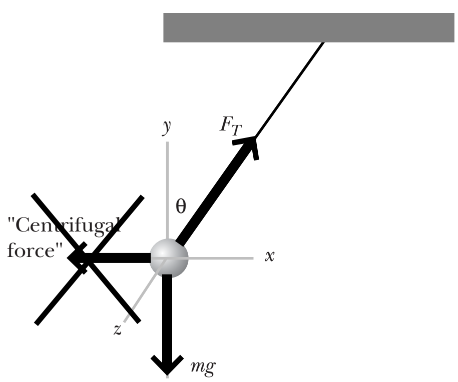


Figure 2b.22 There is no "centrifugal force." There is no interaction associated with such a fictitious force. Moreover, if the net force were zero, the ball would have to move in a straight line!

No "centrifugal force"

We point out that in this analysis there is no role for a radially outward "centrifugal force" (Figure 2b.22). There is no interaction associated with such a fictitious force. Moreover, if the net force were zero, Newton's first law says that the ball would have to move in a straight line!

2b.4.5 Comment: Forward reasoning

A common technique for solving problems is to start by considering what you want to know (or what you have been asked to determine), and to work backward from that to the solution. We have approached the analysis of physical systems in the opposite way. We have begun by organizing our knowledge about the system according to a fundamental principle—the Momentum Principle—and systematically recording our knowledge in equations and diagrams. After we have recorded all the information we have, whatever is missing is what we need to figure out. At this point it is usually quite clear how to figure out this missing information.

This way of working problems, sometimes called "forward reasoning," is typical of the way expert physicists solve problems of this kind—starting each new problem from the fundamental principles. This approach works! For the student, however, approaching problems in this way may require a bit of faith—it is not necessarily obvious how the desired answer will emerge from the analysis. If you practice approaching problems in this way, you will develop confidence in the approach and will gain important experience in explaining complex phenomena starting from fundamental principles.

2b.4.6 Example: An amusement park ride (curving motion)

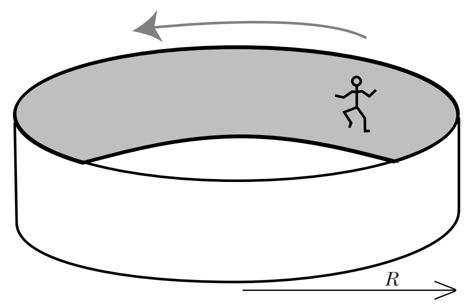


Figure 2b.23 An amusement park ride.

There is an amusement park ride that some people love and others hate in which a bunch of people stand against the wall of a cylindrical room of radius R, and the room starts to rotate at higher and higher speed (Figure 2b.23). When a certain critical speed is reached, the floor drops away, leaving the people stuck against the whirling wall.

Explain why the people stick to the wall without falling down. Include a carefully labeled force diagram of a person (Figure 2b.24), and discuss how the person's momentum changes, and why

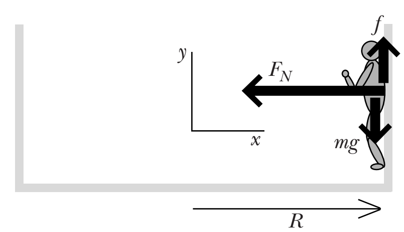


Figure 2b.24 Physics diagram of the person. At this instant the person is moving in the $-z$ direction.

1. Choose a system

   System: person

2. List external objects that interact with the system, with diagram

   the wall, the Earth, and the air (neglect air resistance)
   (represent the system by a circle)

3. Apply the Momentum Principle

$$d\vec{p}/dt = \vec{F}_{net}$$

$$\left\langle -\left(\frac{|\vec{v}|}{R}\right)|\vec{p}|, 0, 0\right\rangle = \langle -F_N, f - mg, 0\rangle$$

4. No need to apply the position update formula

5. Solve for the unknowns

$$-\left(\frac{|\vec{v}|}{R}\right)|\vec{p}| = -F_N \text{ so } F_N = \left(\frac{v}{R}\right)mv$$

$$0 = f - mg \text{ so } f = mg$$

6. Check

   The units check. Also, the faster the ride, the bigger the inward force of the wall, which sounds right.

The wall exerts an unknown force which must have a y component $+f$ (because the person isn't falling) and an x component $-F_N$ normal to the wall (because the person's momentum is changing direction). Note that there is no outward-going "centrifugal" force! There is a momentum change inward; if the net force were zero, the person would move in a straight line.

The vertical component f of the wall force is a frictional force. If the wall has friction that is too low, the person won't be supported. In Chapter 5 we will see that $f \leq \mu F_N$, where the "coefficient of friction" μ has a value for many materials between about 0.1 and 1.0. Basically, the more strongly two objects are squeezed together, the harder it is to slide one along the other.

The speed of the ride has to be large enough that $f \leq \mu F_N$, so we have

$$mg \leq \mu\left(\frac{mv^2}{R}\right)$$

$$\mu \geq \frac{gR}{mv^2}$$

This predicts that the lower the speed v, the greater the coefficient of friction must be to hold the person up on the wall, which sounds right. Conversely, if the ride spins very fast, the wall doesn't have to have a very large coefficient of friction to hold the people against the wall.

2b.5 Systems consisting of several objects

In our applications of the Momentum Principle up to this point, we have usually chosen a single object as the system of interest. We have examined the change of momentum of this single-object system either instantaneously, using the derivative form of the Momentum Principle, or over a finite time interval, using the Momentum Principle in the finite difference form $\vec{p}_f = \vec{p}_i + \vec{F}_{\text{net}}\Delta t$.

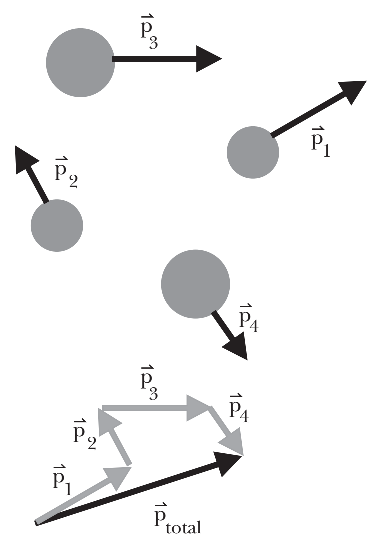


Figure 2b.25 The total momentum of the system of four objects is the sum of the individual momenta of each object.

In some situations, however, it can be very useful to choose a system that consists of two or more interacting objects. This is a legitimate choice of system; the Momentum Principle applied to a system of two or more objects says that the change in the *total* momentum of the system (Figure 2b.25) can be determined by finding the net *external* impulse applied to the system:

MOMENTUM PRINCIPLE FOR MULTIPARTICLE SYSTEMS

$$\Delta\vec{p}_{\text{total}} = (\vec{p}_{\text{total,f}} - \vec{p}_{\text{total,i}}) = \vec{F}_{\text{net,ext}}\Delta t$$

Where:

$\vec{p}_{\text{total}} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ sum of momenta of all objects in the system

$\vec{F}_{\text{net,ext}} \equiv \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$ sum of all external forces on the system

This is an interesting statement of the Momentum Principle; it implies that if we know the external forces acting on a multi-object system, we can draw conclusions about the change of momentum of the system over some time interval without worrying about any of the details of the interactions of the objects with each other. This can greatly simplify the analysis of the motion of some very complex systems.

We have implicitly been using the multiparticle version of the Momentum Principle when we have treated macroscopic objects like humans, spacecraft, and planets as if they were single pointlike objects. In Section 2b.9 there is a full derivation of the multiparticle version of the Momentum Principle, in which it is shown that because of reciprocity, forces between objects inside the system do not change the total momentum of the system.

2b.5.1 Collisions: Negligible external forces

An event is called a "collision" if it involves an interaction that takes place in a short time and has a large effect on the momenta of the objects compared to the effects of other interactions. A collision does not necessarily involve actual physical contact between objects, which may be interacting via long distance forces like the gravitational force or the electric force. Often it can be useful to analyze collisions by choosing a system that includes all of the colliding objects.

As a simple example, consider the case of two sticky balls traveling through the air toward each other, at speeds much less than the speed of light (Figure 2b.26).

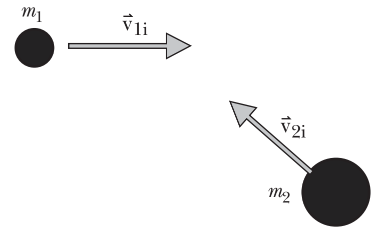


Figure 2b.26 Two sticky balls traveling through the air, just before colliding.

? Choose both balls as the system. What is the total momentum of the system at the instant shown in Figure 2b.26?

Since $v \ll c$, the total momentum of the two-ball system is

$$\vec{p}_{\text{total,i}} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$$

? Suppose that when they collide the balls stick together. What can we conclude about their final velocity?

If we can assume that the effects of gravity and air resistance have a negligible effect during the short time of the actual collision, we know that the momentum of the stuck-together balls (the final total momentum of the system) must be the same as the initial total momentum of the system:

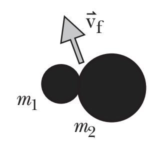


Figure 2b.27 The momentum of the stuck-together balls just after the collision is equal to the sum of the initial momenta of the two balls.

$$\begin{aligned}\vec{p}_{\text{total,f}} &= \vec{p}_{\text{total,i}}\\ (m_1 + m_2)\vec{v}_f &= m_1\vec{v}_{1i} + m_2\vec{v}_{2i}\\ \vec{v}_f &= \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}\end{aligned}$$

If the mass of ball 1 is 2 kg and it initial velocity is $\langle 6, 0, 0\rangle$ m/s, and the mass of ball 2 is 5 kg and its initial velocity is $\langle -5, 4, 0\rangle$ m/s, then the final velocity of the stuck-together balls will be

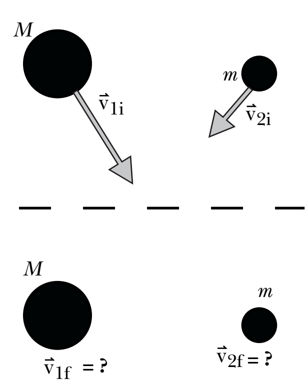
$$\vec{v}_f = \frac{(2\text{ kg})\langle 6, 0, 0\rangle + (5\text{ kg})\langle -5, 4, 0\rangle}{(7\text{ kg})}\text{ m/s} = \langle -1.86, 2.86, 0\rangle\text{ m/s}$$

We reached this conclusion without needing to know anything about the details of the complex forces that the balls exerted on each other during the collision.

Some problems require more than one principle

Because the balls in the previous example had the same final speed, we had enough information to solve for the final velocity of the stuck-together balls—we had one equation with only one unknown (vector) quantity. In more complex situations, we will find that we sometimes do not have enough information to solve for all of the unknown quantities by applying only the Momentum Principle.

For example, consider a collision between two balls that balls bounce off each other, as shown in Figure 2b.28. The Momentum Principle tells us that



$$\begin{aligned}\vec{p}_{\text{total,f}} &= \vec{p}_{\text{total,i}}\\ \vec{p}_{1f} + \vec{p}_{2f} &= \vec{p}_{1i} + \vec{p}_{2i}\end{aligned}$$

but we are left with two unknown (vector) quantities, $\vec{p}_{1f}$ and $\vec{p}_{2f}$, and only one equation. We will not be able to analyze problems of this kind fully until we can invoke the Energy Principle (Chapter 4) and the Angular Momentum Principle (Chapter 9), along with the Momentum Principle.

Figure 2b.28 The initial momenta of the colliding objects are known, but the final momenta after the collision are unknown.

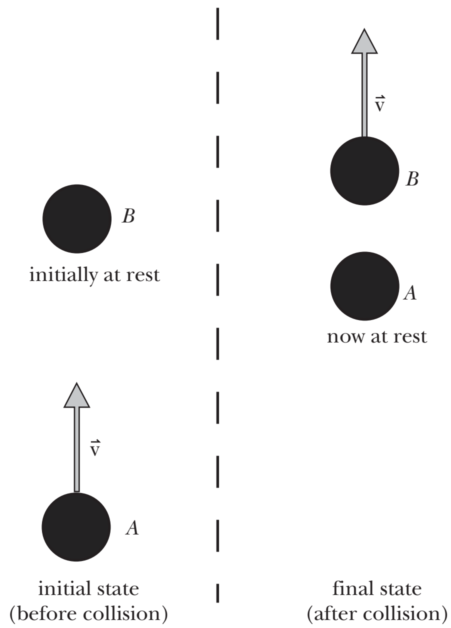


Figure 2b.29 Head on collision between two balls of equal mass. Initially (left side) ball A has velocity $\vec{v}$ and B is at rest. After the collision (right side) A is at rest and B has velocity $\vec{v}$. The total momentum of the two-ball system has not changed.

2b.5.2 Momentum flow within a system

Suppose a bowling ball (ball A) rolls along a smooth surface at almost constant velocity and hits another bowling ball (ball B) of equal mass, which is initially at rest. If the collision is a head-on collision, we will see ball A come to a stop, and ball B roll along with a velocity nearly equal to the initial velocity of ball A (Figure 2b.29).

If we choose a system including both balls, we find that the initial momentum and the final momentum of the system are the same:

$$\vec{p}_i = m\vec{v} + 0$$
$$\vec{p}_f = 0 + m\vec{v}$$

despite the fact that the momentum of each object within the system has changed. Assuming that the effects of air resistance and friction are negligible during the short time when the balls interact with each other, so the net external force applied to the system is approximately zero, this is consistent with the momentum principle:

$$\vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}} = \langle 0, 0, 0\rangle \Delta t = \langle 0, 0, 0\rangle$$

Ex. 2b.14 You and a friend each hold a lump of wet clay. Each lump has a mass of 20 grams. You each toss your lump of clay into the air, where the lumps collide and stick together. Just before the impact, the velocity of one lump was $\langle 5, 2, -3\rangle$ m/s, and the velocity of the other lump was $\langle -3, 0, -2\rangle$ m/s. What was the total momentum of the lumps just before impact? What is the momentum of the stuck-together lump just after the impact? What is its velocity?

Ex. 2b.15 In outer space, far from other objects, two rocks collide and stick together. Before the collision their momenta were $\langle 10, 20, -5\rangle$ kg·m/s and $\langle 8, -6, 12\rangle$ kg·m/s. What was their total momentum before the collision? What must be the momentum of the combined object after the collision?

Ex. 2b.16 At a certain instant, the momentum of a proton is $\langle 3.4\times10^{-21}, 0, 0\rangle$ kg · m/s as it approaches another proton which is initially at rest. The two protons repel each other electrically, without coming close enough to touch. When they are once again far apart, one of the protons now has momentum $\langle 2.4\times10^{-21}, 1.6\times10^{-21}, 0\rangle$ kg · m/s. At this instant, what is the momentum of the other proton?

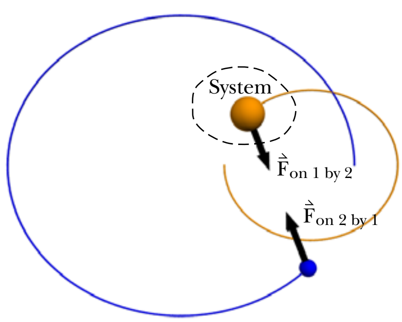


Figure 2b.30 A binary star: choose just one of the stars as the system. The momentum of the system changes due to the external force.

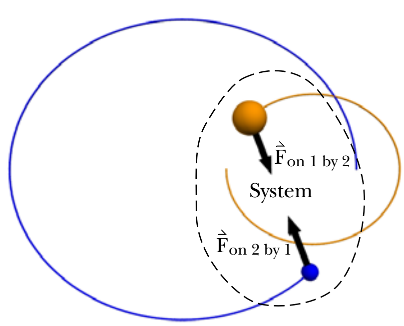


Figure 2b.31 A binary star: choose both stars as the system. The momentum of the combined system doesn't change.

2b.6 Conservation of momentum

The choice of system affects the detailed form of the Momentum Principle. Consider the case of two stars orbiting each other, a "binary star." If you choose as the system of interest just one of the stars (Figure 2b.30), the other star is in the "surroundings" and exerts an external force which changes the first star's momentum.

If on the other hand you choose both stars as your system (Figure 2b.31), the surroundings consist of other stars, which may be so far away as to have negligible effects on the binary star. In that case the net external force acting on the system is nearly zero.

? What happens to the total momentum of the isolated binary star as time goes by?

The Momentum Principle $\vec{p}_f = \vec{p}_i + \vec{F}_{\text{net}}\Delta t$ reduces in this case to $\vec{p}_f = \vec{p}_i$, which predicts that the total momentum of the system $\vec{p} = \vec{p}_1 + \vec{p}_2$ remains constant in magnitude and direction. This is an important special case: the total momentum of an isolated system, a system with negligible interactions with the surroundings, doesn't change but stays constant.

In a later chapter we will see that the total momentum can be expressed as $M_{\text{total}}\vec{v}_{\text{center of mass}}$, where the center of mass is a mathematical point between the two stars, closer to the more massive star. The velocity of the center of mass does not change but is constant as the binary star drifts through space (or is zero if the binary star's total momentum is zero).

One way to think about this result is to say that momentum gained by one star is lost by the other, because as we saw in the previous chapter, the gravitational forces (and impulses) are equal in magnitude but opposite in direction. The effect is that the total momentum doesn't change. Let $\vec{F}$ be the force exerted on star 1 by star 2, so $-\vec{F}$ is the force exerted on star 2 by star 1. After a short time interval Δt the new total momentum is this:

$$(\vec{p}_1 + \vec{F}\Delta t) + (\vec{p}_2 - \vec{F}\Delta t) = \vec{p}_1 + \vec{p}_2$$

Whatever momentum is lost by one star is gained by the other star. This is a simple example of an important principle, the "conservation of momentum," which says that the change of momentum in a system plus the change of momentum in the surroundings adds up to zero.

CONSERVATION OF MOMENTUM

$$\Delta\vec{p}_{\text{system}} + \Delta\vec{p}_{\text{surroundings}} = \vec{0}$$

In the case of the two stars there are no objects in the surroundings, and no external forces, so the momentum of the system doesn't change. Zero net external impulse $(\vec{F}_{\text{net}}\Delta t)$, zero net momentum change. One of the stars can gain momentum (due to a force acting on it), but only if the other star loses the same amount.

Relativistic momentum conservation

Particle accelerators produce beams of particles such as electrons, protons, and pions at speeds very close to the speed of light. When these high-speed particles interact with other particles, experiments show that the total momentum is conserved, but only if the momentum of each particle is defined in the way Einstein proposed, $\vec{p} \equiv m\vec{v}/\sqrt{1-(|\vec{v}|/c)^2}$. It is found that using the low-speed approximation $\vec{p} \approx m\vec{v}$ there is no conservation of momentum when the speed v approaches the speed of light c.

Ex. 2b.17 Consider the head-on collision of two identical bowling balls discussed above and shown in Figure 2b.29.

(a) Choose a system consisting only of ball A. What is the momentum change of the system during the collision? What is the momentum change of the surroundings?

(b) Choose a system consisting only of ball B. What is the momentum change of the system during the collision? What is the momentum change of the surroundings?

(c) Choose a system consisting of both balls. What is the momentum change of the system during the collision? What is the momentum change of the surroundings?

2b.7 Predicting the future of complex gravitating systems

We can use the Newtonian method and numerical integration to try to predict the future of a group of objects that interact with each other mainly gravitationally, such as the Solar System consisting of the Sun, planets, moons, asteroids, comets, etc. There are other, non-gravitational interactions present. Radiation pressure from sunlight makes a comet's tail sweep away from the sun. Streams of charged particles (the "solar wind") from the Sun hit the Earth and contribute to the Northern and Southern Lights (auroras). But the main interactions within the Solar System are gravitational.

The Solar System as a group is orbiting around the center of our Milky Way galaxy, and our galaxy is interacting gravitationally with other galaxies in a local cluster of galaxies (Figure 2.32), but changes in these speeds and directions take place very slowly compared to those within the solar system. To a very good approximation we can neglect these interactions while studying the inner workings of the Solar System itself.

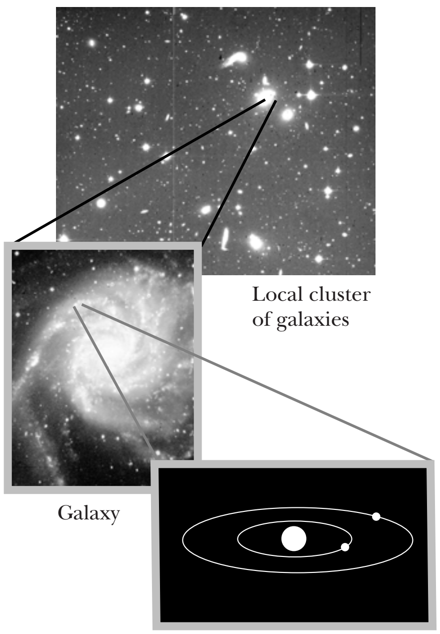


Figure 2.32 Our Solar System orbits around the center of our galaxy, which interacts with other galaxies.

We have modeled the motion of planets and stars using the Momentum Principle and the universal law of gravitation. We can use the same approach to model the gravitational interactions of more complex systems involving three, four, or any number of gravitationally interacting objects. All that is necessary is to include the forces associated with all pairs of objects.

In principle we could use these techniques to predict the future of our entire Solar System. Of course our prediction would be only approximate, because we take finite time steps which introduce numerical errors, and we are neglecting various small interactions. We would need to investigate whether the prediction of our model is a reasonable approximation to the real motion of our real Solar System.

Our Solar System contains a huge number of objects. Most of the nine planets have moons, and Jupiter and Saturn have many moons. There are thousands of asteroids and comets, not to mention an uncounted number of tiny specks of dust. This raises a practical limitation: the fastest computers would take a very long time to carry out even one time step for all the pieces of the Solar System.

2b.7.1 The three-body problem

As we have seen, in order to carry out a numerical integration we need to know the position and velocity of each object at some time t, from which we can calculate the forces on each object at that time. This enables us to calculate the position and momentum for each object at a slightly later time $t + \Delta t$. We need the net force on an object, which we get from Newton's universal law of gravitation and the superposition principle.

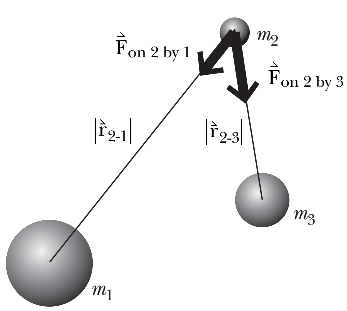


Figure 2b.33 In the "three body problem" each of the three objects interacts with two other objects.

Suppose that there are three gravitating objects, as shown in Figure 2b.33. The force on object m_2, which is acted upon gravitationally by objects m_1 and m_3, is a natural extension of what we did earlier for the analysis of a star and planet. We simply use the superposition principle to include the interaction with m_3 as well as the interaction with m_1:

$$\vec{F}_{\text{net on } m_2} = -G\frac{m_2 m_1}{|\vec{r}_{2\text{-}1}|^2}\hat{r}_{2\text{-}1} - G\frac{m_3 m_1}{|\vec{r}_{2\text{-}3}|^2}\hat{r}_{2\text{-}3}$$

We use this force to update the momentum of object m_2. Similar computations apply to the other masses, m_1 and m_3. These calculations are tedious, but let the computer do them! We can carry out a numerical integration for each mass: given the current locations of all the masses we can calculate the vector net force exerted on each mass at a time t, and then we can calculate the new values of momentum and position at a slightly later time $t + \Delta t$.

While there exist analytical (non-numerical) solutions for two-body gravitational motion, except for some very special cases the “three-body problem” has not been solved analytically. However, in a numerical integration, the computer doesn’t mind that there are lots of quantities and a lot of repetitive calculations to be done. You yourself can solve the three-body problem numerically, by adding additional calculations to your two-body computations (see the three-body homework problem in Chapter 2).

For a single object orbiting a very massive object, or for two objects orbiting each other (with zero net momentum), the possible trajectories are only a circle, ellipse, parabola, hyperbola, or straight line. The motion of a three-body system can be vastly more complex and diverse. Figure 2b.34 shows the numerical integration of a complicated three-body trajectory for one particular set of initial conditions. Adding just one more object opens up a vast range of complex behaviors. Imagine how complex the motion of a galaxy can be!

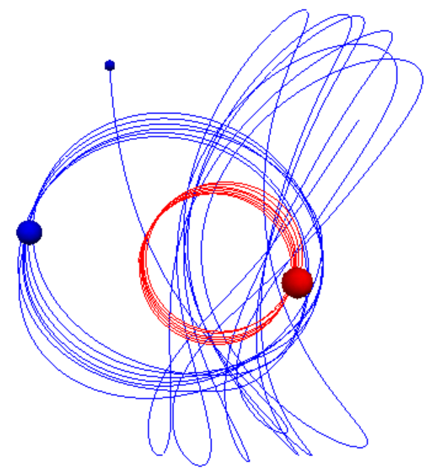


Figure 2b.34 One example of three bodies interacting gravitationally.

2b.7.2 Sensitivity to initial conditions

For a two-body system, slight changes in the initial conditions make only slight changes in the orbit, such as changing a circle into an ellipse, or an ellipse into a slightly different ellipse, but you never get anything other than a simple trajectory.

The situation is very different in systems with three or more interacting objects. Consider a low-mass object orbiting two massive objects which we imagine somehow to be nailed down so that they cannot move (Figure 2b.35). This could also represent two positive charges that are fixed in position, with a low-mass negative charge orbiting around them.

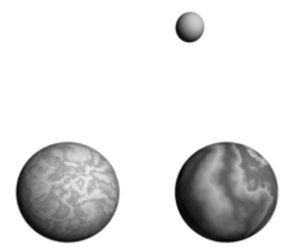


Figure 2b.35 A low mass object orbiting two massive objects.

In Figure 2b.36 are two very different orbits (one shown in black, the other in gray), starting from only slightly different initial conditions. In both of these cases, the low-mass object was released from rest, but at slightly different initial locations. The trajectories are wildly different! (The mass of the object on the left is twice the mass of the object on the right in this computation.)

This sensitivity to initial conditions generally becomes more extreme as you add more objects, and the Solar System contains lots of objects, big and small. Also, we can anticipate that if small errors in specifying the initial conditions can have large effects, so too it is likely that failing to take into account the tiny force exerted by a small asteroid might make a big difference after a long integration time.

The Solar System is actually fairly predictable, because there is one giant mass (the Sun), and the other, much smaller masses are very far apart. This is unlike the example shown here, which deliberately emphasizes the sensitivity observed when there are large masses near each other.

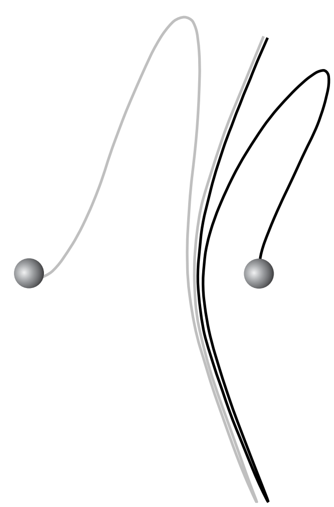


Figure 2b.36 Two different orbits (one gray, the other black), starting from rest but from slightly different positions.

2b.8 Determinism

If you know the net force on an object as a function of its position and momentum (or velocity), you can predict the future motion of the object by simple step-by-step calculations based on the momentum principle. Newton’s demonstration of the power of this approach induced many seventeenth and eighteenth century philosophers and scientists to adopt the view already proposed by Descartes, Boyle, and others, that the Universe is a giant clockwork device, whose future is completely determined by the present positions and momenta of all the macroscopic and microscopic objects in it. Just turn the crank, and predict the future! This point of view is called “determinism,” and taken to its extreme it raises the question of whether humans actually have any free will, or whether all of our actions are predetermined. Scientific and technological advances in the twentieth century, however, have led us to see that although the Newtonian approach can be used to predict the long-term future of a simple system or the short-term future of a complicated system, there are both practical and theoretical limitations on predictions in some systems.

2b.8.1 Practical limitations

One reason we may be unable to predict the long-term future of even a simple system is a practical limitation in our ability to measure its initial conditions with sufficient accuracy. Over time, even small inaccuracies in initial conditions can lead to large cumulative errors in calculations. This is a practical limitation rather than a theoretical one, because in principle if we could measure initial conditions more accurately, we could perform more accurate calculations.

Another practical limitation is our inability to account for all interactions in our model. Every object in the Universe interacts with every other object. In constructing simplified models, we ignore interactions whose magnitude is extremely small. However, over time, even very small interactions can lead to significant effects. Even the tiny "radiation pressure" exerted by sunlight has been shown to affect noticeably the motion of an asteroid over a long time. With larger and faster computers, we can include more and more interactions in our models, but our models can never completely reflect the complexity of the real world.

A branch of current research that focuses on how the detailed interactions of a large number of atoms or molecules lead to the bulk properties of matter is called "molecular dynamics." For example, one way to test our understanding of the nature of the interactions of water molecules is to create a computational model in which the forces between molecules and the initial state of a large number of molecules—several million—are described, and then to see if running the model produces a virtual fluid that actually behaves like water. However, even with an accurate force law for the (electric) interactions between atoms or molecules, it is not feasible to compute and track the motions of the 10^{25} molecules in a glass of water. Significant efforts are underway to build faster and faster computers, and to develop more efficient computer algorithms, to permit realistic numerical integration of more complex systems.

2b.8.2 Chaos

A second kind of limitation occurs in systems that display an extreme sensitivity to initial conditions. We saw something like this in the example of the three-body problem in gravitating systems—small changes in initial conditions produced large changes in behavior. In recent years scientists have discovered systems in which a change in the initial conditions, no matter how small (infinitesimal), can lead to complete loss of predictability: the difference in the two possible future motions of the system diverges exponentially with time. Such systems are called "chaotic." It is thought that over a long time period the weather may be literally unpredictable in this sense.

An interesting popular science book about this new field of research is *Chaos: Making a New Science*, by James Gleick (Penguin 1988). The issue of small changes in initial conditions is a perennial concern of time travellers in science fiction stories. Perhaps the most famous such story is Ray Bradbury's "The Sound of Thunder," in which a time traveller steps on a butterfly during the Jurassic era, and returns to an eerily changed present time.

2b.8.3 Breakdown of Newton's laws

There are other situations of high interest where we cannot usefully apply Newton's laws of motion, because these classical laws do not adequately describe the behavior of physical systems. To model systems composed of very small particles such as protons and electrons, quarks, and photons, it is necessary to use the laws of quantum mechanics. To model in detail the gravitational interactions between massive objects, it is necessary to apply the principles of general relativity. Given these principles, one might assume that it would be possible to follow a procedure similar to the one we have followed in this chapter, and predict in detail the future of these systems. However, there appear to be more fundamental limitations on what we can know about the future.

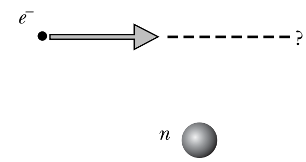


Figure 2.37 The electron may continue to travel in a straight line, but we cannot be certain that it will, because we do not know when the neutron will decay.

2b.8.4 Probability and uncertainty

Our understanding of the atomic world of quantum mechanics suggests that there are fundamental limits to our ability to predict the future, because at the atomic and subatomic levels the Universe itself is non-deterministic. We cannot know exactly what will happen at a given time, but only the probability that certain events will occur within a given time frame.

A simple example may make this clear. A free neutron (one not bound into a nucleus) is unstable and eventually decays into a positively charged proton, a negatively charged electron, and an electrically neutral anti-neutrino:

$$n \rightarrow p^+ + e^- + \bar{\nu}$$

The average lifetime of the free neutron is about 15 minutes. Some neutrons survive longer than this, and some last a shorter amount of time. All of our experiments and all of our theory are consistent with the notion that *it is not possible to predict when a particular neutron will decay*. There is only a (known) probability that the neutron will decay in the next microsecond, or the next, or the next.

If this is really how the Universe works, then there is an irreducible lack of predictability and determinism of the Universe itself. As far as our own predictions using the momentum principle are concerned, consider the following simple scenario. An electron is traveling with constant velocity through nearly empty space, but there is a free neutron in the vicinity (Figure 2.37).

We might predict that the electron will move in a straight line, and some fraction of the time we will turn out to be right (the electrically uncharged neutron exerts no electric force on the electron, there is a magnetic interaction but it is quite small, and the gravitational interaction is tiny). But if the neutron happens (probabilistically) to decay just as our electron passes nearby, suddenly our electron is subjected to large electric forces due to the decay proton and electron, and will deviate from a straight path. Moreover, the directions in which the decay products move are also only probabilistic, so we can't even predict whether the proton or the electron will come closest to our electron (Figure 2.38).

This is a simple but dramatic example of how quantum indeterminacy can lead to indeterminacy even in the context of the Momentum Principle.

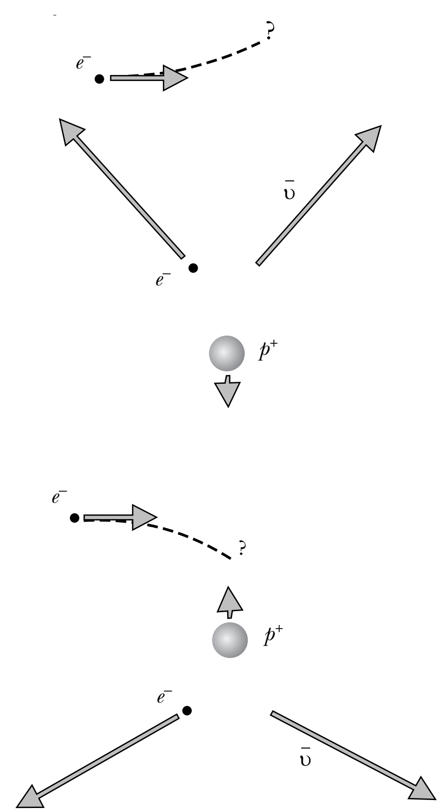


Figure 2.38 If the neutron does decay when the electron is near it, the trajectory of the electron will depend on the directions of the momenta of the decay products.

The Heisenberg uncertainty principle

The Heisenberg uncertainty principle states that there are actual theoretical limits to our knowledge of the state of physical systems. One quantitative formulation states that the position and the momentum of a particle cannot both be simultaneously measured exactly:

$$\Delta x \Delta p_x \geq h$$

This relation says that the product of the uncertainty in position Δx and the uncertainty in momentum Δp_x is equal to a constant h, called Planck's constant. Planck's constant is tiny ($h = 6.6\times10^{-34}$ kg·m^2/s), so this limitation is not noticeable for macroscopic systems, but for objects small enough to require a quantum mechanical description, the uncertainty principle puts fundamental limits on how accurately we can know the initial conditions, and therefore how well we can predict the future.

2b.9 *Derivation of the multiparticle Momentum Principle

In a three-particle system (Figure 2b.39) we show all of the forces acting on each particle, where the lower case $\vec{f}$'s are forces the particles exert on each other (so-called "internal" forces), and the upper case $\vec{F}$'s are forces exerted by other objects that are not shown and are not part of our chosen system (these are so-called "external" forces, such as the gravitational attraction of the Earth, or a force that you exert by pulling on one of the particles).

We will use the following shorthand notation: $\vec{f}_{1,3}$ will denote the force exerted on particle 1 by particle 3; $\vec{F}_{2,\text{ext}}$ will denote the force on particle 2 exerted by external objects, and so on. Here is the Momentum Principle for each of the three particles:

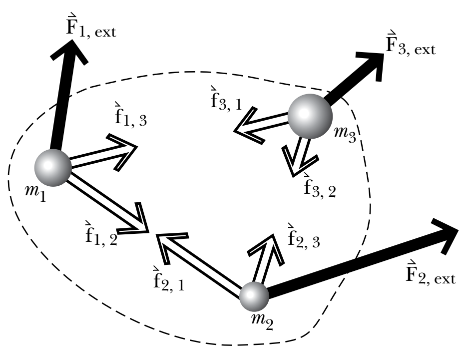


Figure 2b.39 External and internal forces acting on a system of three particles.

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{1,\text{ext}} + \vec{f}_{1,2} + \vec{f}_{1,3}$$

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{2,\text{ext}} + \vec{f}_{2,1} + \vec{f}_{2,3}$$

$$\frac{d\vec{p}_3}{dt} = \vec{F}_{3,\text{ext}} + \vec{f}_{3,1} + \vec{f}_{3,2}$$

Nothing new so far. But now we add up these three equations. That is, we create a new equation by adding up all the terms on the left sides of the three equations, and adding up all the terms on the right sides, and setting them equal to each other:

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_3}{dt} = \vec{F}_{1,\text{ext}} + \vec{f}_{1,2} + \vec{f}_{1,3} + \vec{F}_{2,\text{ext}} + \vec{f}_{2,1} + \vec{f}_{2,3} + \vec{F}_{3,\text{ext}} + \vec{f}_{3,1} + \vec{f}_{3,2}$$

Many of these terms cancel. By the principle of reciprocity (Newton's third law of motion), which is obeyed by gravitational and electric interactions, we have the following:

$$\vec{f}_{1,2} = -\vec{f}_{2,1}$$

$$\vec{f}_{1,3} = -\vec{f}_{3,1}$$

$$\vec{f}_{2,3} = -\vec{f}_{3,2}$$

Thanks to reciprocity, all that remains after the cancellations is this:

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_3}{dt} = \vec{F}_{1,\text{ext}} + \vec{F}_{2,\text{ext}} + \vec{F}_{3,\text{ext}}$$

The total momentum of the system is $\vec{P}_{\text{tot}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$, so we have:

$$\frac{d\vec{P}_{\text{tot}}}{dt} = \vec{F}_{\text{net,ext}}\text{, or in terms of impulse, } \Delta\vec{P}_{\text{tot}} = \vec{F}_{\text{net,ext}}\Delta t$$

The importance of this equation is that reciprocity has eliminated all of the internal forces (the forces that the particles in the system exert on each other); internal forces cannot affect the motion of the system as a whole. All that matters in determining the rate of change of (total) momentum is the net external force. The equation has exactly the same form as the Momentum Principle for a single particle.

Moreover, if the object is a sphere whose density is only a function of radius, the object exerts a gravitational force on other objects as though the sphere were a point particle. We can predict the motion of a star or a planet or an asteroid as though it were a single point particle of large mass.

2b.10 Summary

THE MOMENTUM PRINCIPLE (DERIVATIVE FORM)

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

In words, "the instantaneous time rate of change of the momentum of an object is equal to the net force acting on the object." Or in calculus terms, since the limit of a ratio of infinitesimal quantities is called a derivative, we can say that "the derivative of the momentum with respect to time is equal to the net force acting on the object."

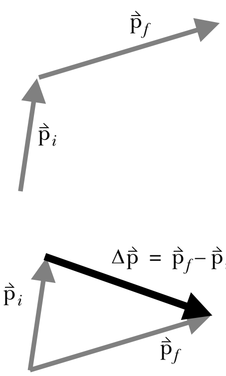


Figure 2b.40 Calculation of $\Delta\vec{p}$.

Finding $d\vec{p}/dt$**:**

1. Draw two arrows (Figure 2b.40),

   one representing $\vec{p}_i$, the momentum of the object a short time before the instant of interest, and

   a second arrow representing $\vec{p}_f$, the momentum of the object a short time after the instant of interest

2. Graphically find $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ by placing the arrows tail to tail, and drawing the resultant arrow starting at the tail of $\vec{p}_i$ and going to the tip of $\vec{p}_f$

3. This arrow indicates the direction of $\Delta\vec{p}$, which is the same as the direction of $d\vec{p}/dt$, provided the time interval involved is sufficiently small

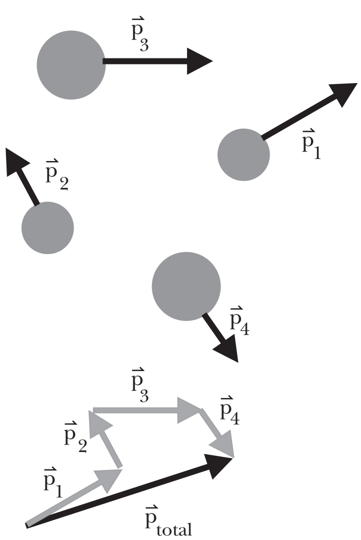


Figure 2b.41 The total momentum of the system of four objects is the sum of the individual momenta of each object.

MOMENTUM PRINCIPLE FOR MULTIPARTICLE SYSTEMS

$$\Delta\vec{p}_{total} = (\vec{p}_{total,f} - \vec{p}_{total,i}) = \vec{F}_{net,ext}\Delta t$$

Where:

$\vec{p}_{total} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ sum of momenta of all objects in the system

$\vec{F}_{net,ext} \equiv \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$ sum of all external forces on the system

CONSERVATION OF MOMENTUM

$$\Delta\vec{p}_{system} + \Delta\vec{p}_{surroundings} = \vec{0}$$

2b.11 Review questions

Circular motion

RQ 2b.1 You are driving an American car, sitting on the left side of the front seat. You make a sharp right turn. You feel yourself “thrown to the left” and your left side hits the left door. Is there a force that pushes you to the left? What object exerts that force? What really happens? Draw a diagram to illustrate and clarify your analysis.

Circular motion

RQ 2b.2 A 30 kg child rides on a playground merry-go-round, 2 m from the center. The merry-go-round makes one complete revolution every 4 seconds. How large is the net force on the child? In what direction does the net force act?

Determinism

RQ 2b.3 List as many limitations as you can on our ability to predict the future.

2b.12 Problems

Problem 2b.1 A spring-like device

A certain spring-like device with uneven windings has the property that to stretch it an amount s from its relaxed length requires a force that is given by $F = bs^3$. You suspend this device vertically, and its unstretched length is 20 cm.

(a) You hang a mass of 15 grams from the device, and you observe that the length is now 24 cm. What is b, including units? Start your analysis from the Momentum Principle.

(b) You hold the 15 gram mass and throw it downward, releasing it when the length of the spring-like device is 27 cm and the speed of the mass is 4 m/s. One millisecond later (10^{-3} s), what is the stretch of the device, and what is the speed of the mass?

Problem 2b.2 Pushing on a block

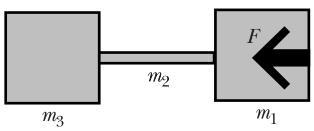


Figure 2b.42 Push on a block (Problem 2b.2).

Two blocks of mass m_1 and m_3, connected by a rod of mass m_2, are sitting on a low-friction surface, and you push to the left on the right block (mass m_1) with a constant force (Figure 2b.42).

(a) What is the acceleration dv_x/dt of the blocks?

(b) What is the compression force in the rod (mass m_2) near its right end? Near its left end?

(c) How would these results change if you *pull* to the left on the *left* block (mass m_3) with the same force, instead of pushing the right block?

Problem 2b.3 Relating radius to period for circular orbits

The planets in our Solar System have orbits around the Sun that are nearly circular, and $v << c$. Calculate the period T (a "year"—the time required to go around the Sun once) for a planet whose orbit radius is r. This is the relationship discovered by Kepler and explained by Newton. (It can be shown by advanced techniques that this result also applies to elliptical orbits if you replace r by the "semi major axis," which is half the longer, "major" axis of the ellipse.) Use this analytical solution for circular motion to predict the Earth's orbital speed, using the data for Sun and Earth on the inside back cover of the textbook.

Problem 2b.4 Experiment—determine *g* with a circular pendulum

Use a circular pendulum to determine g. You can increase the accuracy of the time it takes to go around once by timing N revolutions and then dividing by N. This minimizes errors contributed by inaccuracies in starting and stopping the clock. It is wise to start counting from zero (0, 1, 2, 3, 4, 5) rather than starting from 1 (1, 2, 3, 4, 5 represents only 4 revolutions, not 5). It also improves accuracy if you start and stop timing at a well-defined event, such as when the mass crosses in front of an easily visible mark.

This was the method used by Newton to get an accurate value of g. Newton was not only a brilliant theorist but also an excellent experimentalist. For a circular pendulum he built a large triangular wooden frame mounted on a vertical shaft, and he pushed this around and around while making sure that the string of the circular pendulum stayed parallel to the slanting side of the triangle.

Problem 2b.5 Mass on a spring with circular motion

A small block of mass m is attached to a spring with stiffness k_s and relaxed length L. The other end of the spring is fastened to a fixed point on a low-friction table. The block slides on the table in a circular path of radius $R > L$. How long does it take for the block to go around once?

Problem 2b.6 Circular motion in a magnetic field

When a particle with electric charge q moves with speed v in a plane perpendicular to a magnetic field B, there is a magnetic force at right angles to the motion with magnitude qvB, and the particle moves in a circle of radius r (Figure 2b.43). This formula for the magnetic force is correct even if the speed is comparable to the speed of light. Show that

$$p = \frac{mv}{\sqrt{1-(|\vec{v}|/c)^2}} = qBr$$

even if v is comparable to c. Remember that $\vec{F} = m\vec{a}$ is *not* valid for high speeds.

This result is used to measure relativistic momentum: if the charge q is known, we can determine the momentum of a particle by observing the radius of a circular trajectory in a known magnetic field.

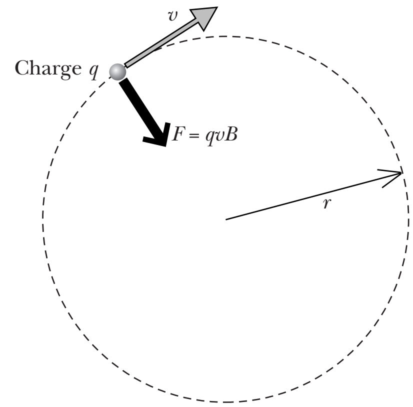
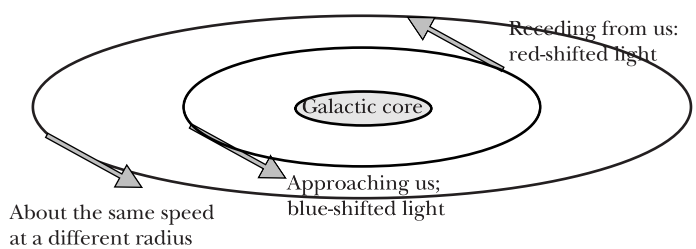


Figure 2b.43 A charge moving in a circle due to a magnetic force (Problem 2b.6).

Problem 2b.7 Dark matter in the Universe

In the 1970's the astronomer Vera Rubin made observations of distant galaxies that she interpreted as indicating that perhaps 90% of the mass in a galaxy is invisible to us ("dark matter"). She measured the speed with which stars orbit the center of a galaxy, as a function of the distance of the stars from the center. The orbital speed was determined by measuring the "Doppler shift" of the light from the stars, an effect which makes light shift toward the red end of the spectrum ("red shift") if the star has a velocity component away from us, and makes light shift toward the blue end of the spectrum if the star has a velocity component toward us. She found that for stars farther out from the center of the galaxy, the orbital speed of the star hardly changes with distance from the center of the galaxy, as is indicated in the diagram below. The visible components of the galaxy (stars, and illuminated clouds of dust) are most dense at the center of the galaxy and thin out rapidly as you move away from the center, so most of the visible mass is near the center.



(a) Predict the speed v of a star going around the center of a galaxy in a circular orbit, as a function of the star's distance r from the center of the galaxy, assuming that almost all of the galaxy's mass M is concentrated at the center.

(b) Construct a logical argument as to why Rubin concluded that much of the mass of a galaxy is not visible to us. Reason from principles discussed in this chapter, and your analysis of part (a). Explain your reasoning. You need to address the following issues:

- Rubin's observations are not consistent with your prediction in (a).
- Most of the *visible* matter is in the center of the galaxy.
- Your prediction in (a) assumed that most of the mass is at the center.

This issue has not yet been resolved, and is still a current topic of astrophysics research. Here is a reference to the original work: "Dark Matter in Spiral Galaxies" by Vera C. Rubin, *Scientific American*, June 1983 (96-108). You can find several graphs of the rotation curves for spiral galaxies on page 101 of this article.

Problem 2b.8 Orbital periods

(a) Many communication satellites are placed in a circular orbit around the Earth at a radius where the period (the time to go around the Earth once) is 24 hours. If the satellite is above some point on the equator, it stays above that point as the Earth rotates, so that as viewed from the rotating Earth the satellite appears to be motionless. That is why you see dish antennas pointing at a "fixed" point in space. Calculate the radius of the orbit of such a "synchronous" satellite. Explain your calculation in detail.

(b) Electromagnetic radiation including light and radio waves travels at a speed of 3×10^{8} m/s. If a phone call is routed through a synchronous satellite to someone not very far from you on the ground, what is the minimum delay between saying something and getting a response? Explain. Include in your explanation a diagram of the situation.

(c) Some human-made satellites are placed in "near-Earth" orbit, just high enough to be above almost all of the atmosphere. Calculate how long it takes for such a satellite to go around the Earth once, and explain any approximations you make.

(d) Calculate the orbital speed for a near-Earth orbit, which must be provided by the launch rocket. (Recently large numbers of near-Earth communications satellites have been launched. Their advantages include making the signal delay unnoticeable, but with the disadvantage of having to track the satellites actively and having to use many satellites to ensure that at least one is always visible over a particular region.)

(e) When the first two astronauts landed on the Moon, a third astronaut remained in an orbiter in circular orbit near the Moon's surface. During half of every complete orbit, the orbiter was behind the Moon and out of radio contact with the Earth. On each orbit, how long was the time when radio contact was lost?

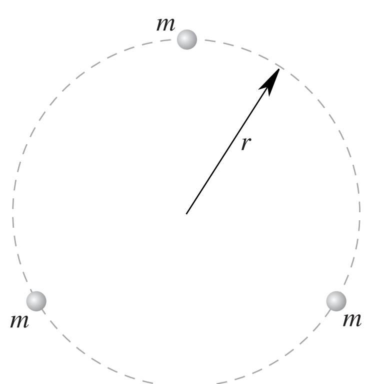


Figure 2b.44 Three stars in a circular orbit (Problem 2b.9).

Problem 2b.9 Analytical 3-body orbit

There is no general analytical solution for the motion of a 3-body gravitational system. However, there do exist analytical solutions for very special initial conditions. Figure 2b.44 shows three stars, each of mass m, which move in the plane of the page along a circle of radius r. Calculate how long this system takes to make one complete revolution. (In many cases 3-body orbits are not stable: any slight perturbation leads to a break-up of the orbit.)

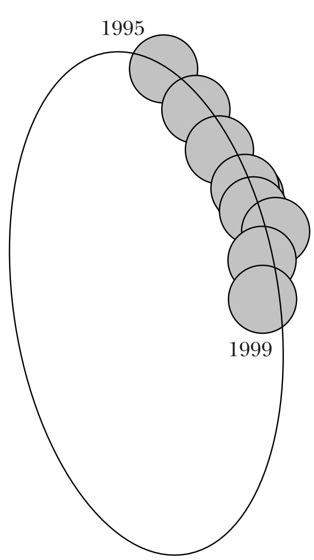


Figure 2b.45 Observed positions of a star from 1995 to 1999 near the center of our Milky Way Galaxy, and an orbit fit to the data (Problem 2b.10).

Problem 2b.10 The center of our galaxy

Remarkable data indicate the presence of a massive black hole at the center of our Milky Way galaxy. One of the two W. M. Keck 10 meter diameter telescopes in Hawaii was used by Andrea Ghez and her colleagues to observe infrared light coming directly through the dust surrounding the central region of our galaxy (visible light is multiply scattered by the dust, blocking a direct view). Stars were observed for several consecutive years to determine their orbits near a motionless center that is completely invisible ("black") in the infrared but whose precise location is known due to its strong output of radio waves, which are observed by radio telescopes. The data were used to show that the object at the center must have a mass that is huge compared to the mass of our own Sun, whose mass is a "mere" 2×10^{30} kg.

(a) The speeds of the stars as a function of distance from the center were consistent with Newtonian predictions for orbits around a massive central object. How should the speed of each star depend on its distance from the center, and the mass of the central object? Express your result symbolically.

(b) The star nearest the center of our galaxy was observed to be about 1×10^{14} m from the center, with a period of about 15 years. What is the speed of this star in m/s? Also express the speed as a fraction of the speed of light. (The actual orbit is an ellipse as reproduced in Figure 2b.45, but for purposes of this analysis you may approximate it as being circular.)

(c) This is an extraordinarily high speed for a star. Is it reasonable to approximate the star's momentum as mv?

(d) Calculate the mass of the massive black hole about which this star is orbiting.

(e) How many of our Suns does this represent?

It is thought that all galaxies may have such a black hole at their centers, as a result of long periods of mass accumulation. When many bodies orbit each other, sometimes an object happens in an interaction to acquire enough speed to escape from the group, and the remaining objects are left closer together. Some simulations show that over time, as much as half the mass may be ejected, with agglomeration of the remaining mass. This could be part of the mechanism for forming massive black holes.

For more information, see the home page of Andrea Ghez, http://www.astro.ucla.edu/faculty/ghez.htm. The papers available there refer to "arcseconds," which is an angular measure of how far apart objects appear on the sky, and to "parsecs," which is a distance equal to 3.3 light-years (a light-year is the distance light goes in one year).

Problem 2b.11 Looping roller coaster

What is the minimum speed v that a roller coaster car must have in order to make it around an inside loop and just barely lose contact with the track at the top of the loop (Figure 2b.46)? The center of the car moves along a circular arc of radius R. Include a carefully labeled force diagram. State briefly what approximations you make. Design a plausible roller coaster loop, including numerical values for v and R.

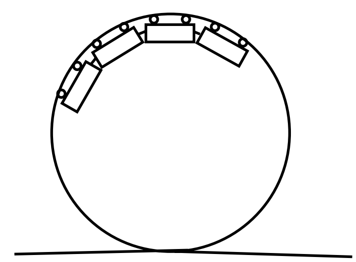


Figure 2b.46 Make a loop in a roller coaster (Problem 2b.11).

Problem 2b.12 Your weight on a bathroom scale

If you stand on a bathroom scale at the North Pole, the scale shows your "weight" as an amount Mg (actually, it shows the force F_N that the scale exerts on your feet). At the North Pole you are 6357 km from the center of the Earth. If instead you stand on the scale at the equator, the scale reads a different value due to two effects: 1) The Earth bulges out at the equator (due to its rotation), and you are 6378 km from the center of the Earth. 2) You are moving in a circular path due to the rotation of the Earth (one rotation every 24 hours). Taking into account both of these effects, what does the scale read at the equator?

Problem 2b.13 Space station

An engineer whose mass is 70 kg holds onto the outer rim of a rotating space station whose radius is 14 m and which takes 30 s to make one complete rotation. What is the magnitude of the force the engineer has to exert in order to hold on? What is the magnitude of the net force acting on the engineer?

Problem 2b.14 NEAR spacecraft orbits Eros

After the NEAR spacecraft passed Mathilde, on several occasions rocket propellant was expelled to adjust the spacecraft's momentum in order to follow a path that would approach the asteroid Eros, the final destination for the mission. After getting close to Eros, further small adjustments made the momentum just right to give a circular orbit of radius 45 km (45×10^3 m) around the asteroid. So much propellant had been used that the final mass of the spacecraft while in circular orbit around Eros was only 500 kg. The spacecraft took 1.04 days to make one complete circular orbit around Eros. Calculate what the mass of Eros must be.

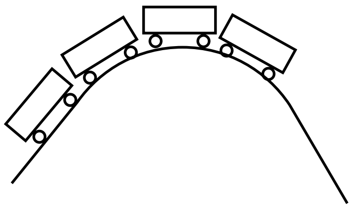


Figure 2b.47 Go over the top in a roller coaster (Problem 2b.15a).

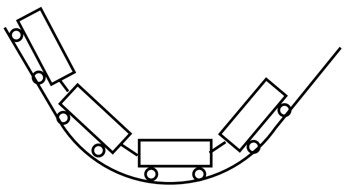


Figure 2b.48 Go through a dip in a roller coaster (Problem 2b.15b).

Problem 2b.15 Weightlessness

By "weight" we usually mean the gravitational force exerted on an object by the Earth. However, when you sit in a chair your own perception of your own "weight" is based on the contact force the chair exerts upward on your rear end rather than on the gravitational force. The smaller this contact force is, the less "weight" you perceive, and if the contact force is zero, you feel peculiar and "weightless" (an odd word to describe a situation when the only force acting on you is the gravitational force exerted by the Earth!). Also, in this condition your internal organs no longer press on each other, which presumably contributes to the odd sensation in your stomach.

(a) How fast must a roller coaster car go over the top of a circular arc for you to feel "weightless"? The center of the car moves along a circular arc of radius R (Figure 2b.47). Include a carefully labeled force diagram.

(b) How fast must a roller coaster car go through a circular dip for you to feel three times as "heavy" as usual, due to the upward force of the seat on your bottom being three times as large as usual? The center of the car moves along a circular arc of radius R (Figure 2b.48). Include a carefully labeled force diagram.

Problem 2b.16 At the bottom of a Ferris wheel

A Ferris wheel is a vertical, circular amusement ride. Riders sit on seats that swivel to remain horizontal as the wheel turns. The wheel has a radius R and rotates at a constant rate, going around once in a time T. At the bottom of the ride, what are the magnitude and direction of the force exerted by the seat on a rider of mass m? Include a diagram of the forces on the rider.

Problem 2b.17 Swing a ball on a spring

A ball of unknown mass m is attached to a spring. In outer space, far from other objects, you hold the other end of the spring and swing the ball around in a circle of radius 1.5 m at constant speed.

(a) You time the motion and observe that going around 10 times takes 6.88 seconds. What is the speed of the ball?

(b) Is the momentum of the ball changing or not? How can you tell?

(c) If the momentum is changing, what interaction is causing it to change? If the momentum is not changing, why isn't it?

(d) The relaxed length of the spring is 1.2 m, and its stiffness is 1000 N/m. While you are swinging the ball, since the radius of the circle is 1.5 m, the length of the spring is also 1.5 m. What is the magnitude of the force that the spring exerts on the ball?

(e) What is the mass m of the ball?

Problem 2b.18 Car going over a hill

A sports car (and its occupants) of mass M, is moving over the rounded top of a hill of radius R. At the instant when the car is at the very top of the hill, the car has a speed v. You can safely neglect air resistance.

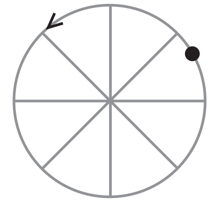
(a) Taking the sports car as the system of interest, what object(s) exert non-negligible forces on this system?

(b) At the instant when the car is at the very top of the hill, draw a diagram showing the system as a dot, with force vectors whose tails are at the location of the dot. Label the force vectors (that is, give them algebraic names). Try to make the lengths of the force vectors be proportional to the magnitudes of the forces.

(c) Starting from the momentum principle, calculate the force exerted by the road on the car.

(d) Under what conditions will the force exerted by the road on the car be zero? Explain.

Problem 2b.19 Ferris wheel



A person of mass 70 kg rides on a Ferris wheel whose radius is 4 m. The person's speed is constant at 0.3 m/s. What is the magnitude of the *net* force acting on the person at the instant shown? Draw the *net* force vector on the diagram at this instant, with the tail of the vector on the person (shown as a dot on the diagram). Show your work clearly.

Problem 2b.20 Outer space collision

In outer space a small rock with mass 5 kg traveling with velocity $\langle 0, 1800, 0 \rangle$ m/s strikes a stationary large rock head-on and bounces straight back with velocity $\langle 0, -1500, 0 \rangle$ m/s. After the collision, what is the vector momentum of the large rock?

Problem 2b.21 Two rocks collide

Two rocks collide in outer space. Before the collision, one rock had mass 9 kg and velocity $\langle 4100, -2600, 2800 \rangle$ m/s. The other rock had mass 6 kg and velocity $\langle -450, 1800, 3500 \rangle$ m/s. A 2 kg chunk of the first rock breaks off and sticks to the second rock. After the collision the rock whose mass is 7 kg has velocity $\langle 1300, 200, 1800 \rangle$ m/s. After the collision, what is the velocity of the other rock, whose mass is 8 kg?

Problem 2b.22 Two rocks collide

Two rocks collide with each other in outer space, far from all other objects. Rock 1 with mass 5 kghas velocity $\langle 30, 45, -20 \rangle$ m/s before the collision and $\langle -10, 50, -5 \rangle$ m/s after the collision. Rock 2 with mass 8 kg has velocity $\langle -9, 5, 4 \rangle$ m/s before the collision. Calculate the final velocity of rock 2.

Problem 2b.23 Two rocks stick together

In outer space two rocks collide and stick together. Here are the masses and initial velocities of the two rocks:

Rock 1: mass = 15 kg, initial velocity = $\langle 10, -30, 0 \rangle$ m/s

Rock 2: mass = 32 kg, initial velocity = $\langle 15, 12, 0 \rangle$ m/s

What is the velocity of the stuck-together rocks after the collision?

Problem 2.24 The car and the mosquito

A car of mass M moving in the x direction at high speed v strikes a hovering mosquito of mass m, and the mosquito is smashed against the windshield.

(a) What is the approximate momentum change of the mosquito? Give magnitude and direction. Explain any approximations you make.

(b) At a particular instant during the impact, when the force exerted on the mosquito by the car is F, what is the magnitude of the force exerted on the car by the mosquito?

(c) What is the approximate momentum change of the car? Give magnitude and direction. Explain any approximations you make.

(d) Qualitatively, why is the collision so much more damaging to the mosquito than to the car?

Problem 2.25 The bouncing ball

A steel ball of mass m falls from a height h onto a scale calibrated in newtons. The ball rebounds repeatedly to nearly the same height h. The scale is sluggish in its response to the intermittent hits and displays an *average* force F_{avg}, such that

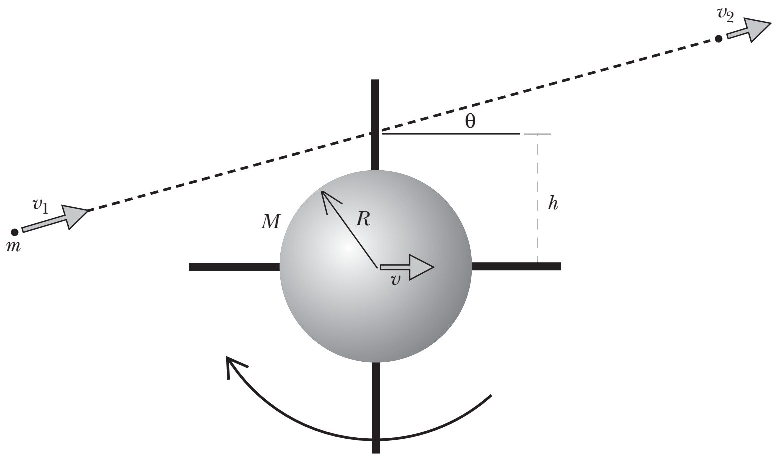
$$F_{avg} T = F \Delta t$$

where $F \Delta t$ is the brief impulse that the ball imparts to the scale on every hit, and T is the time between hits.

Calculate this average force in terms of m, h, and physical constants. Compare your result with what the scale reads if the ball merely rests on the scale. Explain your analysis carefully (but briefly).

Problem 2b.26 Meteor hits a spinning satellite

A satellite which is spinning clockwise has four low-mass solar panels sticking out as shown. A tiny meteor traveling at high speed rips through one of the solar panels and continues in the same direction but at reduced speed. Afterwards, calculate the v_x and v_y components of the center-of-mass velocity of the satellite. Initial data are provided on the diagram.



Problem 2.27 Moving the Earth

Suppose all the people of the Earth go to the North Pole and, on a signal, all jump straight up. Estimate the recoil speed of the Earth. The mass of the Earth is 6×10^{24} kg, and there are about 6 billion people (6×10^{9}).

Problem 2b.28 Bullet embeds in block

A bullet of mass m traveling horizontally at a very high speed v embeds itself in a block of mass M that is sitting at rest on a nearly frictionless surface. What is the speed of the block after the bullet embeds itself in the block?

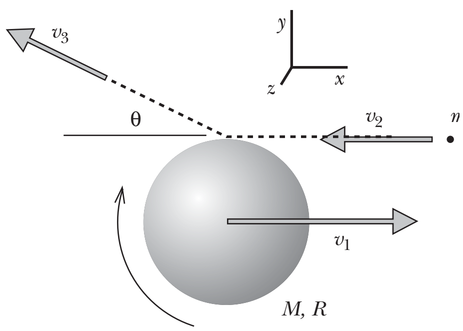


Figure 2b.49 Space junk strikes a spinning satellite (Problem 2b.29).

Problem 2b.29 Space junk

A tiny piece of space junk of mass m strikes a glancing blow to a spinning satellite (Figure 2b.49). After the collision the space junk is traveling in a new direction and moving more slowly. The space junk had negligible rotation both before and after the collision. The velocities of the space junk before and after the collision are shown in the diagram. The satellite has mass M and radius R,. Before the collision the satellite was moving and rotating as shown in the diagram. Just after the collision, what are the components of the center-of-mass velocity of the satellite (v_x and v_y)?

Problem 2b.30 Two balls collide

A ball of mass 0.05 kg moves with a velocity $\langle 17, 0, 0\rangle$ m/s. It strikes a ball of mass 0.1 kg which is initially at rest. After the collision, the heavier ball moves with a velocity of $\langle 3, 3, 0\rangle$ m/s.

(a) What is the velocity of the lighter ball after impact?

(b) What is the impulse delivered to the 0.05 kg ball by the heavier ball?

(c) If the time of contact between the balls is 0.03 sec, what is the force exerted by the heavier ball on the lighter ball?

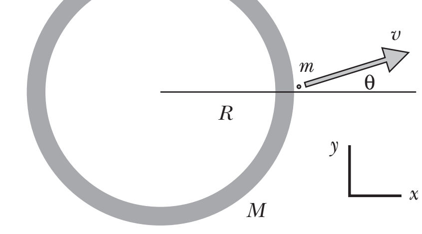


Figure 2b.50 Launch a package from a space station (Problem 2b.31).

Problem 2b.31 Space station

A space station has the form of a hoop of radius R, with mass M (Figure 2b.50). Initially its center of mass is not moving, but it is spinning. Then a small package of mass m is thrown by a spring-loaded gun toward a nearby spacecraft as shown; the package has a speed v after launch. Calculate the center-of-mass velocity of the space station (v_x and v_y) after the launch.

2b.13 Answers to exercises

2b.1 (page 115) 4 N

2b.2 (page 115) $\vec{0}$, $\vec{0}$

2b.3 (page 115) $\vec{0}$, $\vec{0}$

2b.4 (page 115) $\langle 2, 0, -4\rangle$ m/s^2, $\langle 1.6, 0, -3.2\rangle$ (kg · m/s)/s,

$\langle 1.6, 0, -3.2\rangle$ N

2b.5 (page 119) nonzero, downward

2b.6 (page 119) nonzero, downward

2b.7 (page 119) nonzero, in direction of final momentum

2b.8 (page 121) 0.975 (kg · m/s)/s toward center

0.975 N toward center

2b.9 (page 121) Both $d\vec{p}/dt$ and $\vec{F}_{net}$ point toward the center

2b.10 (page 121) 3.57×10^{22} (kg · m/s)/s toward the Sun

3.57×10^{22} N toward the Sun

2b.11 (page 128) 8000 m/s (8 km/s)

2b.12 (page 128) 22.5 N, toward center of circle

2b.13 (page 129) 3×10^{4} m/s

2b.14 (page 134) $\langle 0.04, 0.04, -0.1\rangle$ kg · m/s,

$\langle 0.04, 0.04, -0.1\rangle$ kg · m/s,

$\langle 1, 1, -2.5\rangle$ m/s

2b.15 (page 134) $\langle 18, 14, 7\rangle$ kg · m/s, $\langle 18, 14, 7\rangle$ kg · m/s

2b.16 (page 134) $\langle 1\times10^{-21}, -1.6\times10^{-21}, 0\rangle$ kg · m/s

2b.17 (page 135) (a) $-m\vec{v}$, $+m\vec{v}$; (b) $+m\vec{v}$, $-m\vec{v}$; (c) $\vec{0}$, $\vec{0}$