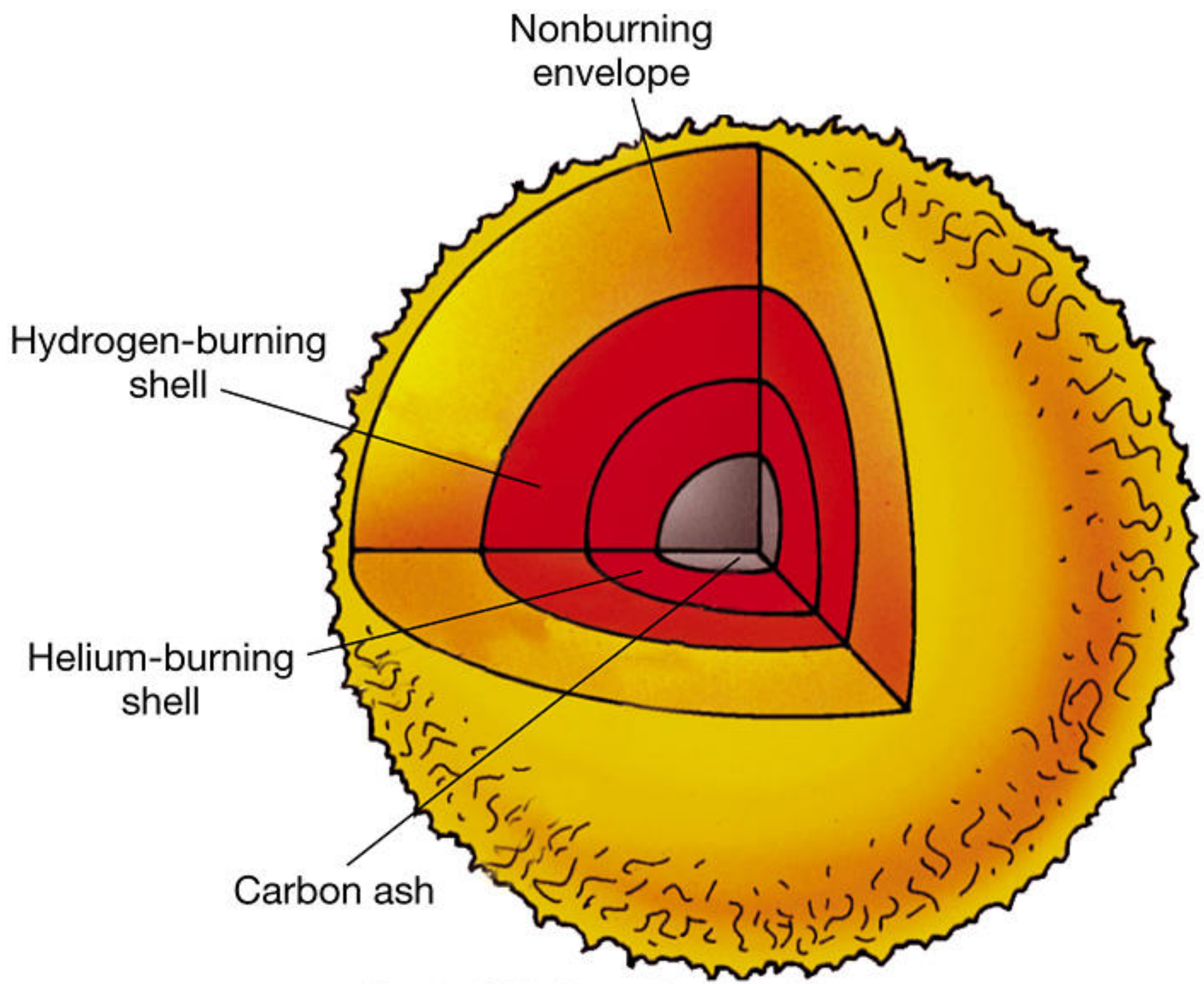
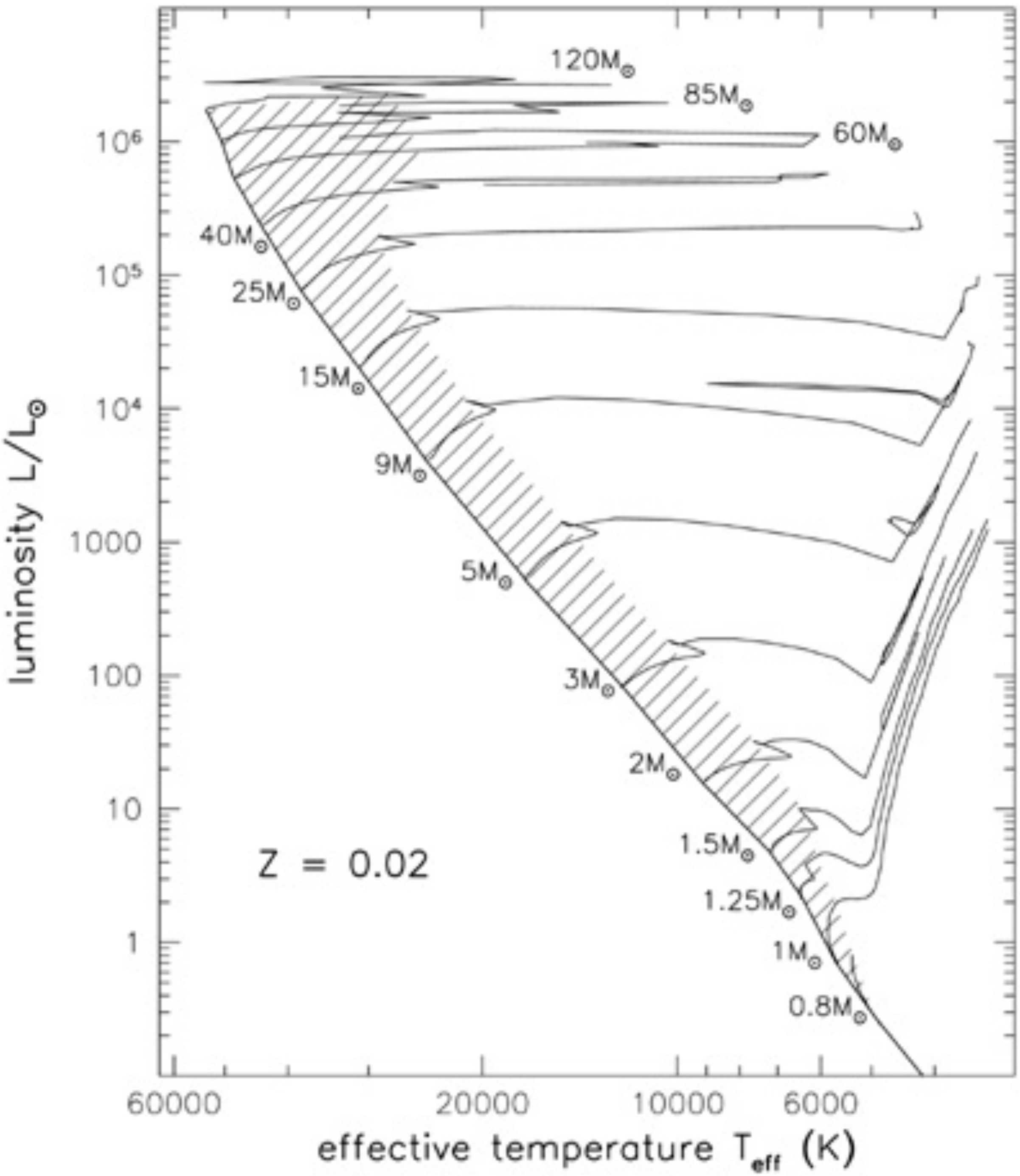
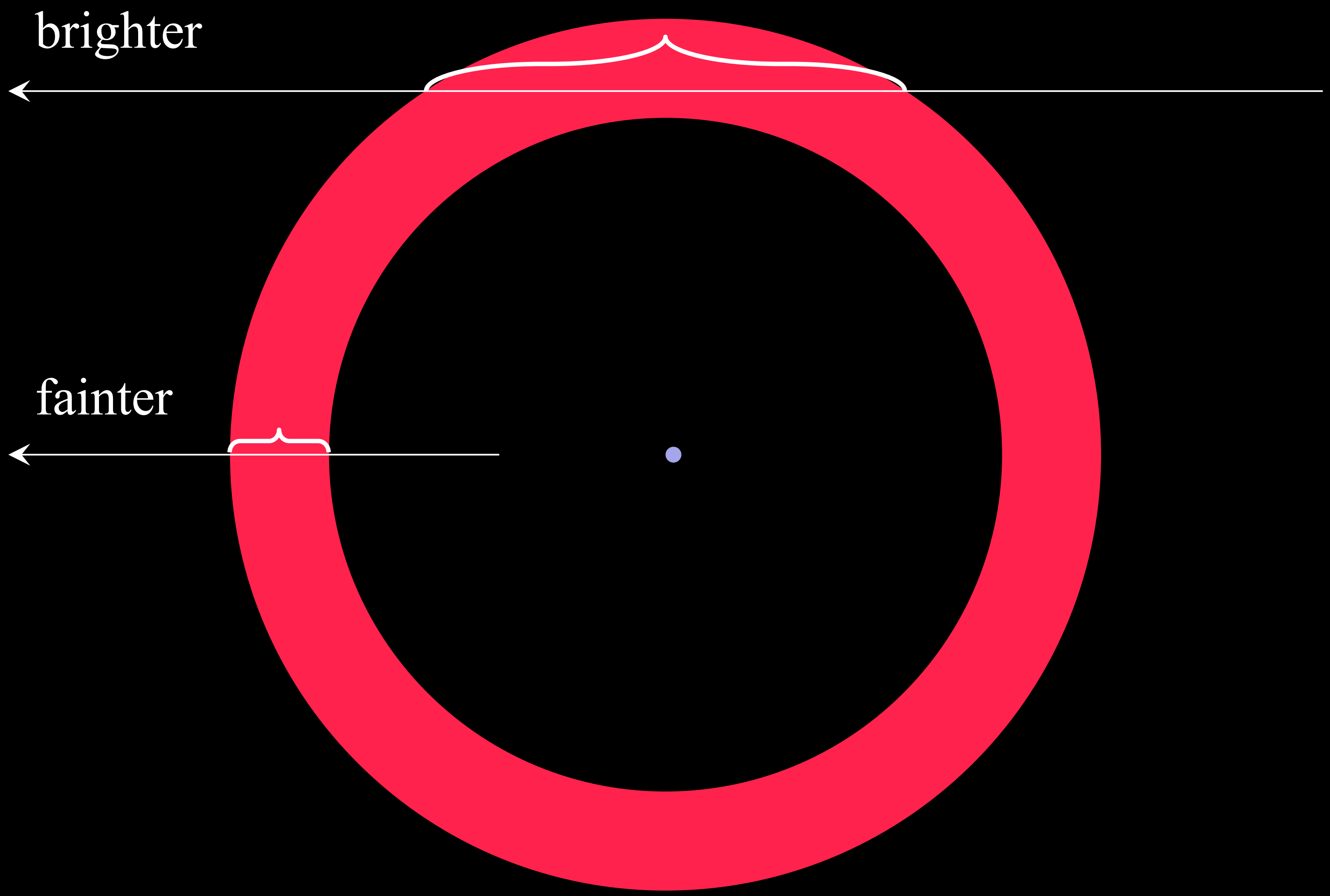
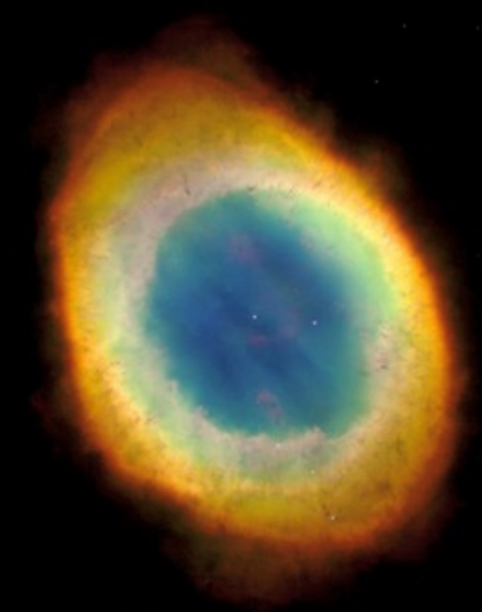

Nonburning
envelope
Hydrogen-burning
shell
Helium-burning
shell
Carbon ash



120M⊙
85M⊙
60M⊙
40M⊙
25M⊙
15M⊙
9M⊙
5M⊙
3M⊙
2M⊙
1.5M⊙
1.25M⊙
1M⊙
0.8M⊙
Z = 0.02
luminosity L/L⊙
10^6
10^5
10^4
1000
100
10
1
60000
20000
10000
6000
effective temperature T_eff (K)

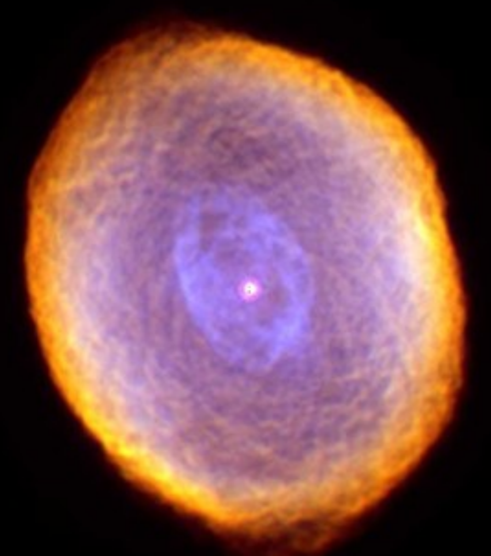

brighter
fainter

Eskimo Nebula



Ring Nebula

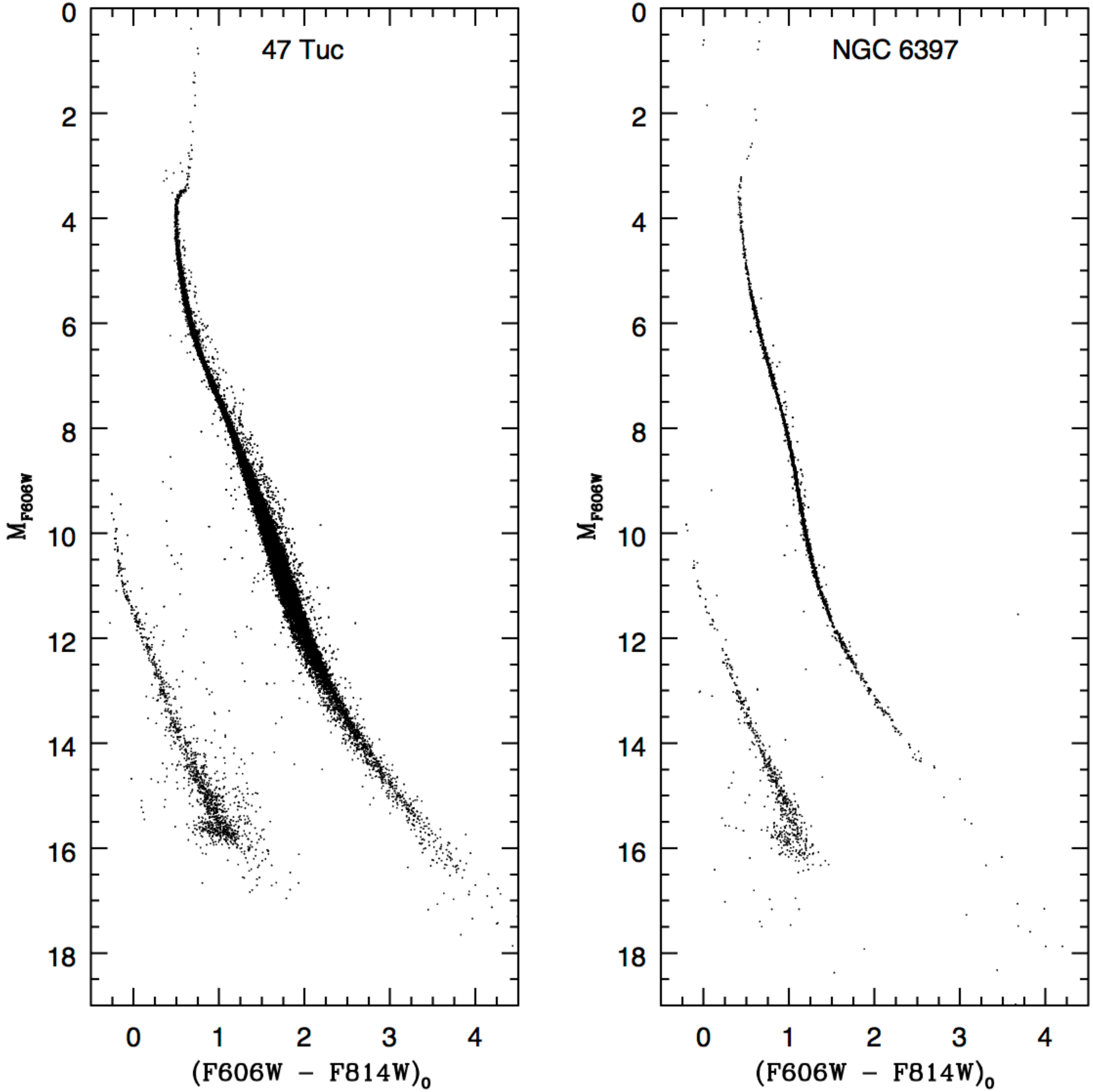
Necklace Nebula



Spirograph Nebula (IC 418)

Cat’s Eye Nebula

Hour Glass Nebula



47 Tuc
NGC 6397
M_F606W
(F606W − F814W)_0
0
2
4
6
8
10
12
14
16
18
0
1
2
3
4

Quantum gas

de Broglie wavelength of an electron:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}$$

non-classical behavior when

$$n^{-1/3} < \lambda/2$$

$m_e \ll m_p$
affects <u>electrons</u> first

$$\Longrightarrow \rho > \rho_q = \frac{8m_p}{h^3}(3m_e kT)^{3/2}$$

$$= 3.5\times10^5 \text{ kg m}^{-3} \left(\frac{T}{10^7\ K}\right)^{3/2}$$

Heisenberg Uncertainty Principle

$$\Delta x \, \Delta p_x \; > \; h$$

$$\Delta x \, \Delta y \, \Delta z \, \Delta p_x \, \Delta p_y \, \Delta p_z > \; h^3$$

$$dV \; d^3p \; > \; h^3$$

NB position well defined $\Longrightarrow$ ΔV small

$\Longrightarrow$ d^3p large

$\Longrightarrow$ large pressure

"electron degeneracy pressure"

Pauli Exclusion Principle

Two electrons cannot occupy the same quantum state in a quantum system

Fermi-Dirac Distribution

$dN = f_{FD}(E)\, dV\, d^3p$, where

spin: $s = \frac{1}{2}$

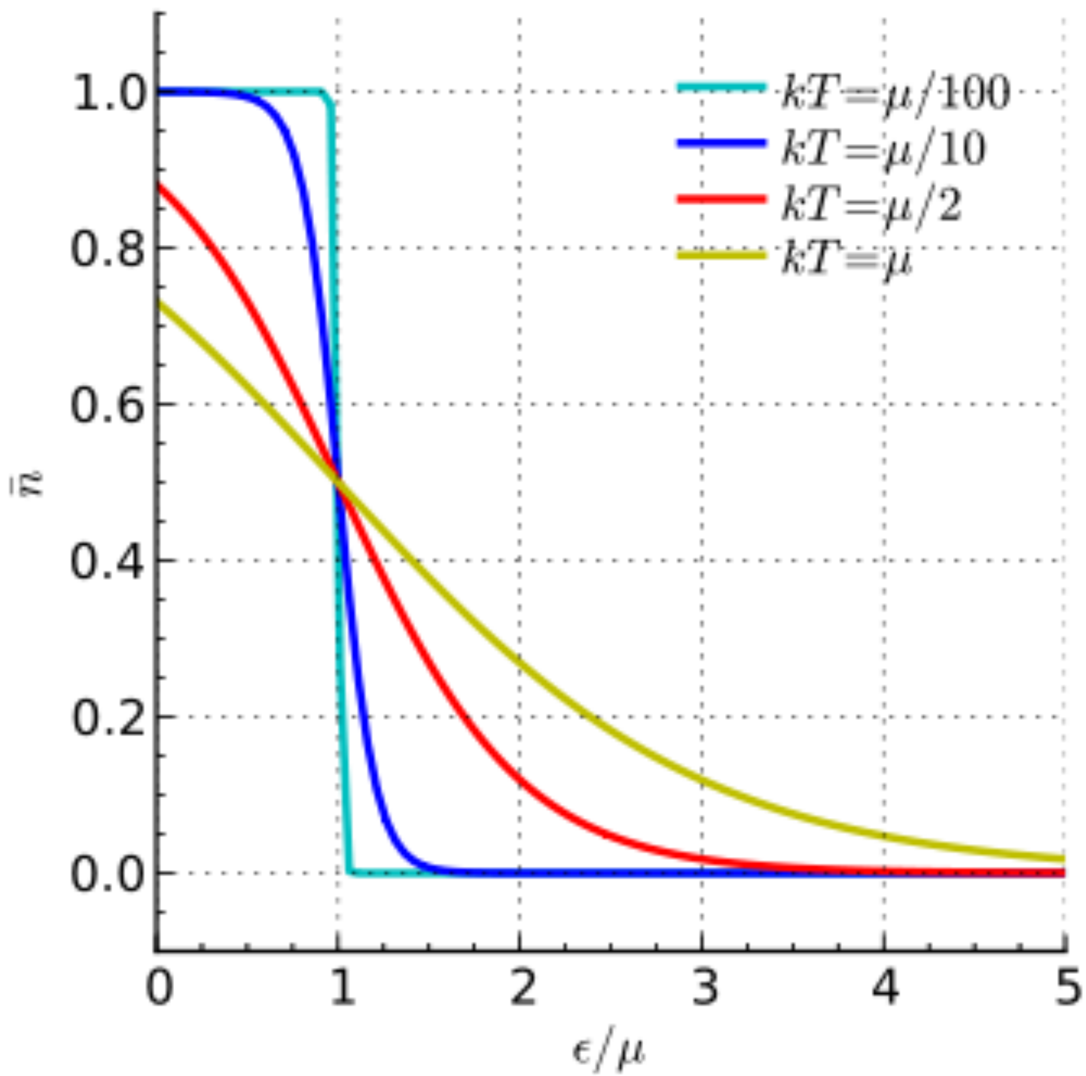
$$f_{FD}(E) = \frac{2s+1}{e^{[E-\mu(T)]/kT}+1}\,\frac{1}{h^3}$$

chemical potential

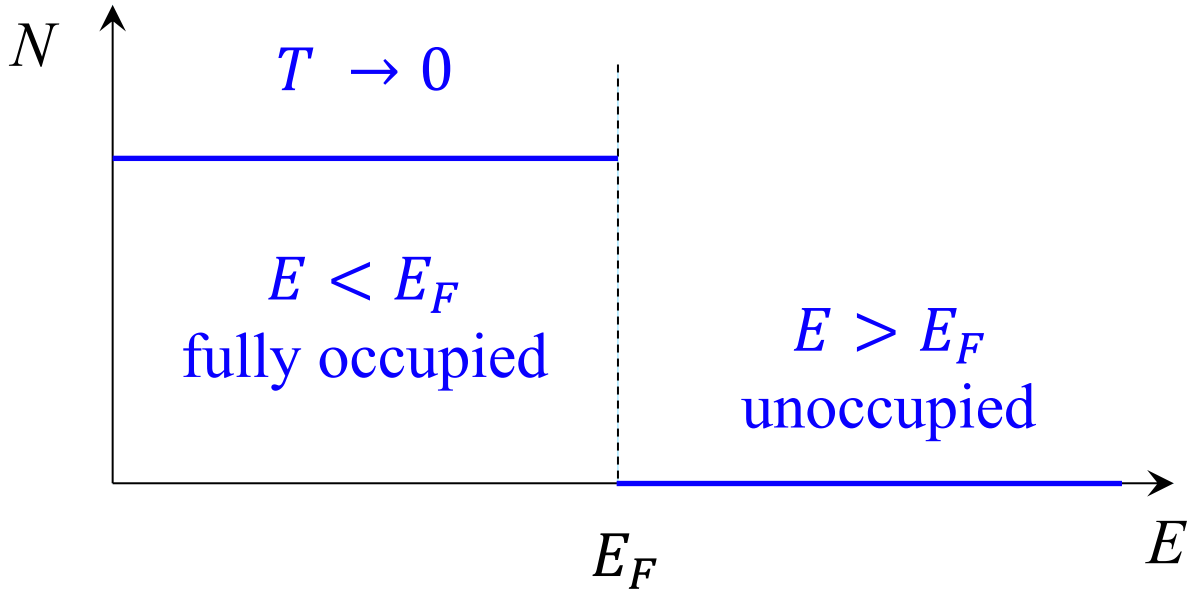
$\mu(T) \rightarrow E_F$ as $T \rightarrow 0$

$E_F =$ Fermi energy

$$f_{FD} = \frac{2s+1}{e^{[E-\mu(T)]/kT}+1}\,\frac{1}{h^3}$$



Fermi-Dirac Distribution



Fermi momentum:

$$p_F = \sqrt{2mE_F}$$

isotropic:

$$d^3p = 4\pi p^2 dp$$

$$\Longrightarrow \quad dN = \begin{cases} \dfrac{8\pi p^2}{h^3}\, dV\, dp, & p < p_f \\ 0, & \text{otherwise} \end{cases}$$

Fermi-Dirac Distribution

$$n_e = \int_0^{p_F} f(p)\,dp$$

$$= \int_0^{p_F} \frac{8\pi}{h^3}\, p^2\,dp$$

$$= \frac{8\,\pi}{3h^3}\, p_F^3$$

$$\Longrightarrow \quad p_F = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

Fermi-Dirac Distribution

$$P_e = \int_0^{p_F} \frac{1}{3} v\, p\, f(p)\, dp$$

$(v = p/m_e)$

$$= \int_0^{p_F} \frac{8\pi}{3h^3} \frac{p^4}{m_e} dp$$

$$= \frac{8\pi}{15h^3 m_e} p_F^5$$

independent of T!

$$= \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{5\, m_e} n_e^{5/3}$$

$n_e = Z\left(\frac{\rho}{Am_p}\right)$

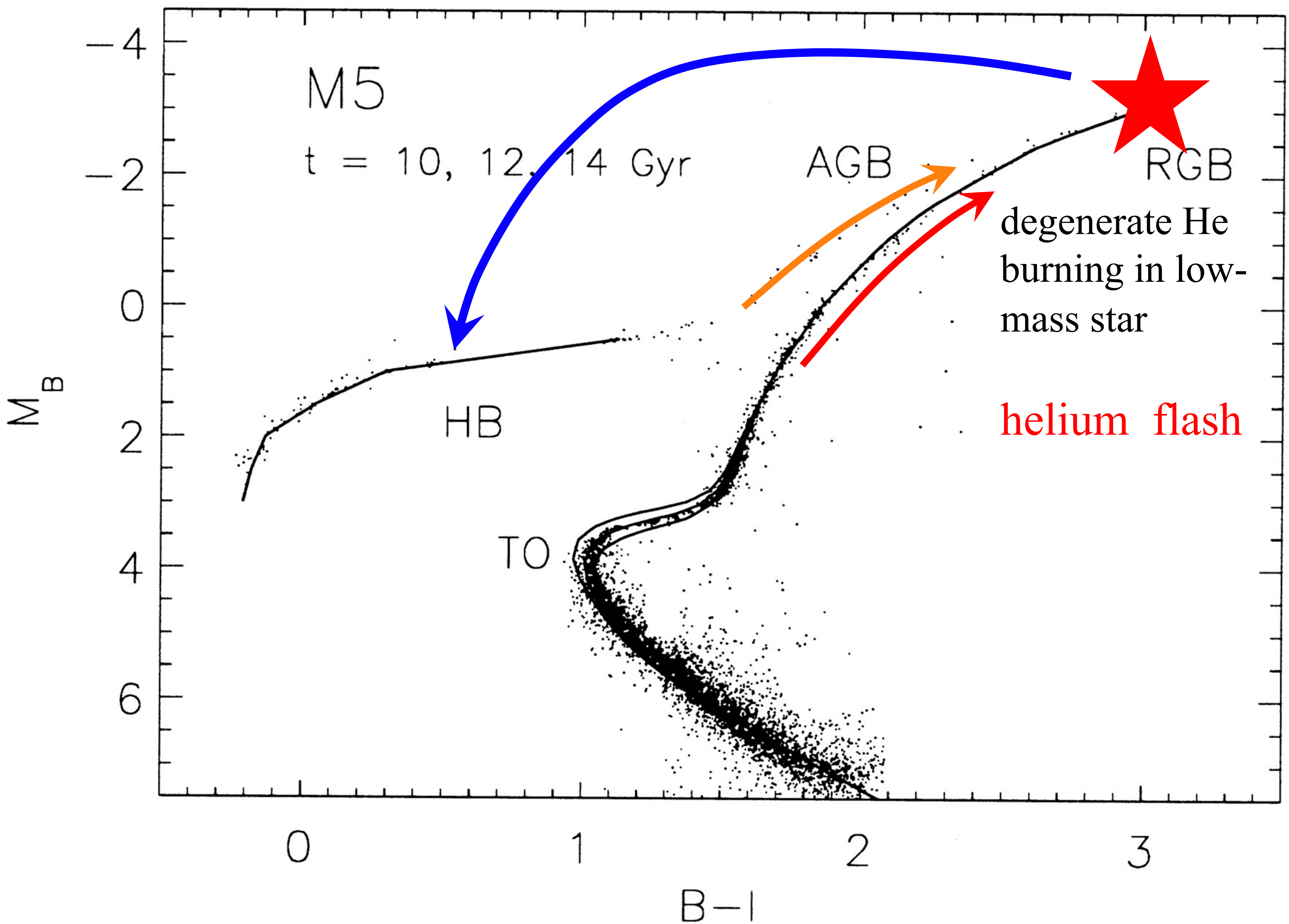
$$= \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

Typical White Dwarf

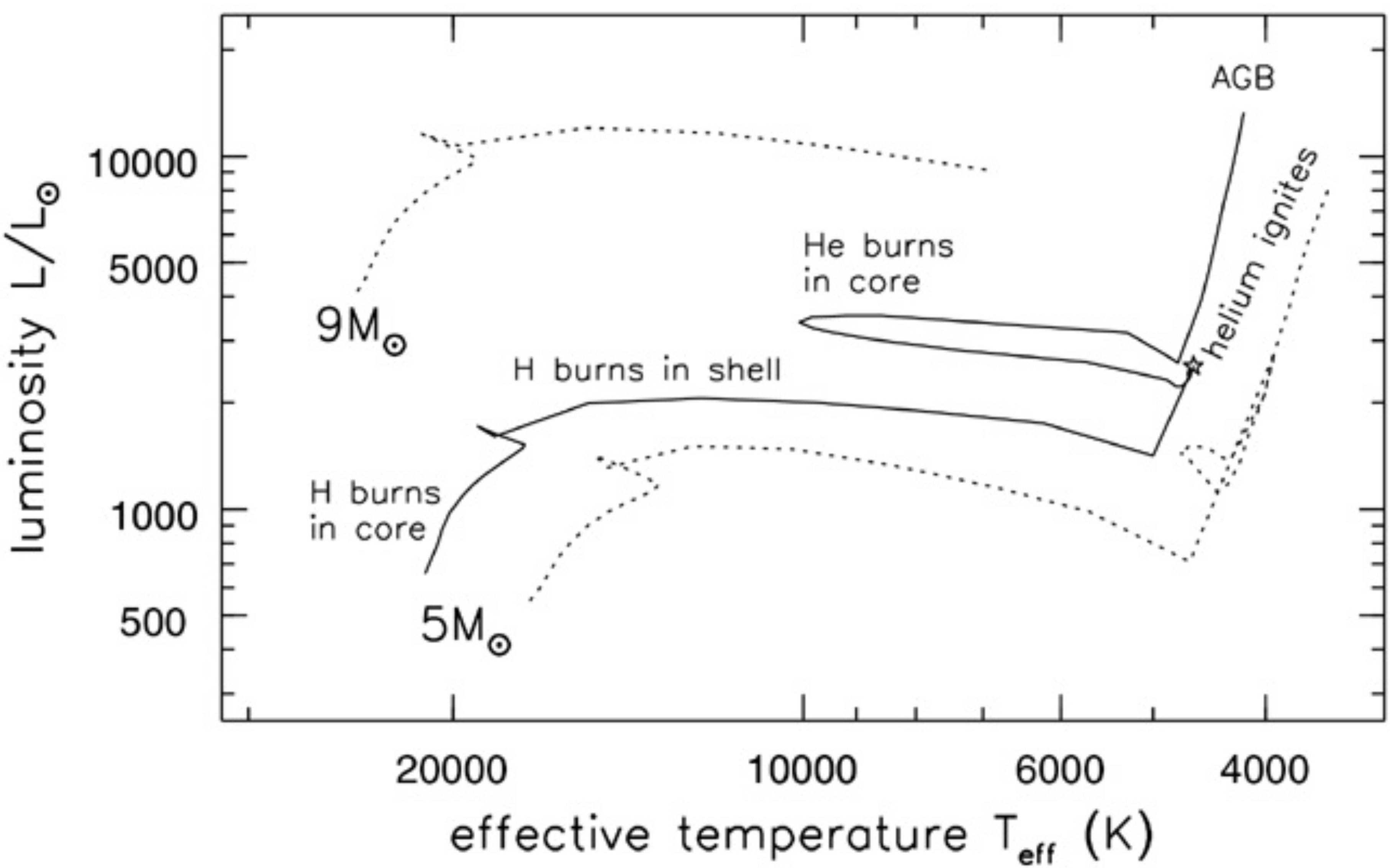
$$\rho \sim 10^9 \text{ kg m}^{-3}$$
$$T \sim 10^7 \text{ K}$$
$$Z/A \sim 0.5$$

$$\Longrightarrow \quad P_e \sim 3\times10^{21} \text{ Pa}$$

thermal: $$P_{th} = n_{tot}\, k\, T$$
$$\sim 2\times10^{19} \text{ Pa}$$
$$\ll P_e$$



M5
t = 10, 12, 14 Gyr
AGB
RGB
HB
TO
degenerate He burning in low-mass star
helium flash
M_B
B−I
−4
−2
0
2
4
6
0
1
2
3



AGB
10000
5000
1000
500
luminosity L/L⊙
He burns
in core
helium ignites
9M⊙
H burns in shell
H burns
in core
5M⊙
20000
10000
6000
4000
effective temperature Teff (K)

Stellar Evolution Outcomes

$< 0.08\ M_\odot$	brown dwarf (degenerate H)	
$0.08-0.25\ M_\odot$	He white dwarf	
$0.25-8\ M_\odot$	CO white dwarf	$t \sim 10^{10}$ yr $(M/M_\odot)^{-3}$
$8-12\ M_\odot$	NeO white dwarf	
$12-20\ M_\odot$	supernova/neutron star	$t \sim 3\text{-}5 \times 10^6$ yr
$>20\ M_\odot$	supernova/black hole	

White Dwarf Mass-Radius Relation

scaling: $\rho \sim \frac{M}{R^3}$

$P \sim \rho^{5/3} \sim \frac{M^{5/3}}{R^5}$

$P \sim \frac{M^2}{R^4}$

$\Longrightarrow R \sim M^{-1/3}$

WD shrinks, $\Delta V \downarrow$, $d^3p \uparrow$ as mass increases

details: $R_{WD} \approx 2.3 \times 10^4 \text{ km} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3}$

Ultrarelativistic White Dwarf

$$P_e = \int_0^{p_F} \frac{1}{3} v\, p\, f(p)\, dp$$

$(v = c)$

$$= \int_0^{p_F} \frac{8\pi c}{3h^3} p^3\, dp$$

$$= \frac{2\pi c}{3h^3}\, p_F^4$$

$$= \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4}\; n_e^{4/3}$$

$n_e = Z\left(\frac{\rho}{Am_p}\right)$

$$= \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

independent of m_e

Ultrarelativistic White Dwarf Mass-Radius Relation

scaling: $\rho \sim \frac{M}{R^3}$

$$P \sim \rho^{4/3} \sim \frac{M^{4/3}}{R^4}$$

$$P \sim \frac{M^2}{R^4}$$

inconsistent!

try: $P \sim \rho^{(4+\epsilon)/3}$

$$\Longrightarrow \frac{M^2}{R^4} \sim \frac{M^{(4+\epsilon)/3}}{R^{4+\epsilon}}$$

$$\Longrightarrow R^{\epsilon} \sim M^{(\epsilon-2)/3}$$

$$\Longrightarrow R \sim M^{(\epsilon-2)/3\epsilon} \rightarrow 0 \text{ as } \epsilon \rightarrow 0$$

white dwarf collapses

Maximum White Dwarf Mass

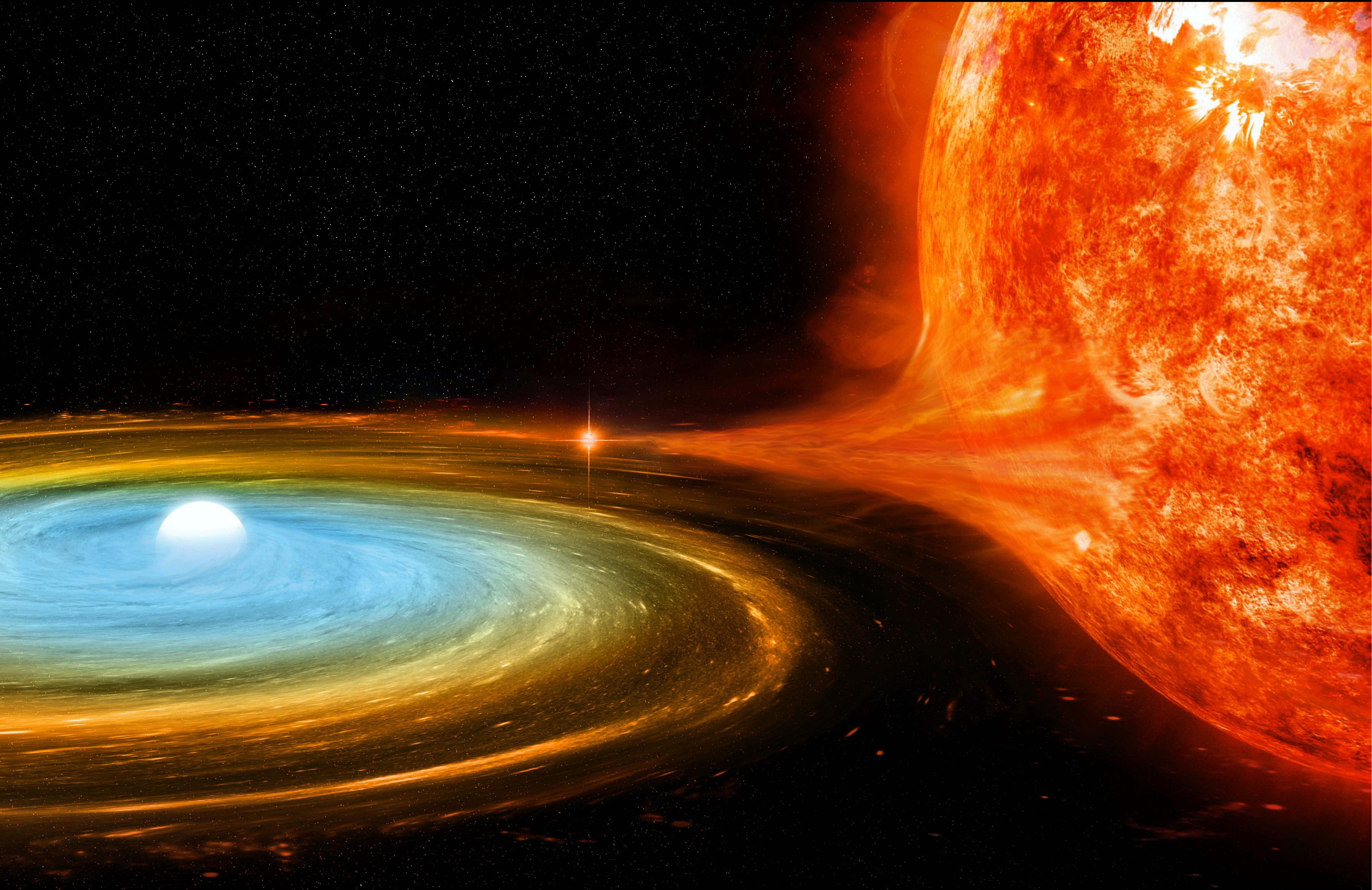
once the electrons in a white dwarf become relativistic, the star must collapse

critical mass is the Chandrasekhar mass:

$$M_{ch} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

$$= 1.4\, M_\odot \quad \text{for} \quad \frac{Z}{A} = 0.5$$

independent of m_e



White Dwarf Collapse

collapse $\Longrightarrow$ $T\uparrow \Longrightarrow C/O$ burning throughout

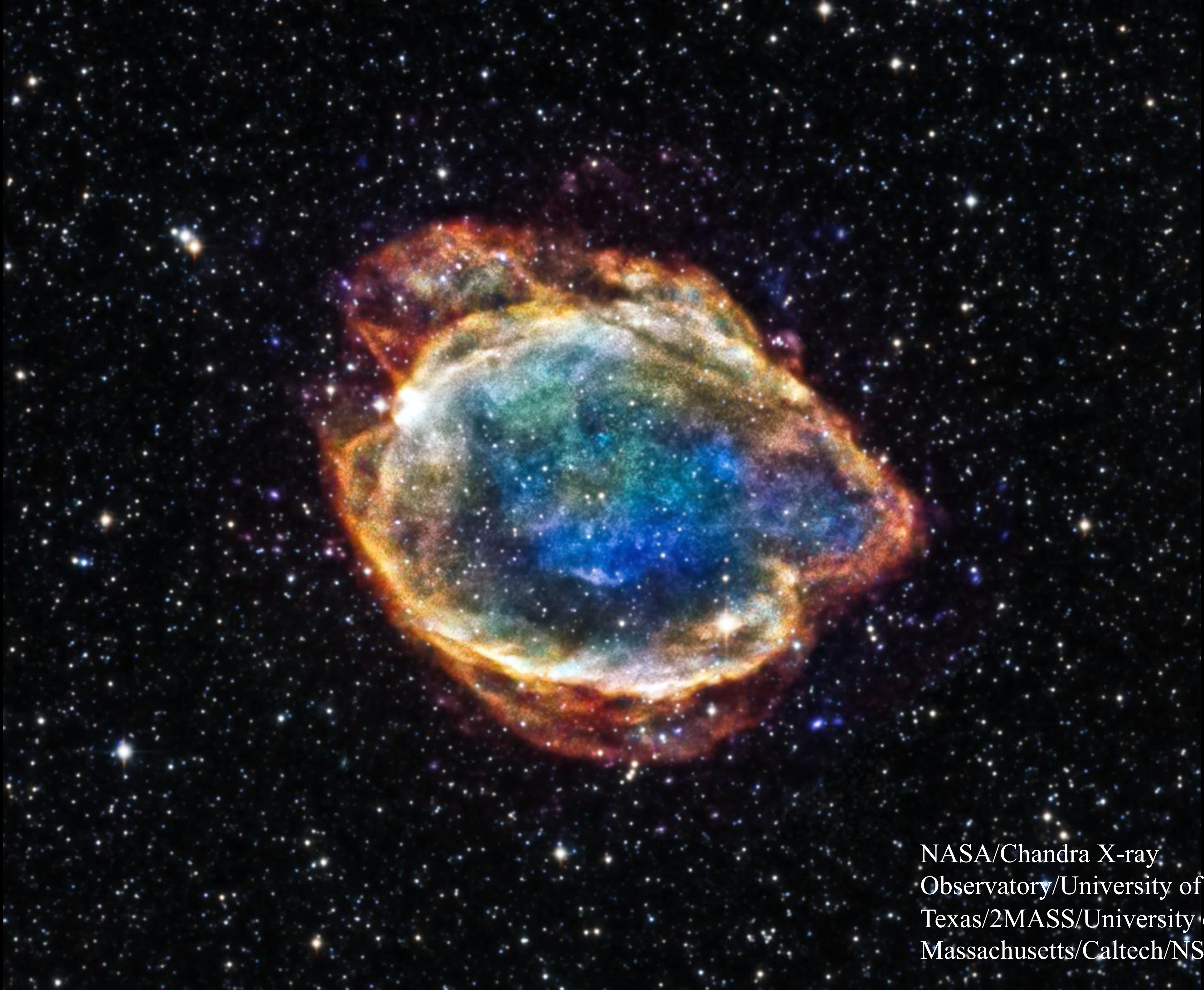
$\Longrightarrow$ deflagration/explosion

$\Longrightarrow$ <u>supernova</u> (Type Ia)

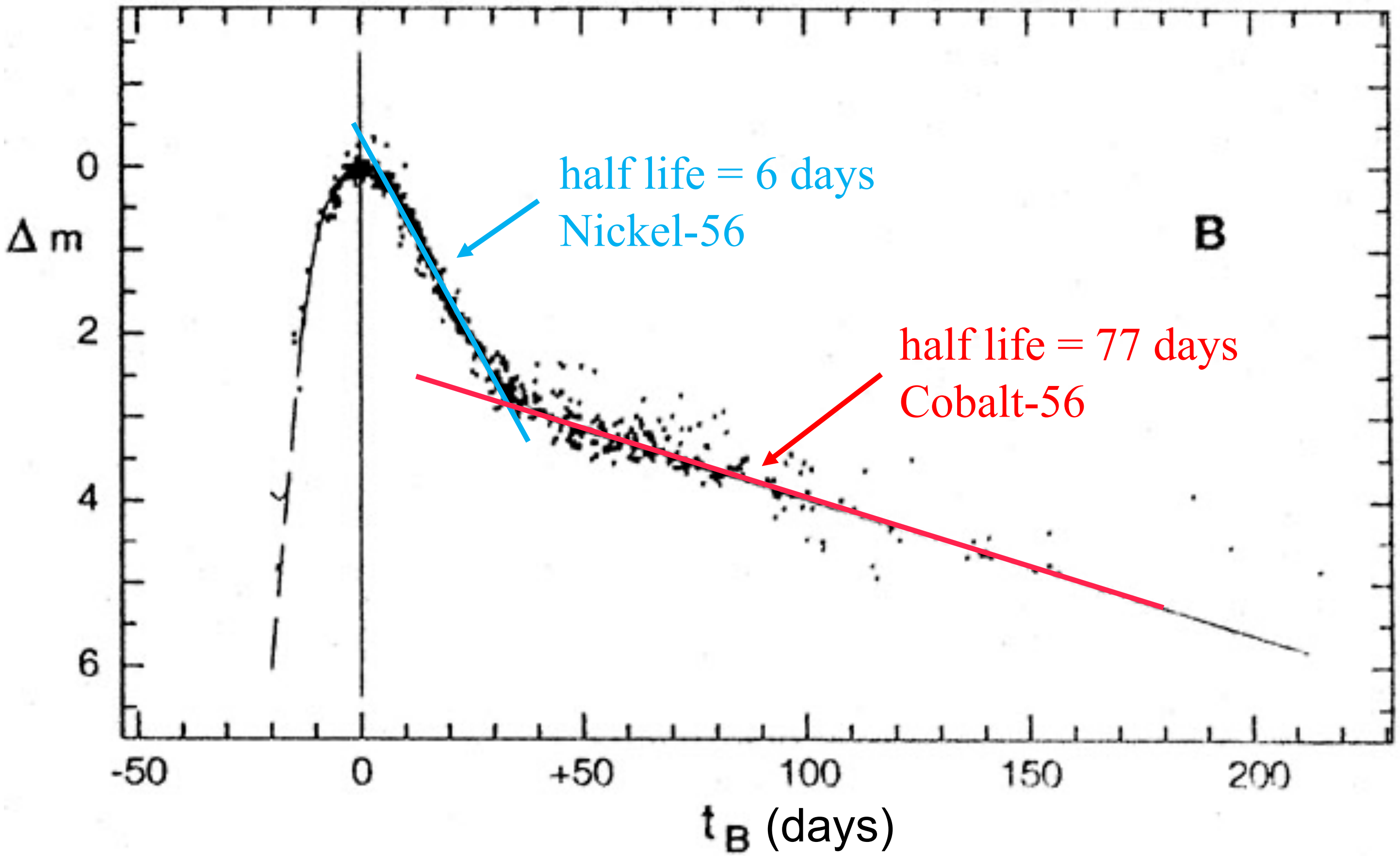
energetics: $\sim 10^{44}$ J total ($\sim L_\odot \times 10^{10}$ yr)

light curve: radioactive decay (Ni, Co, …)

peak luminosity $\sim 5\times10^{9}\ L_\odot$



NASA/Chandra X-ray Observatory/University of Texas/2MASS/University of Massachusetts/Caltech/NSF



Cadonau 1987

White Dwarf Deflagration

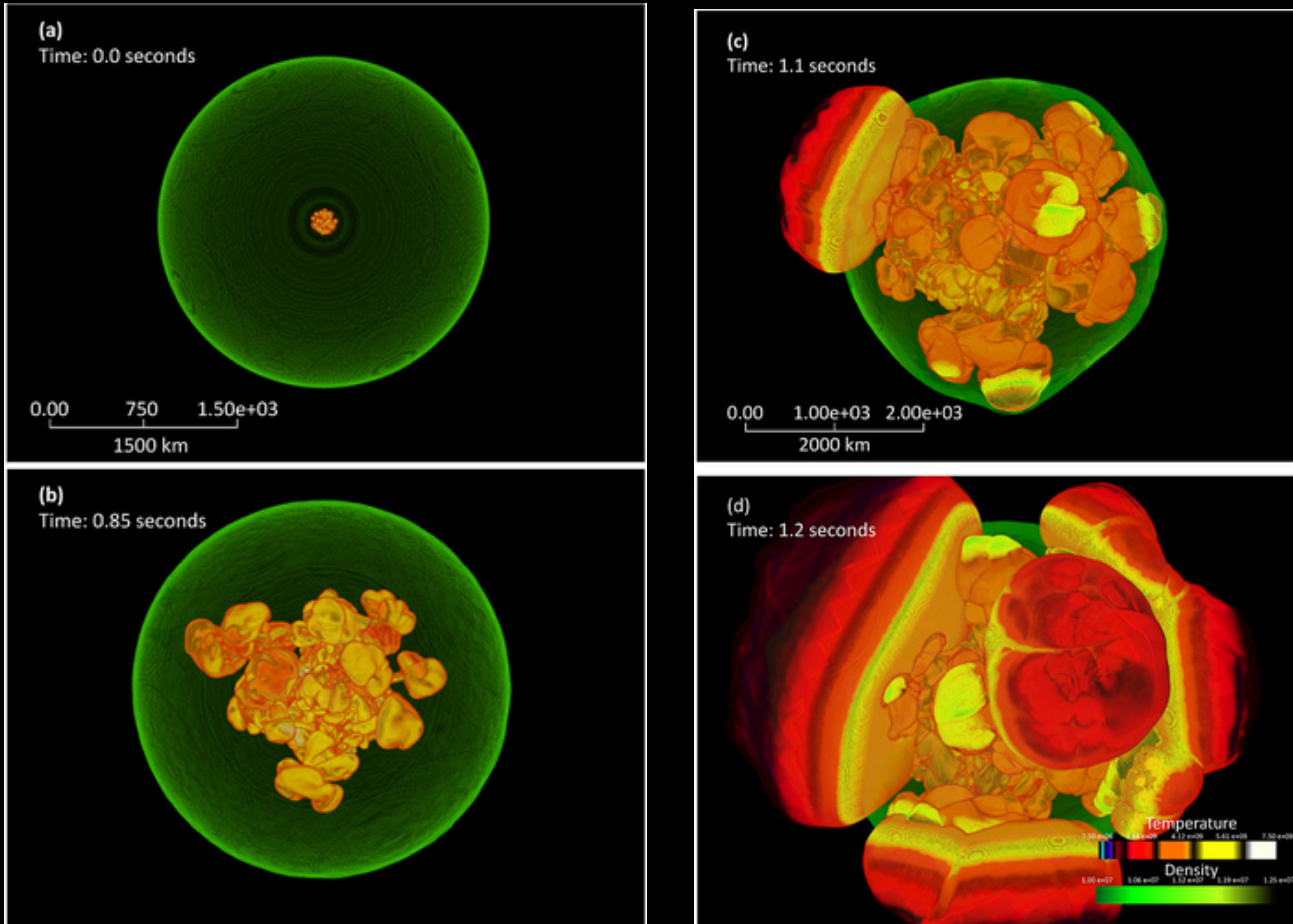
Resolution: 6 km

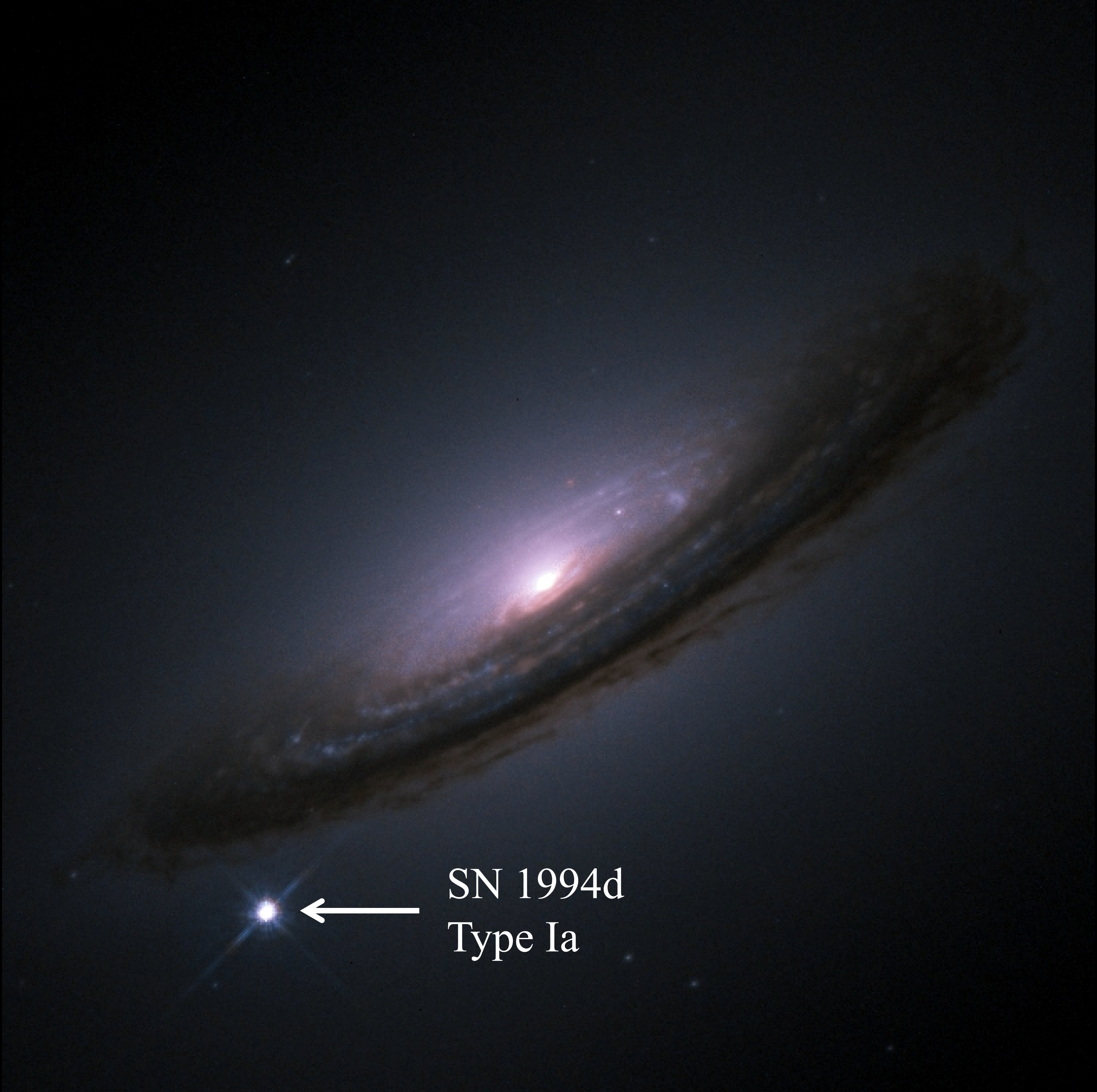
Initial Bubble Radius: 18 km

Ignition Offset: 42 km

Variable 1: Density [1.5e+07 - 2.0e+07]

Variable 2: Reaction Progress [0.0 - 1.0]


(a)
Time: 0.0 seconds
0.00
750
1.50e+03
1500 km
(b)
Time: 0.85 seconds
(c)
Time: 1.1 seconds
0.00
1.00e+03
2.00e+03
2000 km
(d)
Time: 1.2 seconds
Temperature
Density


SN 1994d
Type Ia

High-Mass Stars

$He, C, O, \ldots$ fuse non-degenerately

alpha capture:

$$^4He + {}^{12}C \rightarrow {}^{16}O + \gamma$$

$$^4He + {}^{16}O \rightarrow {}^{20}Ne + \gamma$$

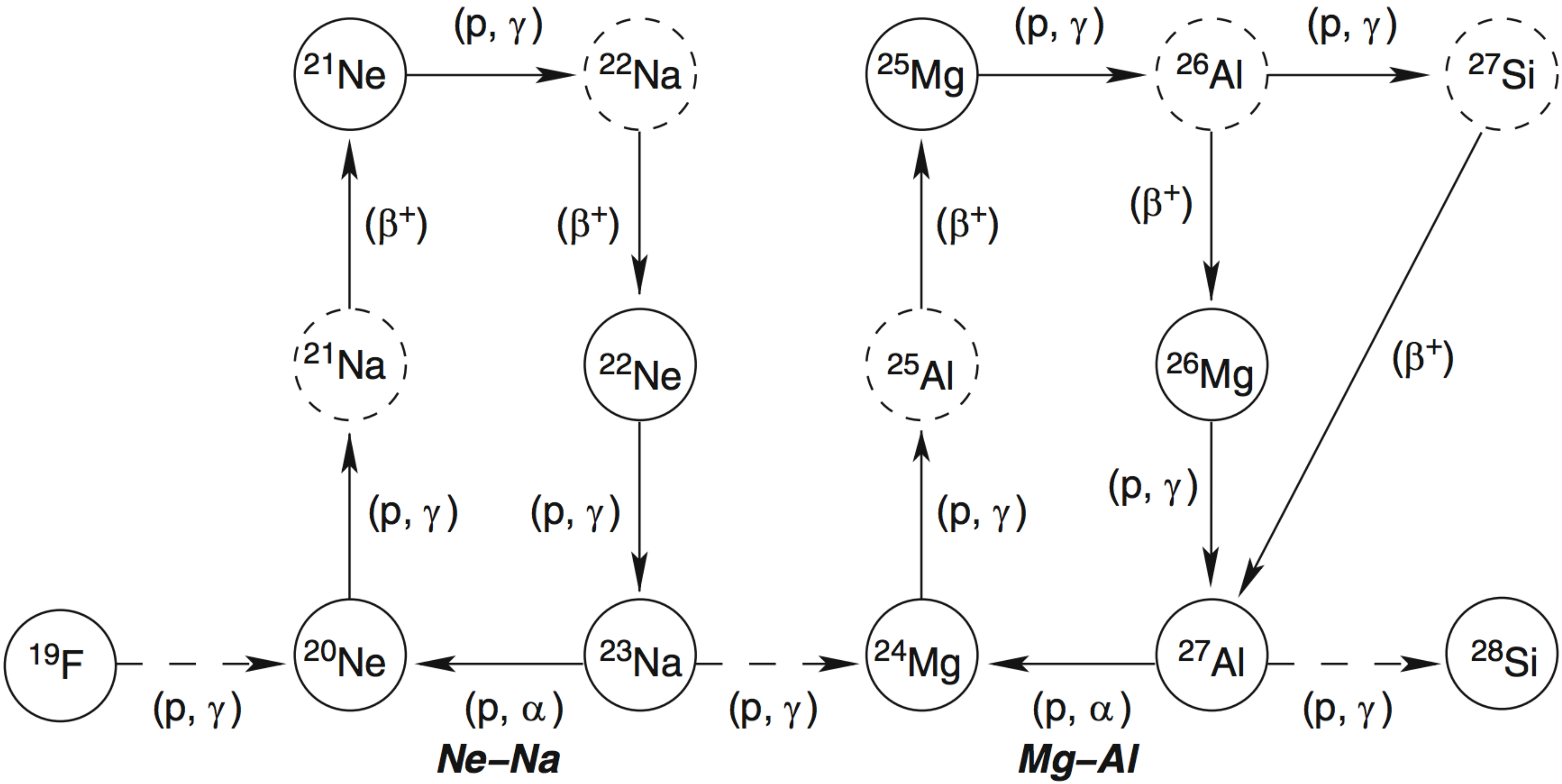
$$^4He + {}^{20}Ne \rightarrow {}^{24}Mg + \gamma$$

C fusion:

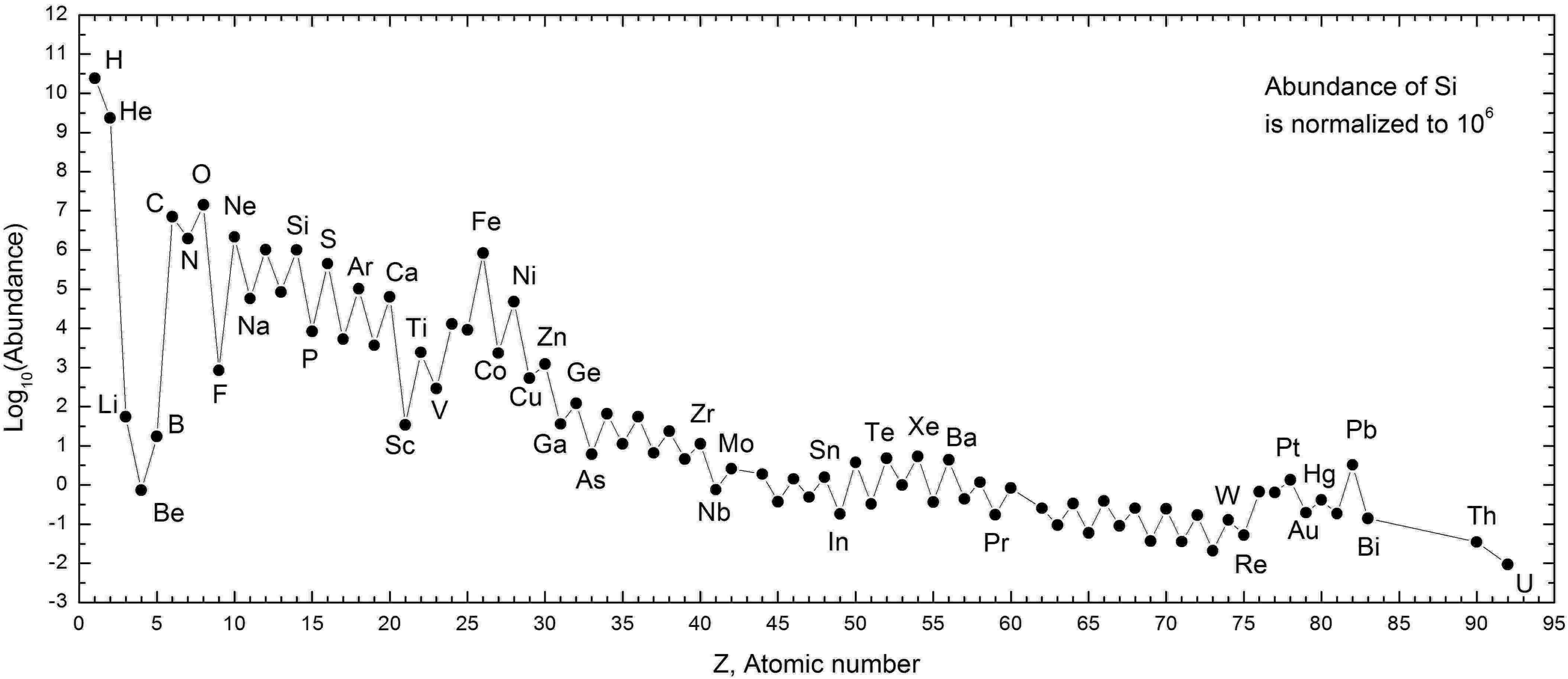
$$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^4He + \gamma$$

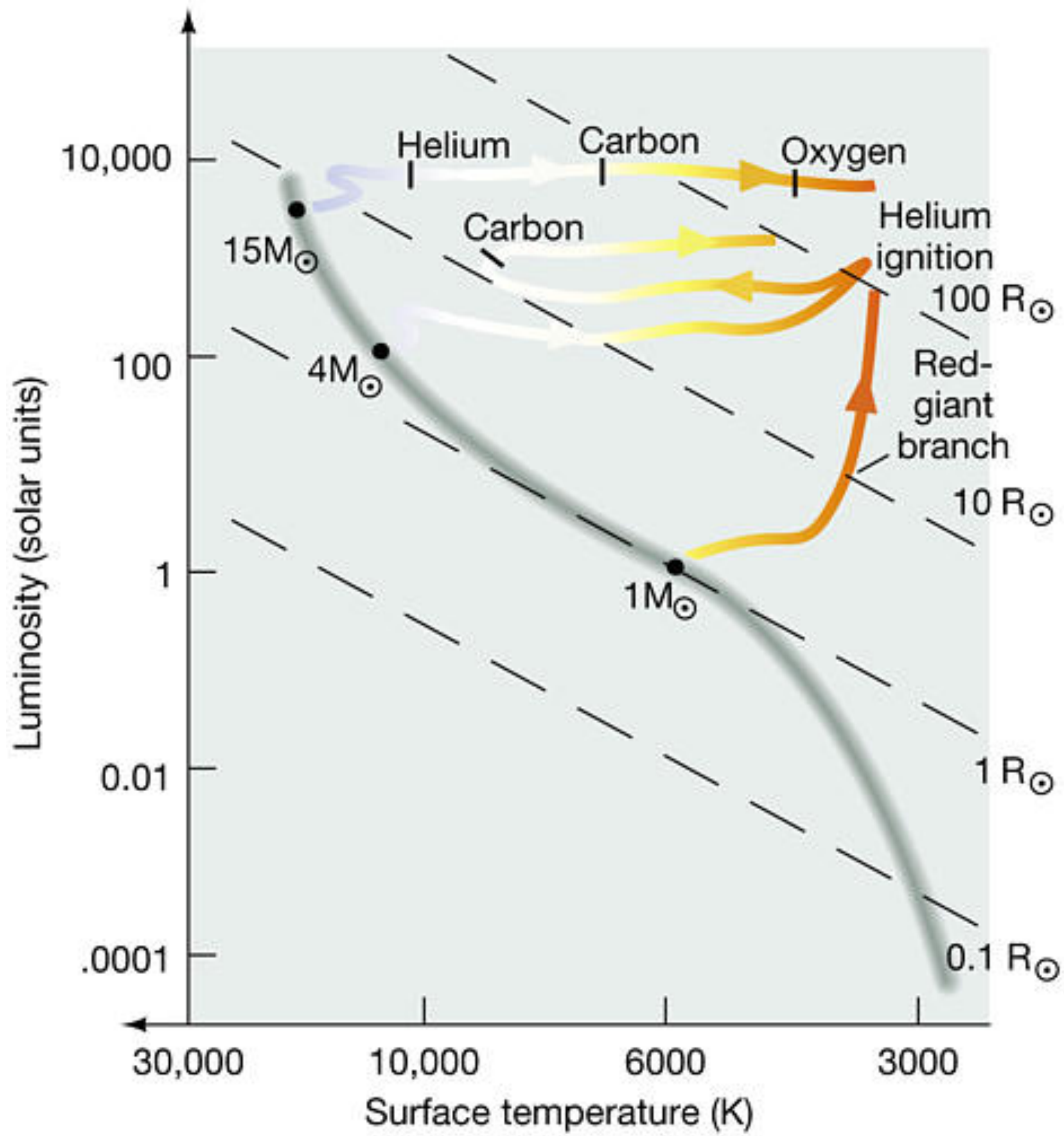
or $^{23}Na + p$

or $^{23}Mg + n$

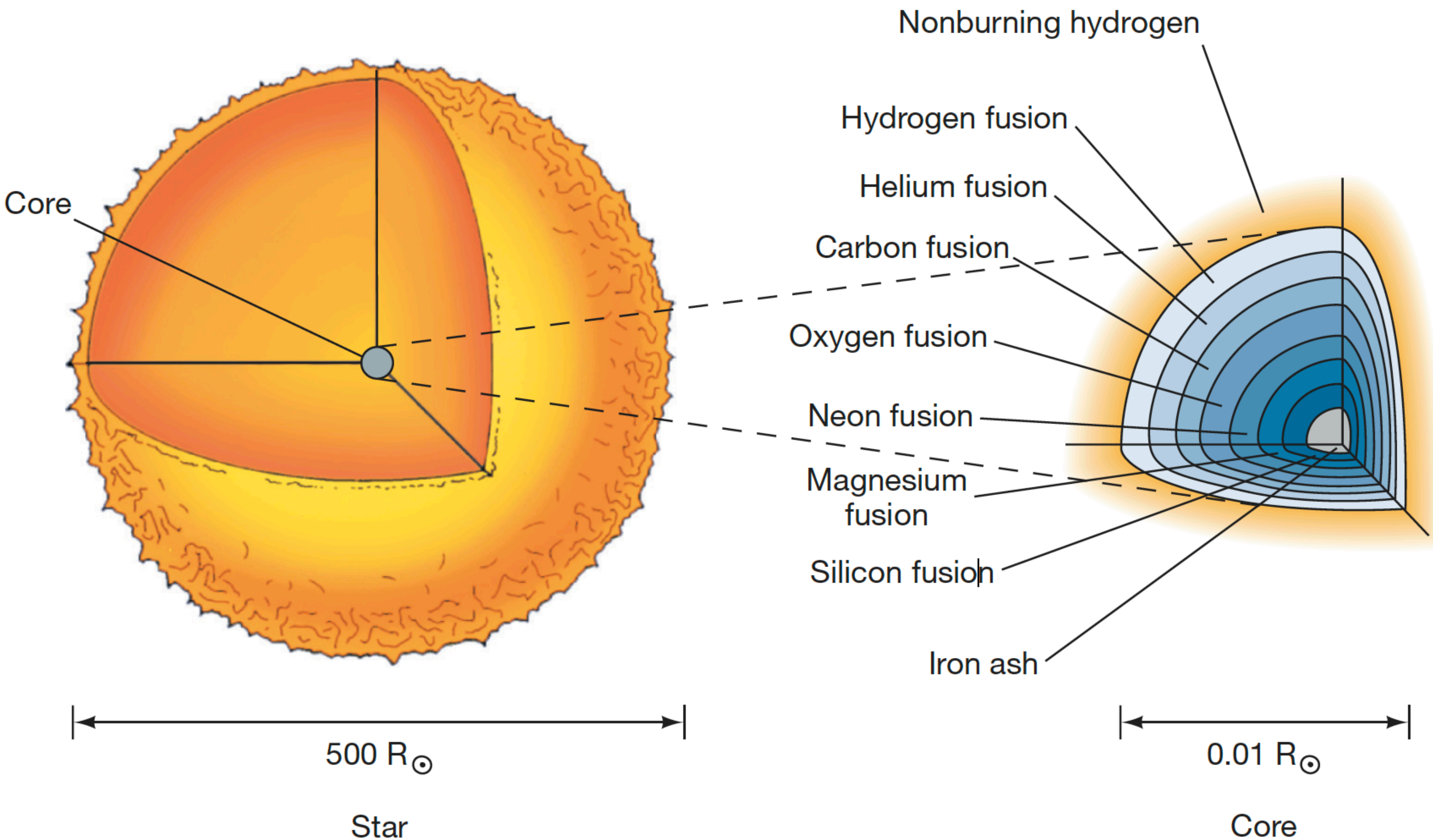


^{19}F
^{20}Ne
^{21}Na
^{21}Ne
^{22}Na
^{22}Ne
^{23}Na
^{24}Mg
^{25}Al
^{25}Mg
^{26}Al
^{26}Mg
^{27}Al
^{27}Si
^{28}Si
(p, γ)
(p, α)
(β+)
Ne–Na
Mg–Al


Abundance of Si
is normalized to 10^6
Log_{10}(Abundance)
Z, Atomic number
H
He
Li
Be
B
C
N
O
F
Ne
Na
Si
P
S
Ar
Ca
Sc
Ti
V
Fe
Co
Ni
Cu
Zn
Ga
Ge
As
Zr
Nb
Mo
Sn
In
Te
Xe
Ba
Pr
W
Re
Pt
Au
Hg
Pb
Bi
Th
U



10,000
100
1
0.01
.0001
Luminosity (solar units)
Helium
Carbon
Oxygen
Carbon
Helium ignition
15M⊙
4M⊙
1M⊙
100 R⊙
Red-giant branch
10 R⊙
1 R⊙
0.1 R⊙
30,000
10,000
6000
3000
Surface temperature (K)



layered core structure

High-Mass Stars

Faster and faster burning stages ($\sim 25\ M_\odot$ star)

$$H \sim 5\times10^6 \text{ yr}$$
$$He \sim 5\times10^5 \text{ yr}$$
$$C \sim 500 \text{ yr}$$
$$Ne \sim 1 \text{ yr}$$
$$Si \sim 1 \text{ day}$$

$$\rightarrow\ {}^{56}Fe$$

Problem:

Iron won't fuse!