Weighing the Earth

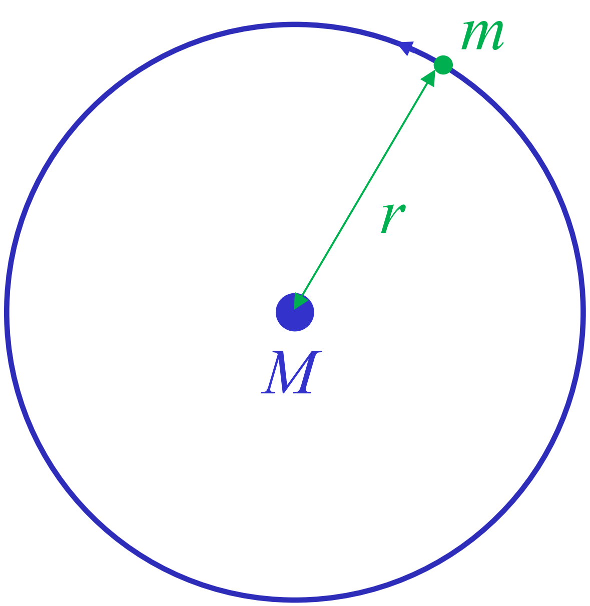
$$F_{grav} = \frac{GMm}{r^2} = ma$$

$$\Rightarrow \quad a = \frac{GM}{r^2}$$

$$g = \frac{GM_\oplus}{R_\oplus^2} \approx 9.80\,\mathrm{m/s^2}$$

$$G = 6.67 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$$

$$M_\oplus = \frac{gR_\oplus^2}{G} = 5.97 \times 10^{24}\,\mathrm{kg}$$




D = 384,400 km
3000 km

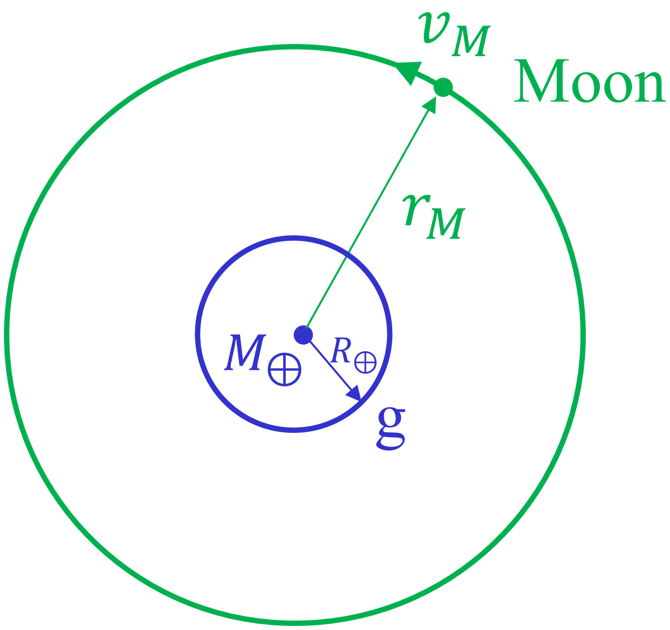
The Moon and the Inverse-Square Law

$$r_M = 384,000\,\mathrm{km}$$

$$v_M = \frac{2\pi r_M}{27.3\,\mathrm{days}} = 1.02\,\mathrm{km/s}$$

$$a_M = \frac{v_M^2}{r_M} = 2.7 \times 10^{-3}\,\mathrm{m/s^2}$$

$$\frac{a_M}{g} = 2.8 \times 10^{-4} = \frac{R_\oplus^2}{r_M^2}$$



→ inverse square!

Circular Orbital Motion

$$a = \frac{v^2}{r} = \frac{GM}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$P = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} \propto r^{3/2}$$ (Kepler III)

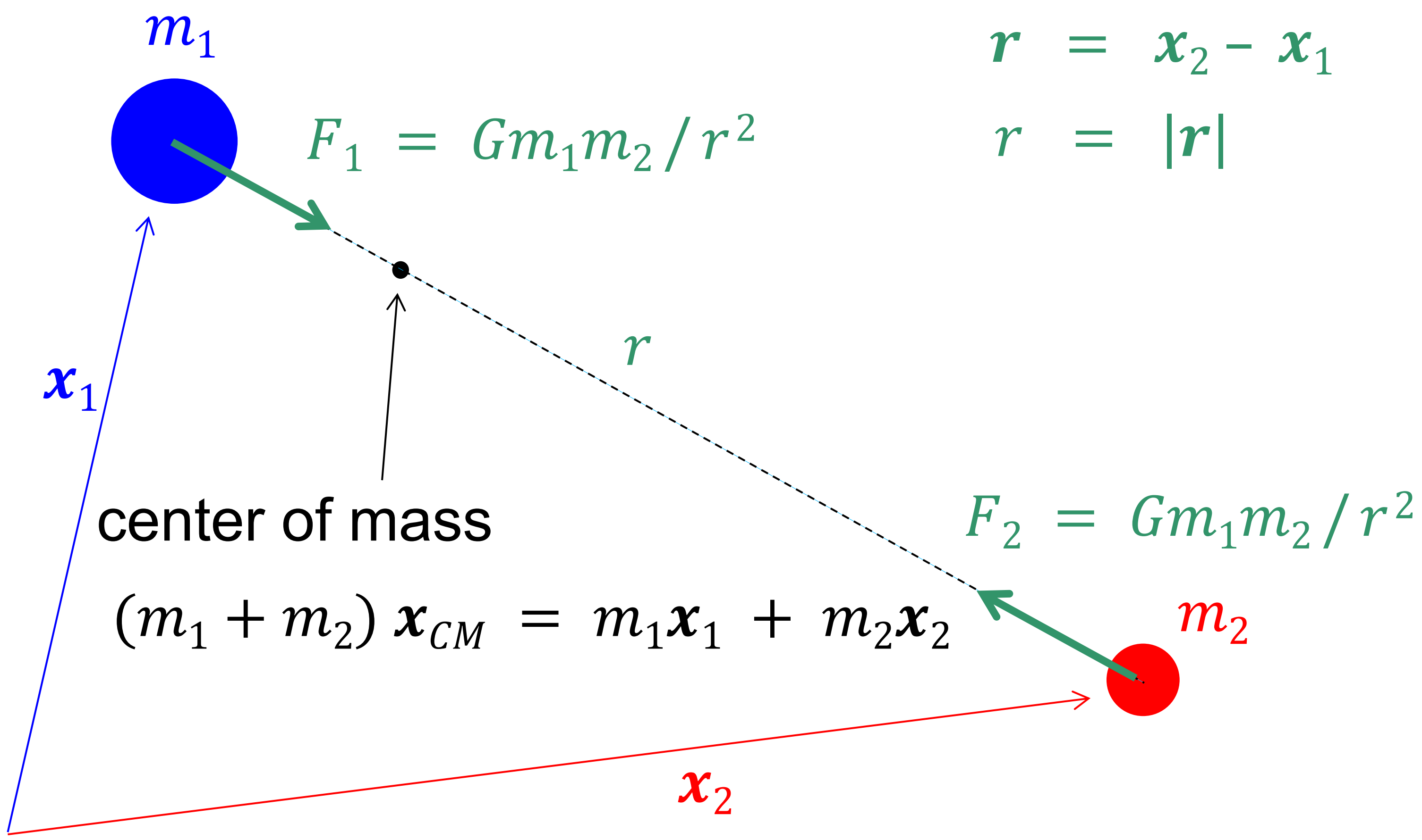
HST: $r = 6500\,\mathrm{km}, \quad v = 7.83\,\mathrm{km/s}, \quad P = 1.45\,\mathrm{hours}$

GPS: $P = 12\,\mathrm{hr}, \; r = 26,600\,\mathrm{km}$

Geo: $P = 24\,\mathrm{hr}, \; r = 42,200\,\mathrm{km}$

→ Earth: $P = 1\,\mathrm{year}, \; r = 1\,\mathrm{AU} \quad \Rightarrow \quad M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$

Two-body Motion under Gravity



The Two-Body problem

$$\mathbf{a}_1 = \frac{Gm_2\left(\mathbf{x}_2 - \mathbf{x}_1\right)}{r^3}$$

$$\mathbf{a}_2 = \frac{Gm_1\left(\mathbf{x}_1 - \mathbf{x}_2\right)}{r^3}$$

$$r = \left|\mathbf{x}_2 - \mathbf{x}_1\right|$$

$$\mathbf{x}_{CM} = \frac{m_1\mathbf{x}_1 + m_2\mathbf{x}_2}{m_1 + m_2}$$

$$\begin{aligned}
\mathbf{a}_{CM} &= \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2}{m_1 + m_2} \\
&= \frac{\mathbf{F}_1 + \mathbf{F}_2}{m_1 + m_2} \\
&= \mathbf{0}
\end{aligned}$$

The Two-Body problem

Relative motion: $\mathbf{r} = \mathbf{x}_2 - \mathbf{x}_1, \quad \mathbf{a} = \mathbf{a}_2 - \mathbf{a}_1$

$$
\begin{aligned}
\Rightarrow \qquad \mathbf{a} &= -\frac{Gm_1\mathbf{r}}{r^3} - \frac{Gm_2\mathbf{r}}{r^3} \\
&= -\frac{GM\mathbf{r}}{r^3}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{r}_1 &= \mathbf{x}_1 - \mathbf{x}_{CM} \\
\mathbf{r}_2 &= \mathbf{x}_2 - \mathbf{x}_{CM}
\end{aligned}
$$

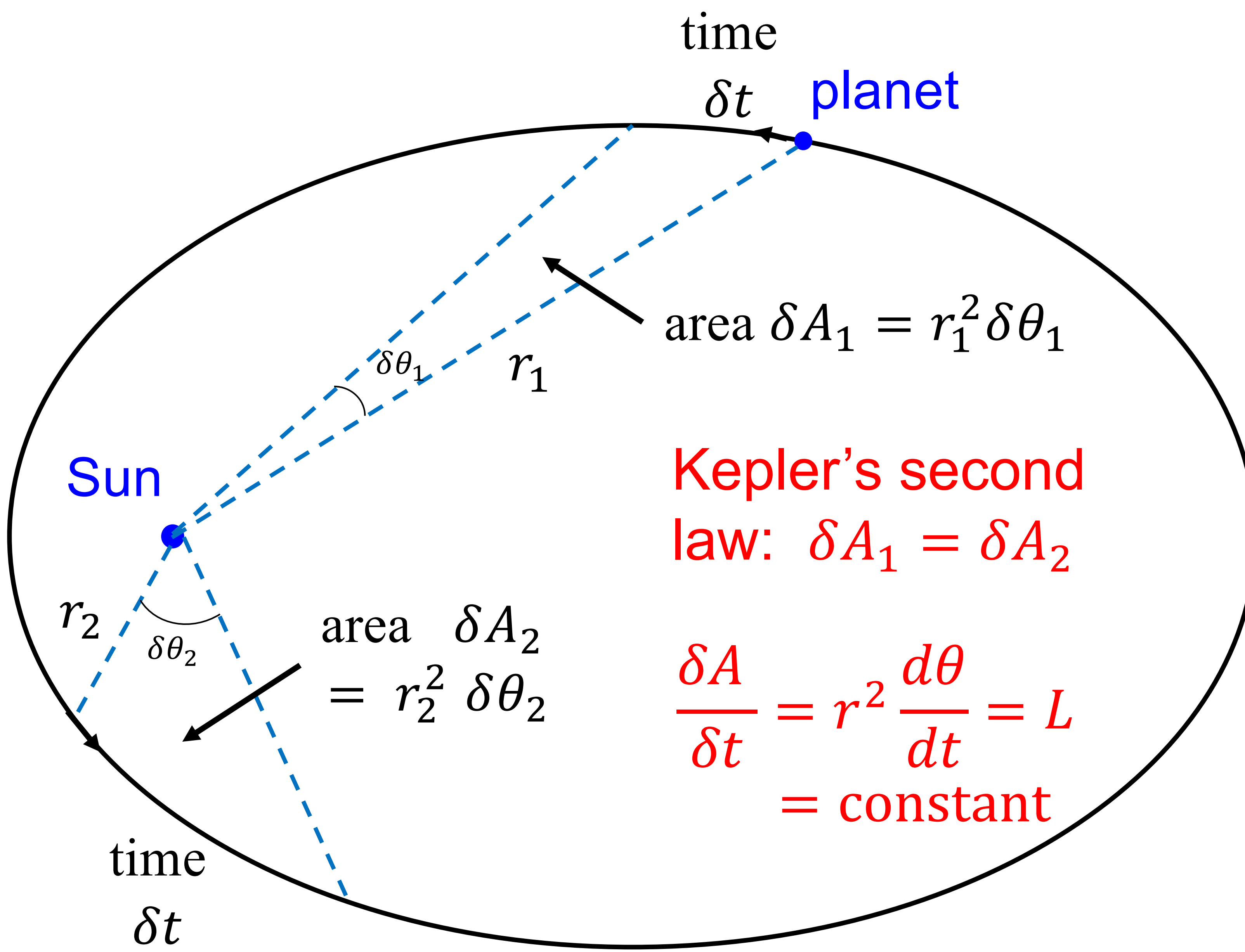
$$
\begin{aligned}
m_1\mathbf{r}_1 + m_2\mathbf{r}_2 &= \mathbf{0} \\
\mathbf{r}_1 &= \frac{m_2}{m_1 + m_2}\mathbf{r} \\
\mathbf{r}_2 &= -\frac{m_1}{m_1 + m_2}\mathbf{r}
\end{aligned}
$$

Conservation Laws

$$\mathbf{a} = -\nabla\phi, \quad \text{where} \quad \phi = -\frac{GM}{r}$$

$$E = \tfrac{1}{2}v^2 - \frac{GM}{r}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{v} \quad \Rightarrow \quad \begin{aligned} \frac{d\mathbf{L}}{dt} &= \frac{d\mathbf{r}}{dt} \times \mathbf{v} + \mathbf{r} \times \frac{d\mathbf{v}}{dt} \\ &= \mathbf{v} \times \mathbf{v} + \mathbf{r} \times \mathbf{a} \\ &= \mathbf{0} \end{aligned}$$


time
δt
planet
area $\delta A_1 = r_1^2 \delta\theta_1$
$\delta\theta_1$
r_1
Sun
Kepler's second
law: $\delta A_1 = \delta A_2$
r_2
$\delta\theta_2$
area δA_2
$= r_2^2 \ \delta\theta_2$
$\frac{\delta A}{\delta t} = r^2 \frac{d\theta}{dt} = L$
$= \text{constant}$
time
δt

Bound and Unbound Orbits

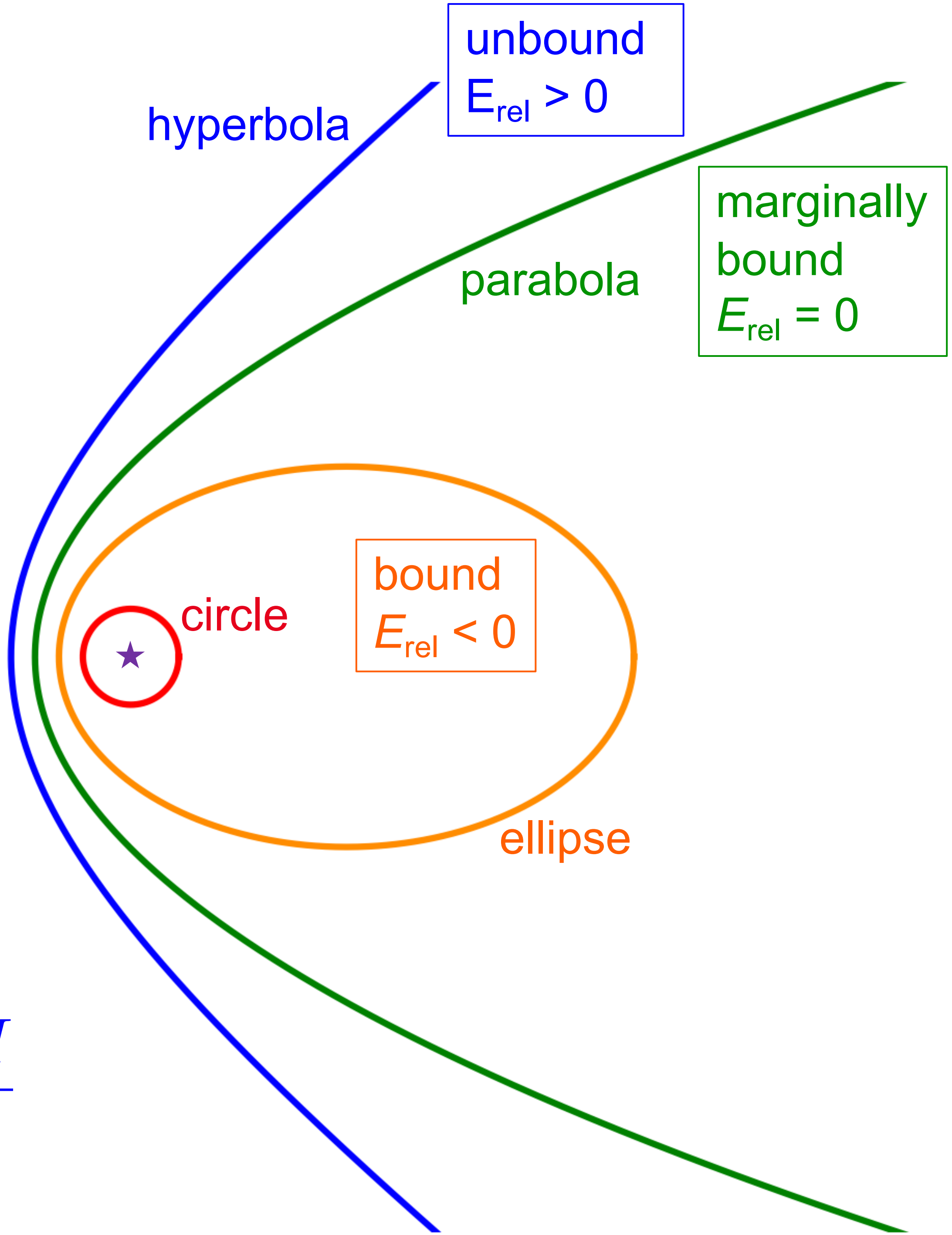
$$E = \tfrac{1}{2}v^2 - \frac{GM}{r}$$

$$E > 0 \quad \Rightarrow \quad E = \tfrac{1}{2}v_\infty^2$$ unbound

$$E < 0 \quad \Rightarrow \quad r_{circ} = -\frac{GM}{2E}$$ bound

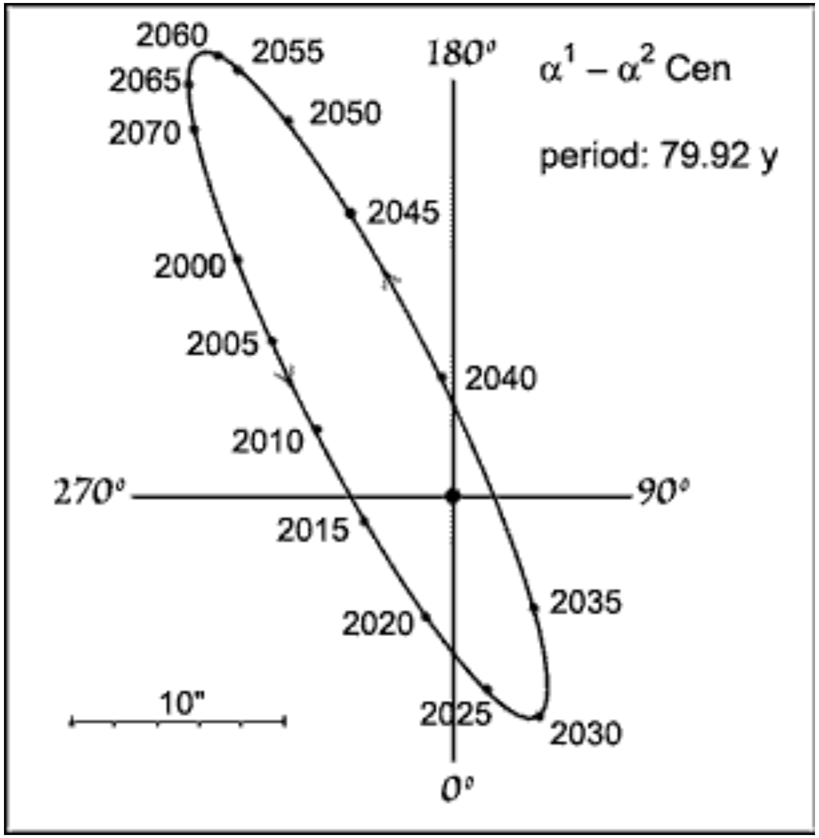
$$E = 0 \quad \Rightarrow \quad v_{esc}^2 = \frac{2GM}{r}$$ marginal

Conic Sections

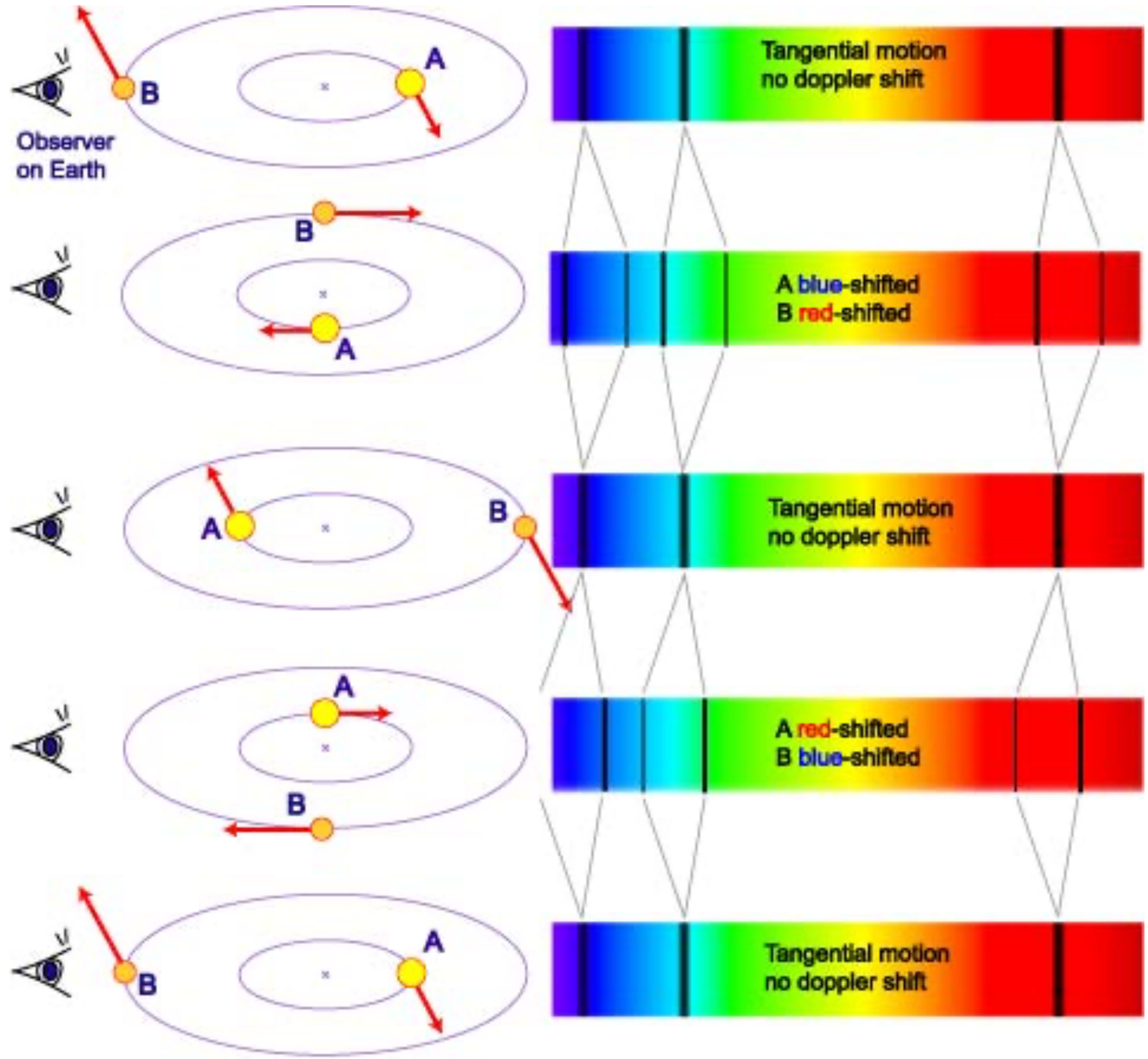


$$E_{rel} = \frac{1}{2}v^2 - \frac{GM}{r}$$

Visual Binary



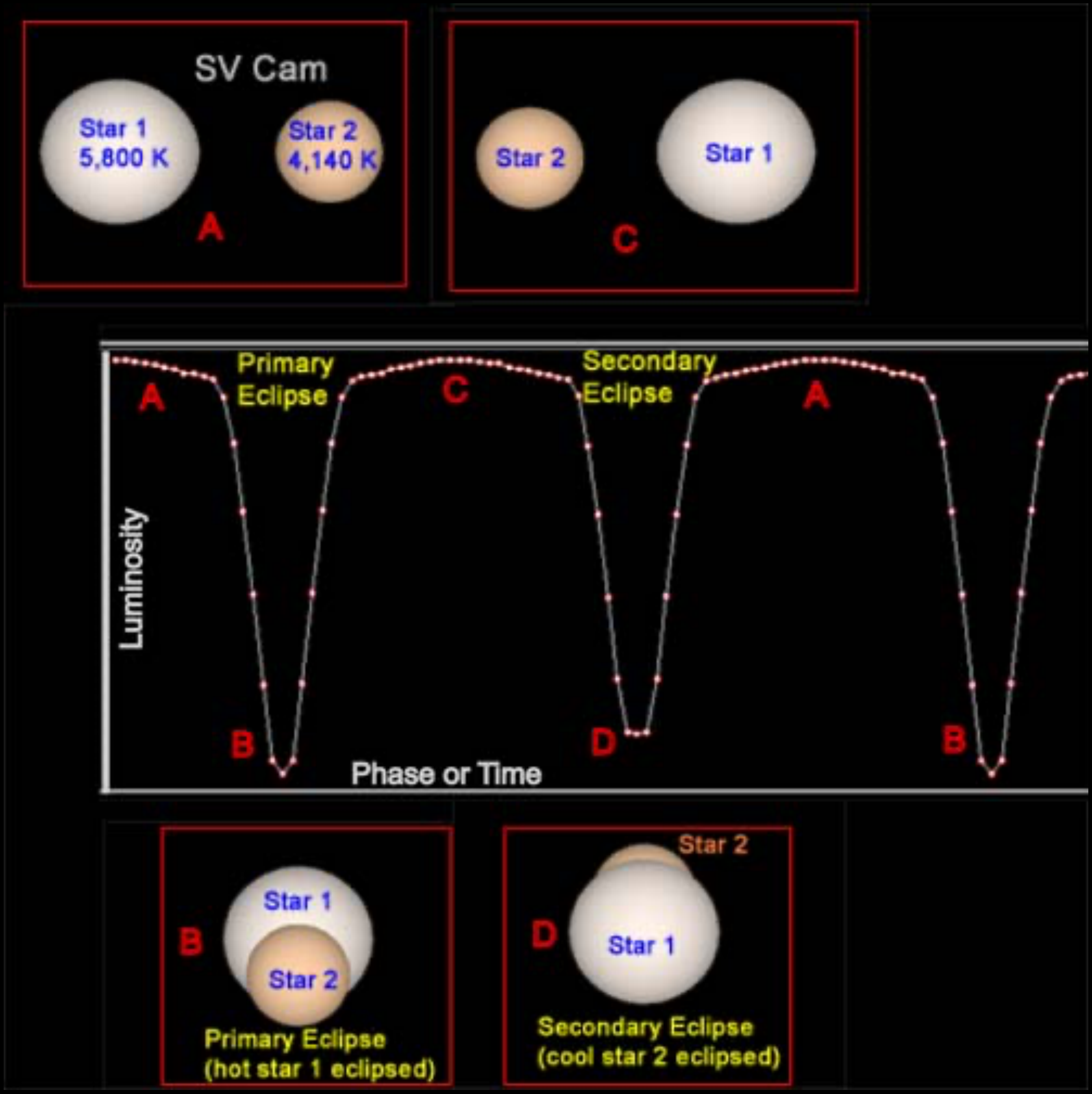
Spectroscopic Binary



A Spectroscopic Binary System

High-mass star A and lower-mass B orbit around a common centre of mass. The observed combined spectrum shows periodic splitting and shifting of spectral lines. The amount of shift is a function of the alignment of the system relative to us and the orbital speed of the stars.

Eclipsing Binary



Measuring Stellar Masses

Visual binary with velocities $\Longrightarrow$ M_1, M_2

Spectroscopic binary with 2 velocities $\Longrightarrow$ $\frac{M_1}{M_2}$, $M \sin i$

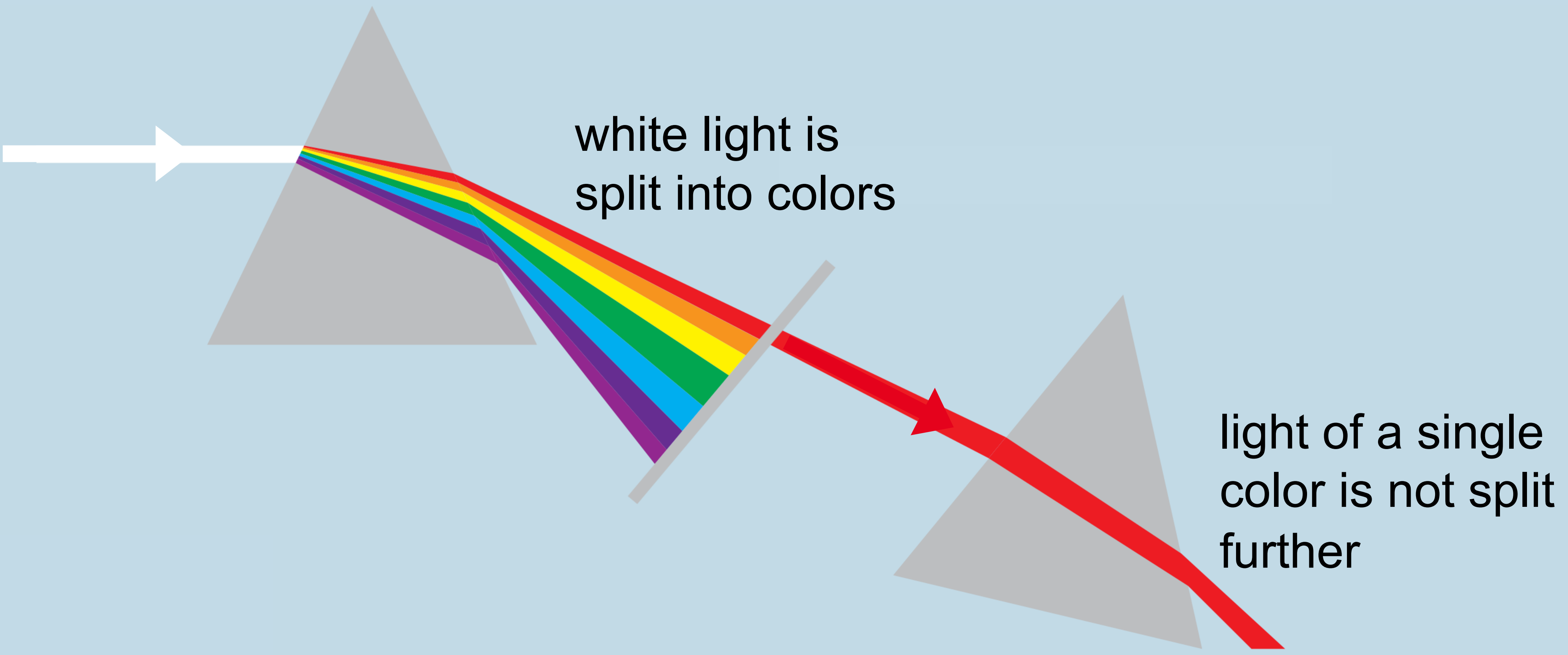
Eclipsing binary: $i = 90°$, so $\sin i = 1$

Spectroscopic binary with 1 velocity

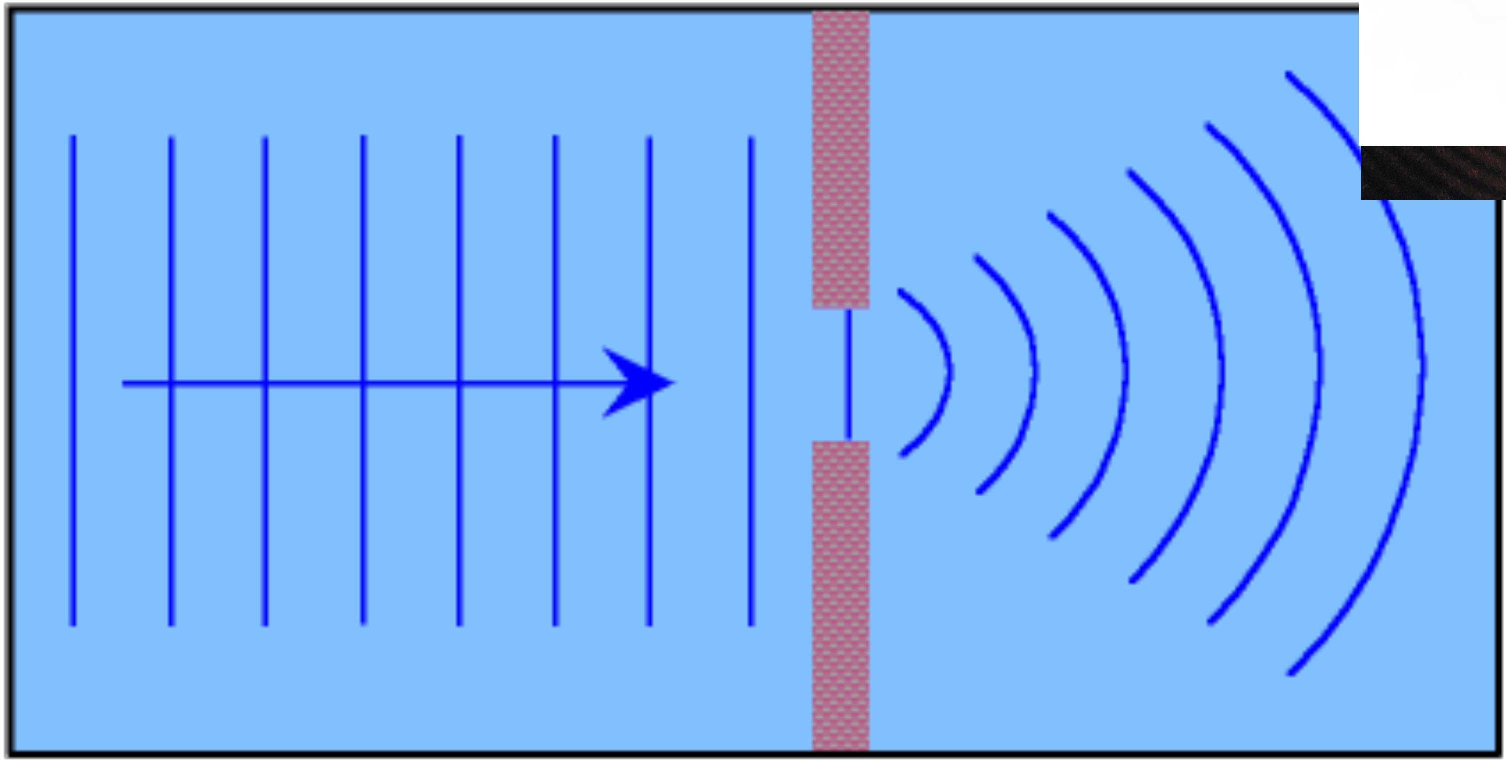
$$\Longrightarrow \text{ mass function } f(M_1, M_2, i) = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2}$$

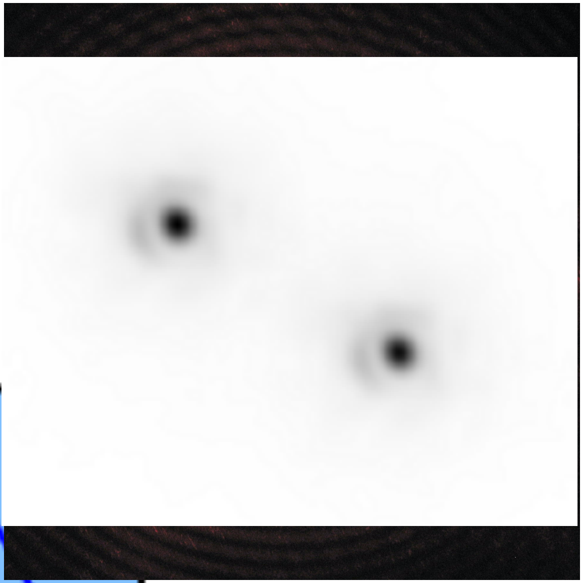
Typical measured masses: $0.1 - 100\ M_\odot$

Double Prism Experiment



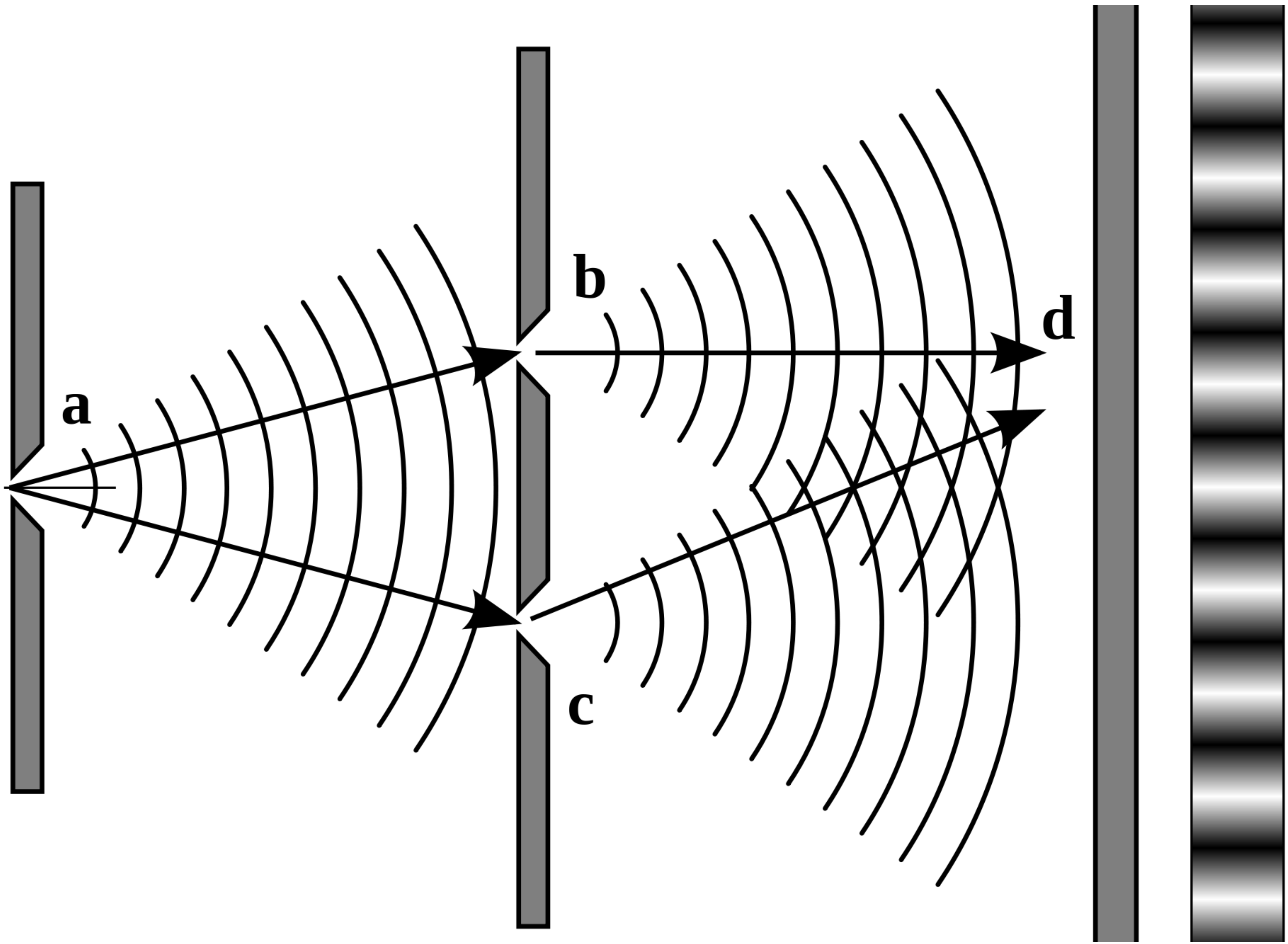
Diffraction

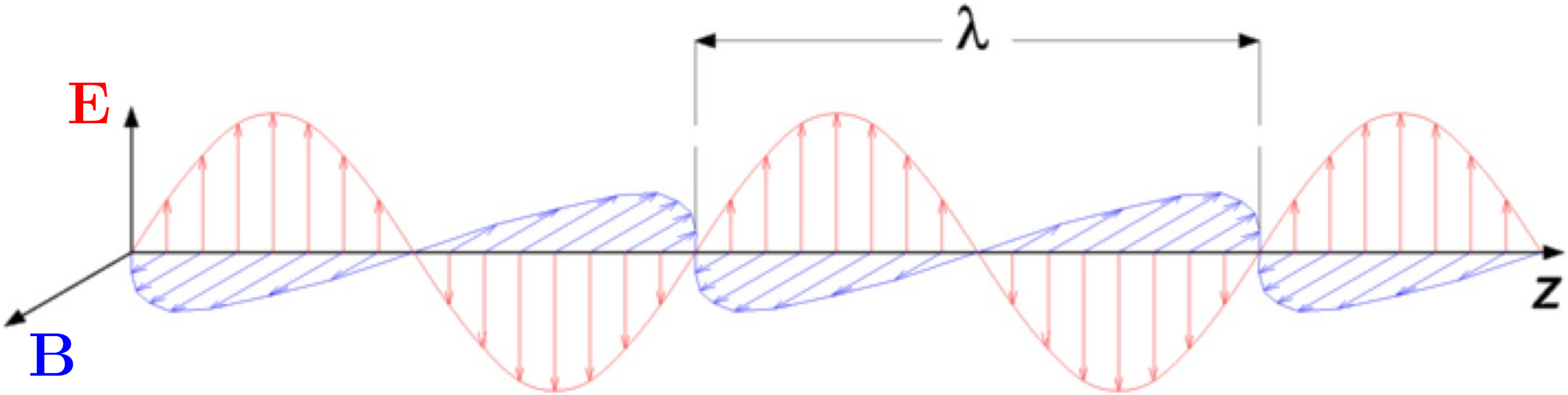




$$\text{Diffraction angle} \propto \frac{\text{wavelength}}{\text{gap width}}$$

Double Slit Experiment



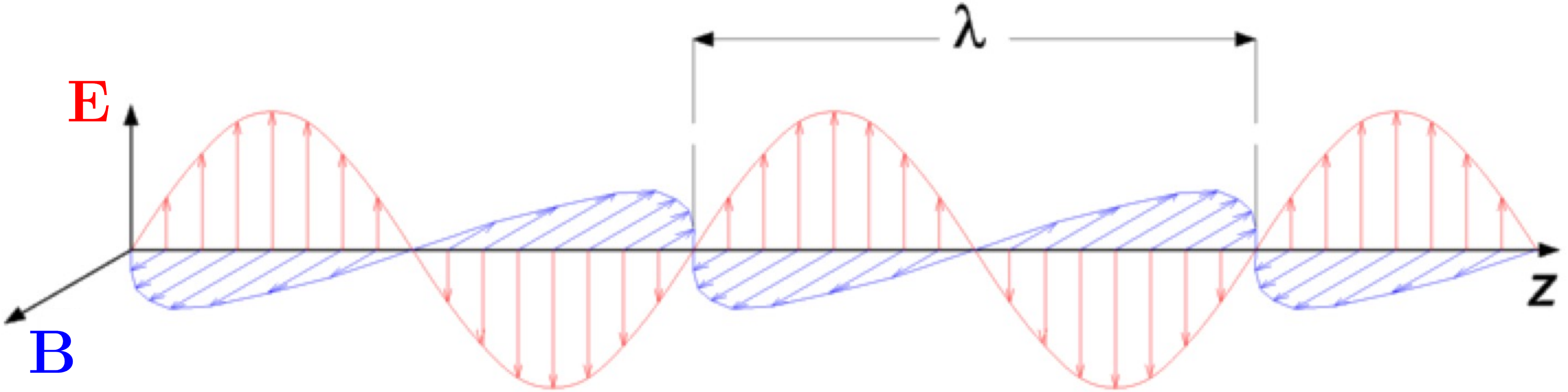


$$\mathbf{E} = (E_x(z,t),\ 0,\ 0))$$

$$\mathbf{B} = (0,\ B_y(z,t),\ 0))$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

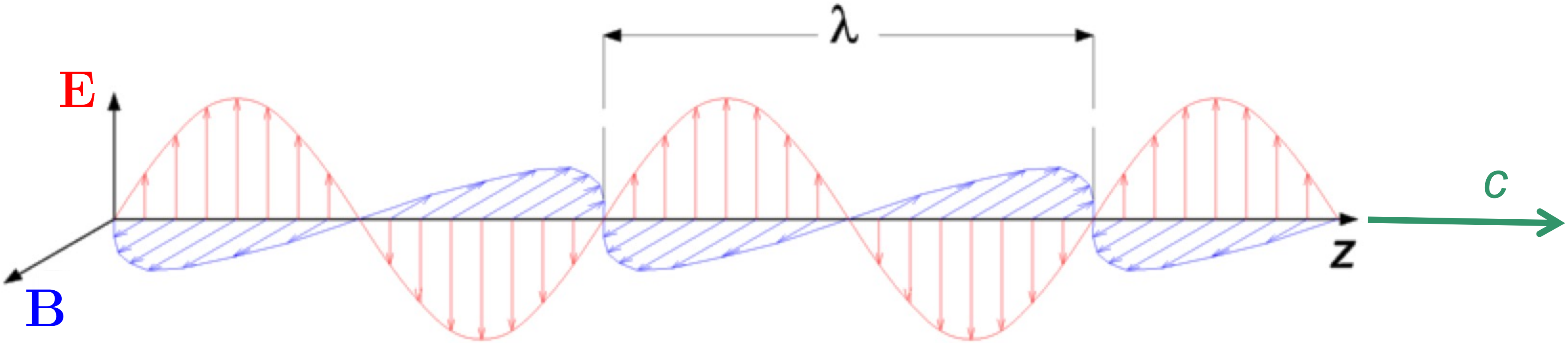


$$\mathbf{E} = (E_x(z,t),\ 0,\ 0))$$
$$\mathbf{B} = (0,\ B_y(z,t),\ 0))$$

$$-\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$
$$-\frac{\partial B_y}{\partial z} = \epsilon_0\mu_0\,\frac{\partial E_x}{\partial t}$$

$$\epsilon_0\mu_0\,\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z\partial t}$$
$$= -\frac{\partial^2 E_x}{\partial z^2}$$

$$\frac{\partial^2 E_x}{\partial t^2} + \frac{1}{\epsilon_0\mu_0}\,\frac{\partial^2 E_x}{\partial z^2} = 0$$

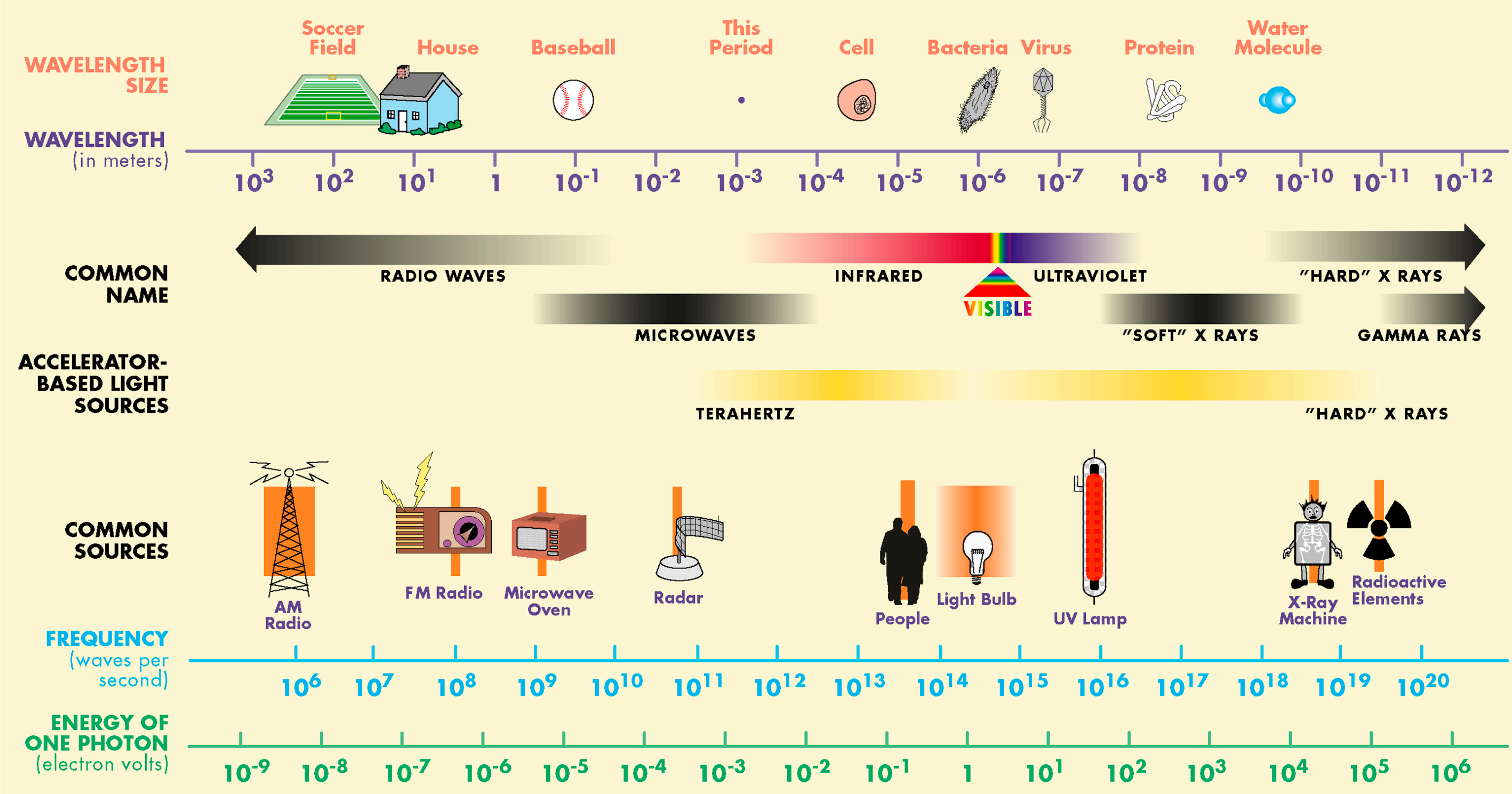


$$E_x = E_0 \cos 2\pi \left(\frac{z}{\lambda} - ft\right)$$

$$B_y = \frac{E_0}{c} \cos 2\pi \left(\frac{z}{\lambda} - ft\right)$$

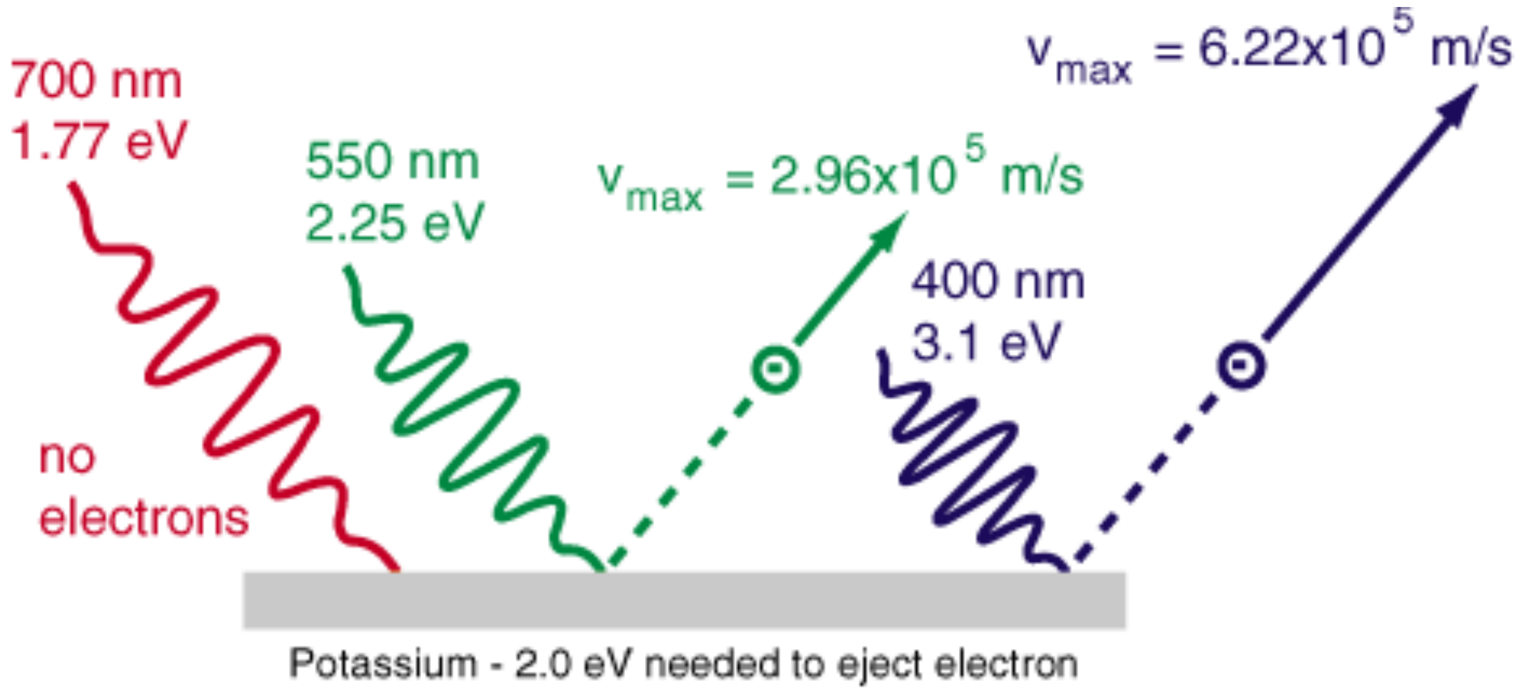
$$f\lambda = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c = 3 \times 10^8 \text{ m/s}$$

Electromagnetic Spectrum



Photoelectric Effect

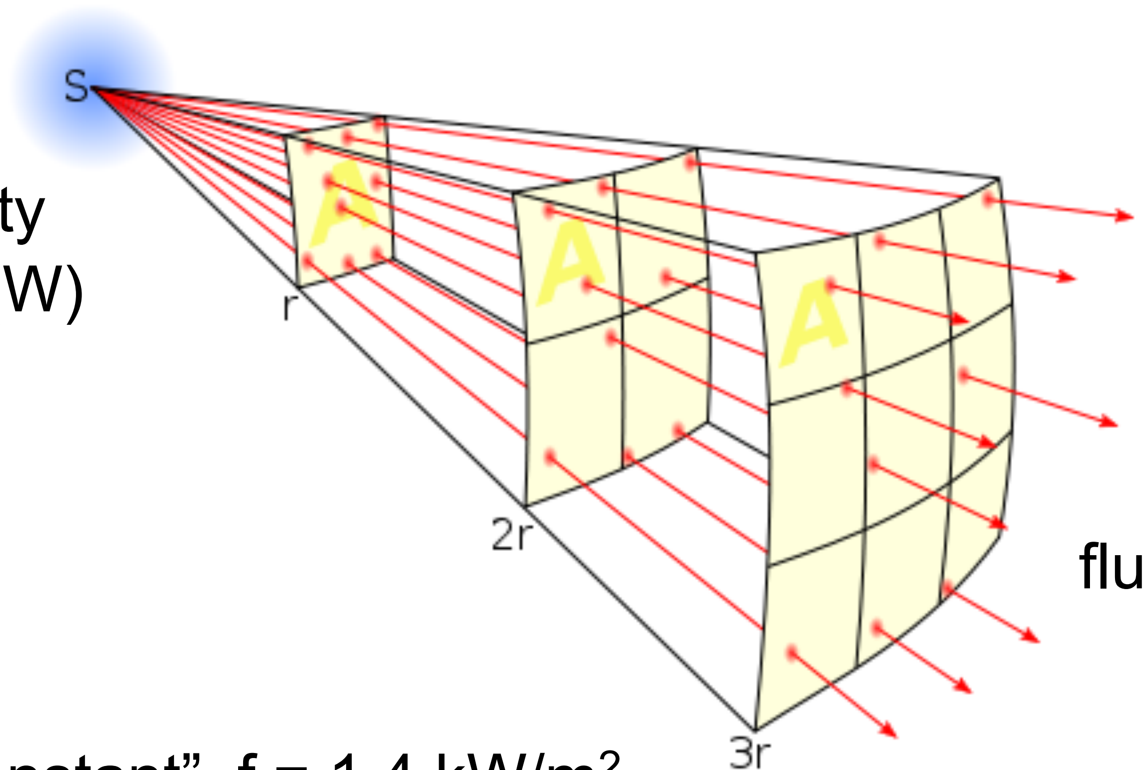
$$E_{photon} = h\nu$$



Planck's constant $h = 6.63 \times 10^{-34}$ J.s

Inverse Square Law

source
luminosity
L watts (W)



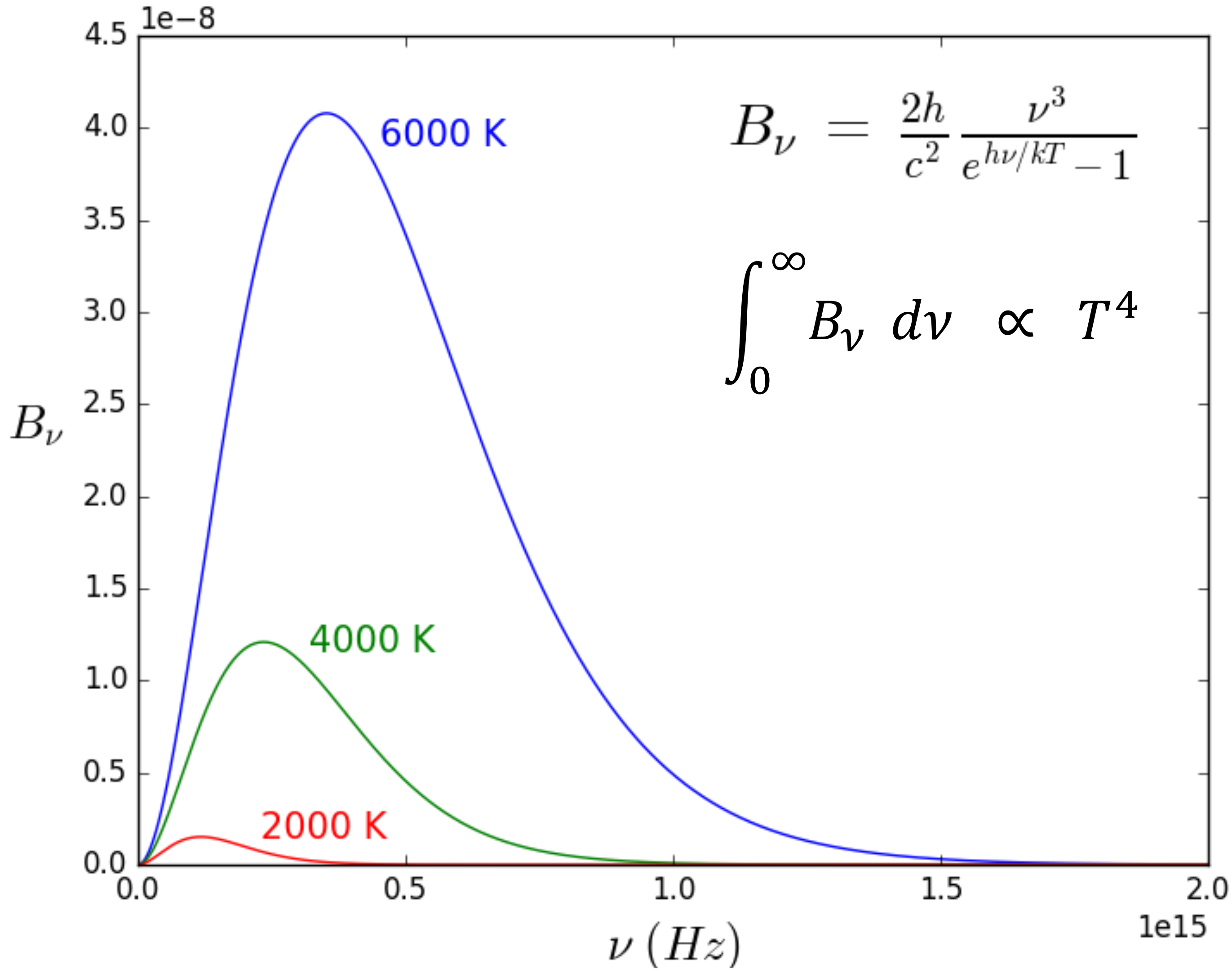
flux = $\dfrac{L}{4\pi r^2}$ (W/m^2)

"solar constant" f = 1.4 kW/m^2
distance r = 1 AU
solar luminosity L = 3.96 x 10^{26} W

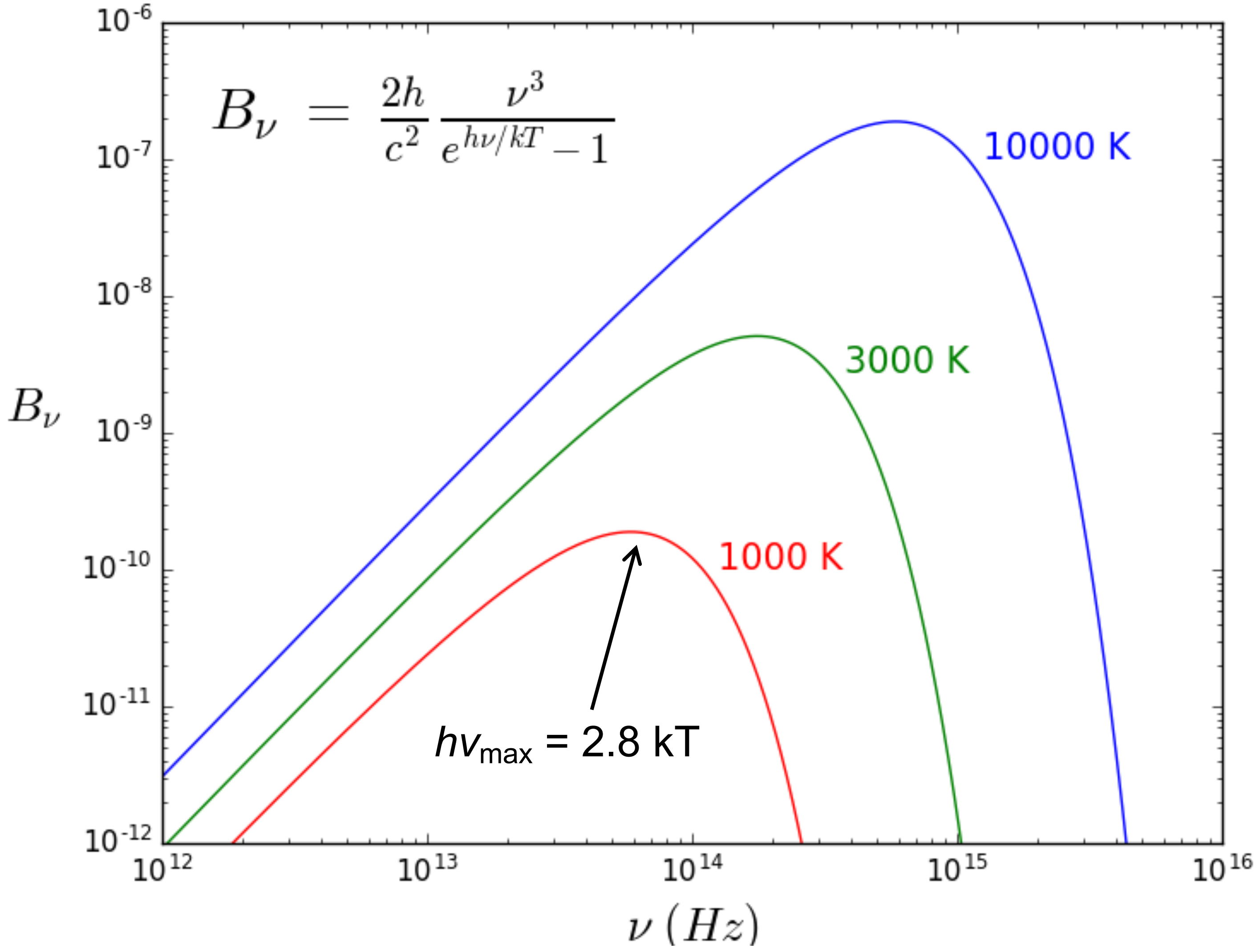
standard candle

$$r = \sqrt{\frac{L}{4\pi f}}$$

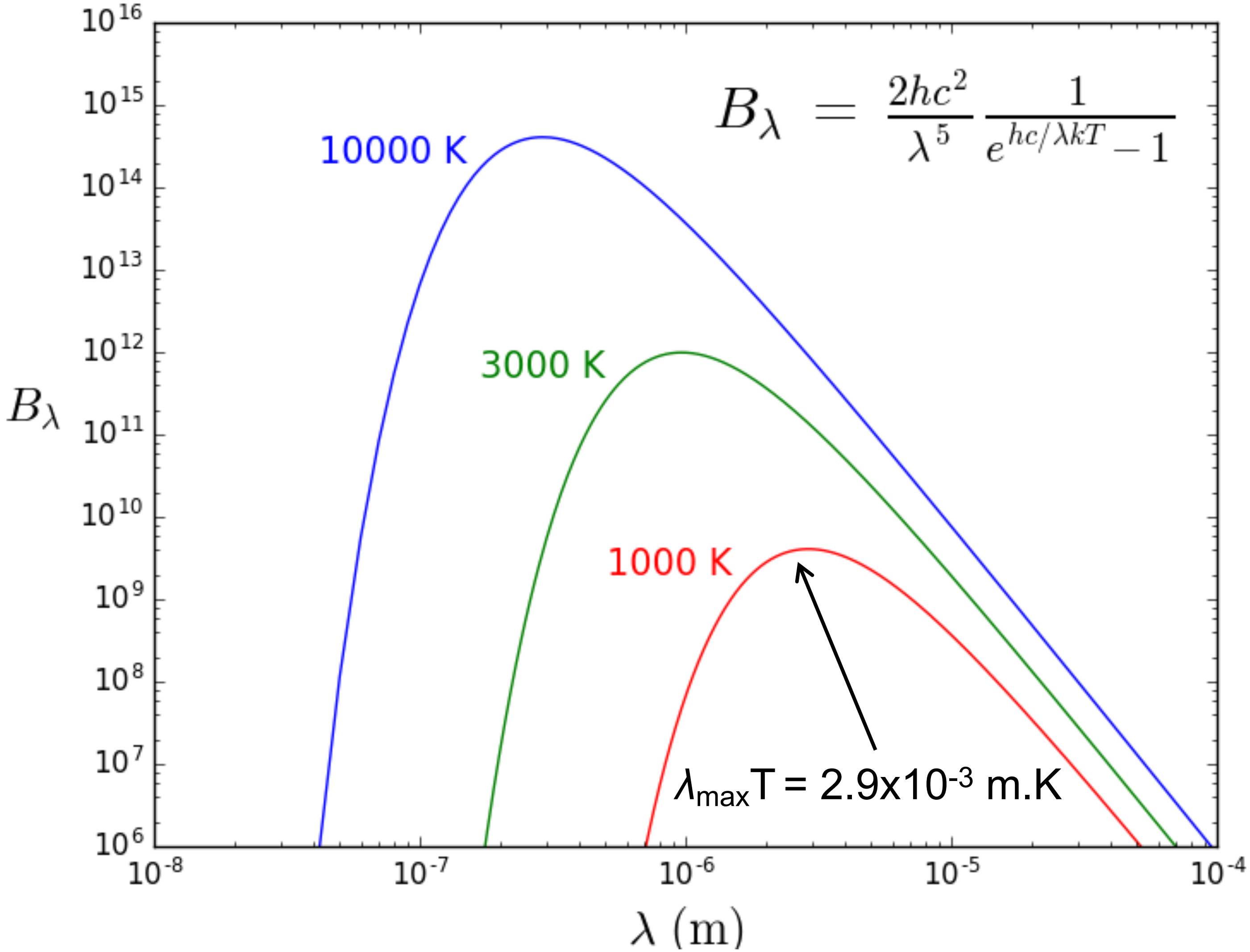
Blackbody Radiation



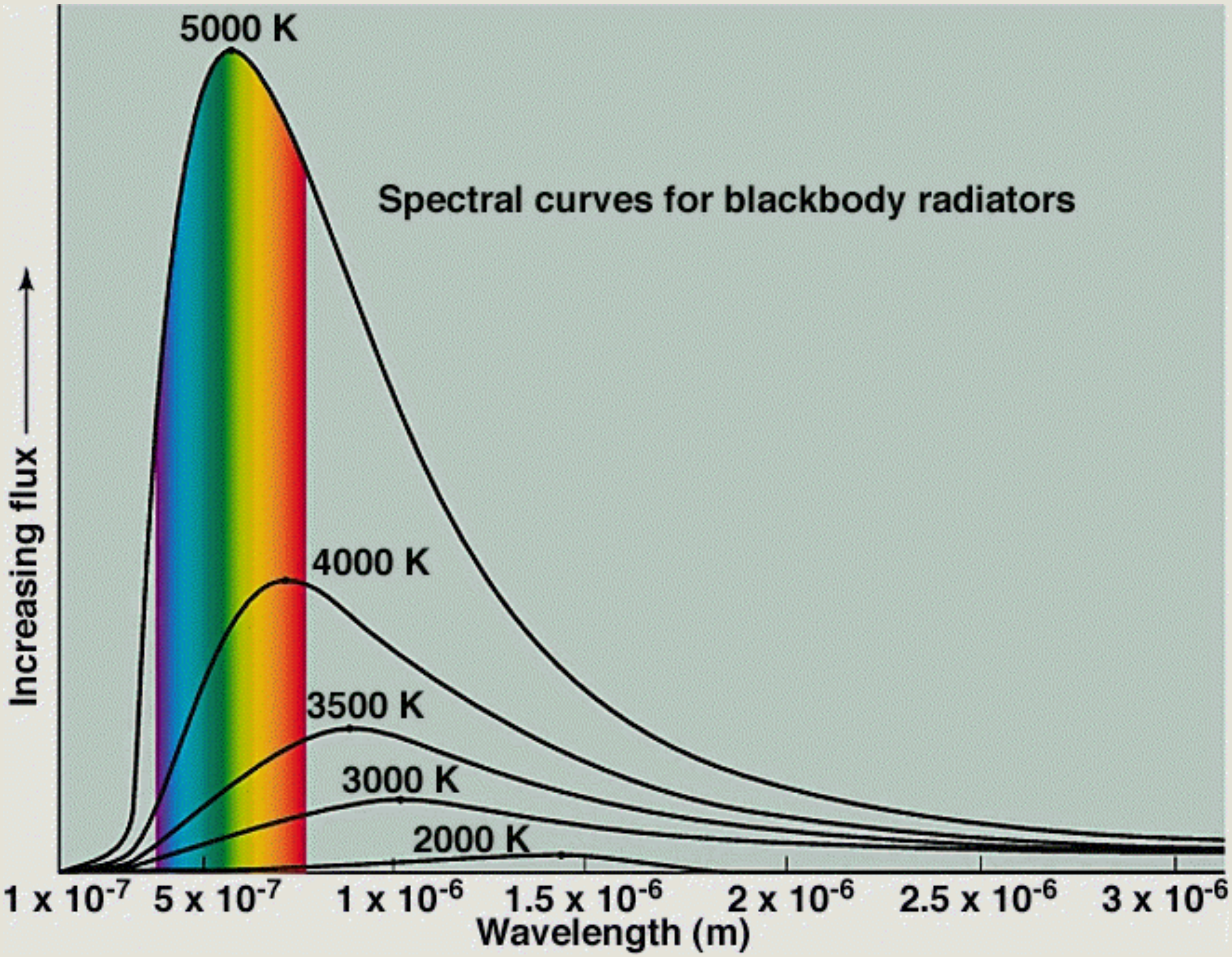
Blackbody Radiation



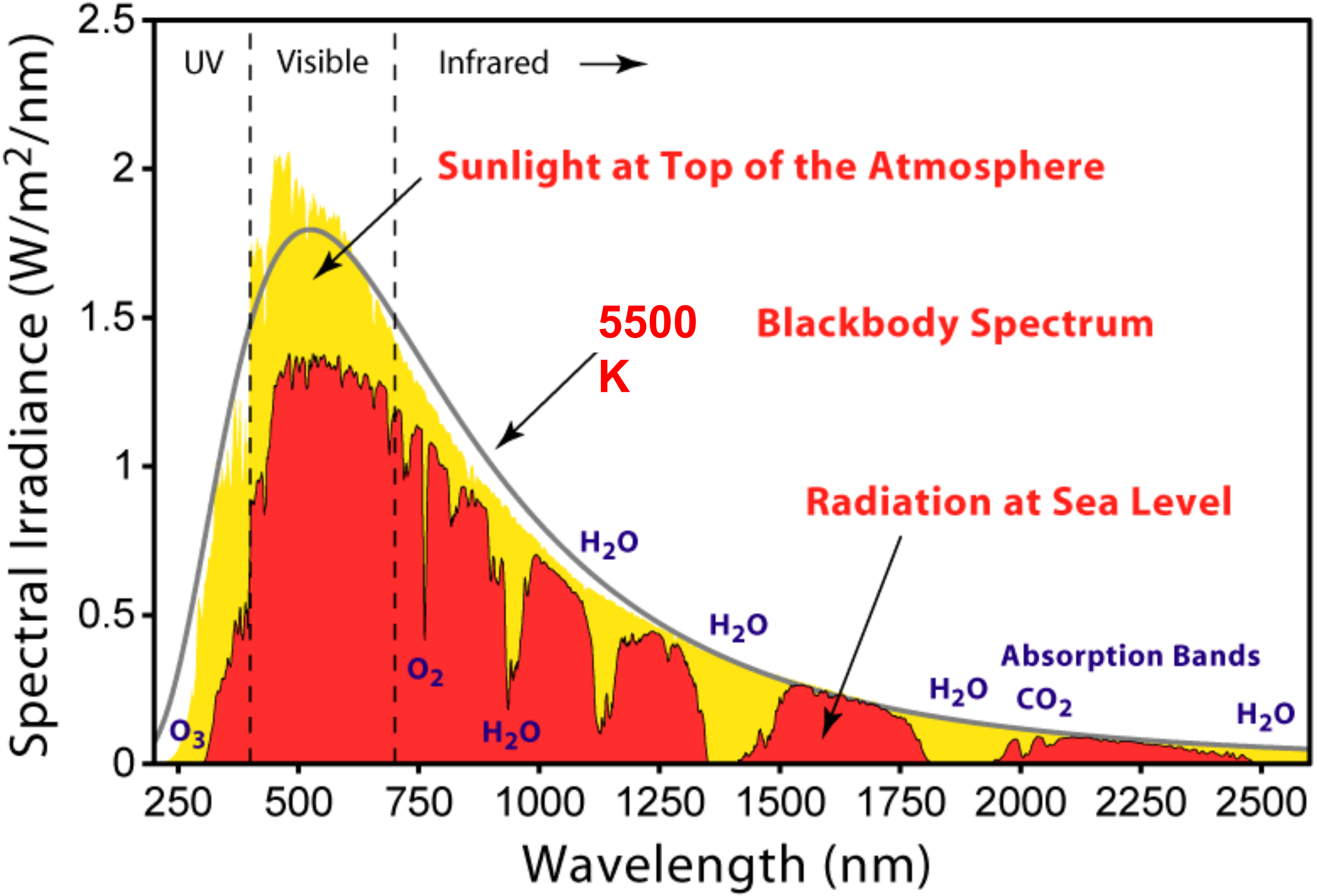
Blackbody Radiation

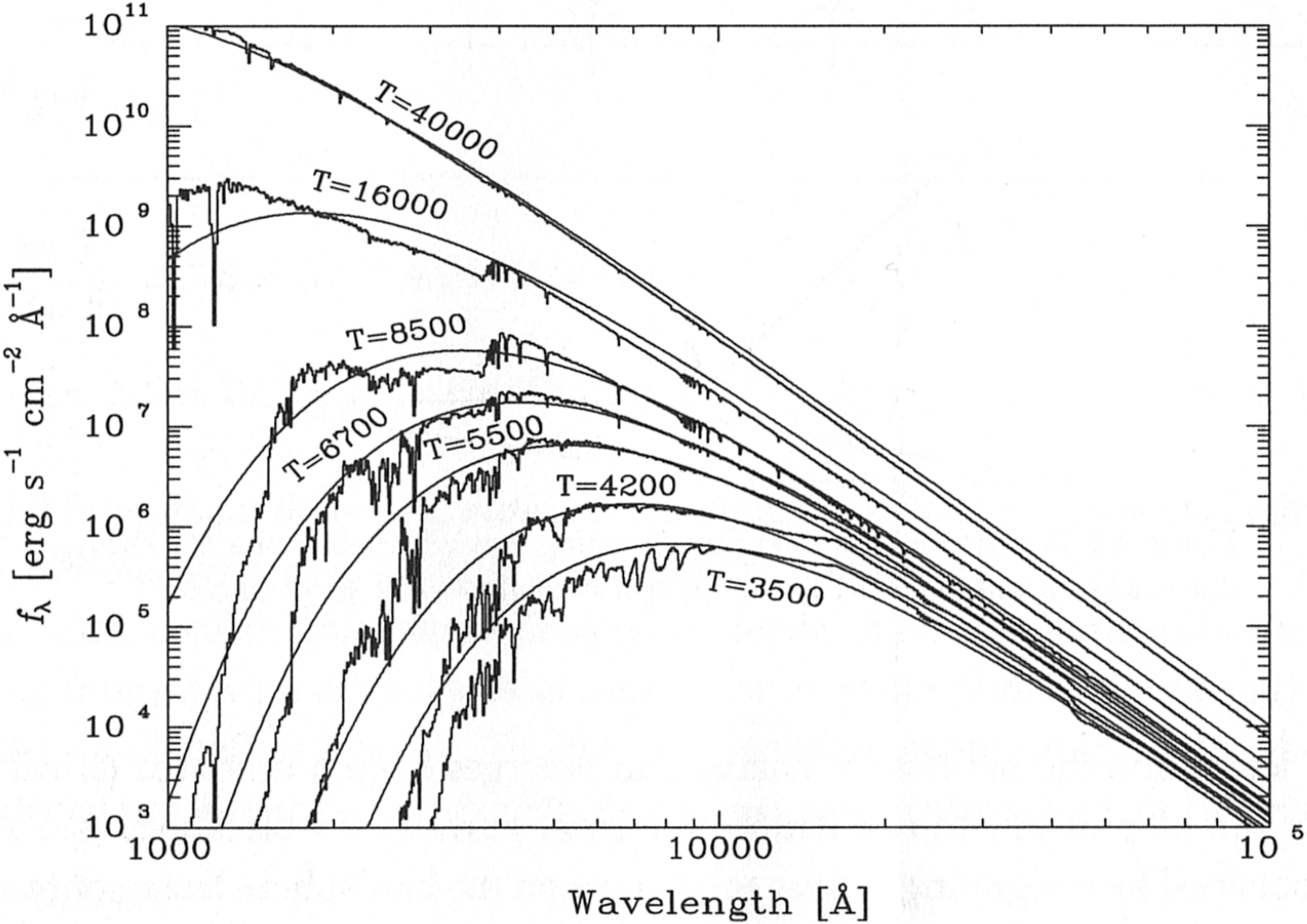


Blackbody Radiation



Solar Radiation Spectrum





T=40000
T=16000
T=8500
T=6700
T=5500
T=4200
T=3500
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
1000
10000
10^{5}
f_λ [erg s^{-1} cm^{-2} Å^{-1}]
Wavelength [Å]