A Study of HII Region Emission Lines

Danny Pan
Quantum Physics II

Abstract: This paper studies the effects of electron recombination in HII regions in space. These regions are rich in ionized atoms that recombine with electrons and decay down into the ground state. The possible emission lines and emission line intensities are studied and explored here. Of particular interest in this paper is the study of electrons recombining in Hydrogen atoms and the presence of strong H-alpha lines in the emission spectra.

I. INTRODUCTION

Every once in a while, dust and gas combine to form large massive stars. The most massive of which are O/B type stars, labeled for their abnormally large masses and their high luminosities. These stars are very short-lived, having a typical age of 10-100 million years, and will eventually become Black Holes. While it is still alive, however, they wreak havoc upon the region of space they inhabit.

O/B stars have very high luminosities, meaning they produce large amounts of radiation at all energies. It is fairly typical for them to produce radiation at energies greater than 13.6eV, the electron binding energy. Regions of gas surrounding the central star, the same region that condensed to form the star, are affected by this radiation and is in turn heated. There are various mechanisms for heating this region of gas.

- Photoionization: This is the process of electrons being stripped off of Hydrogen atoms and causing ionized hydrogen to form, giving it the name HII region.
- Photoelectric heating: Far Ultra-Violet photons get absorbed by dust grains and heats up the ambient gas.
- Collisional heating: Collisions between dust and gas can heat up the region.

Likewise, there are various ways for the same HII regions to cool down.

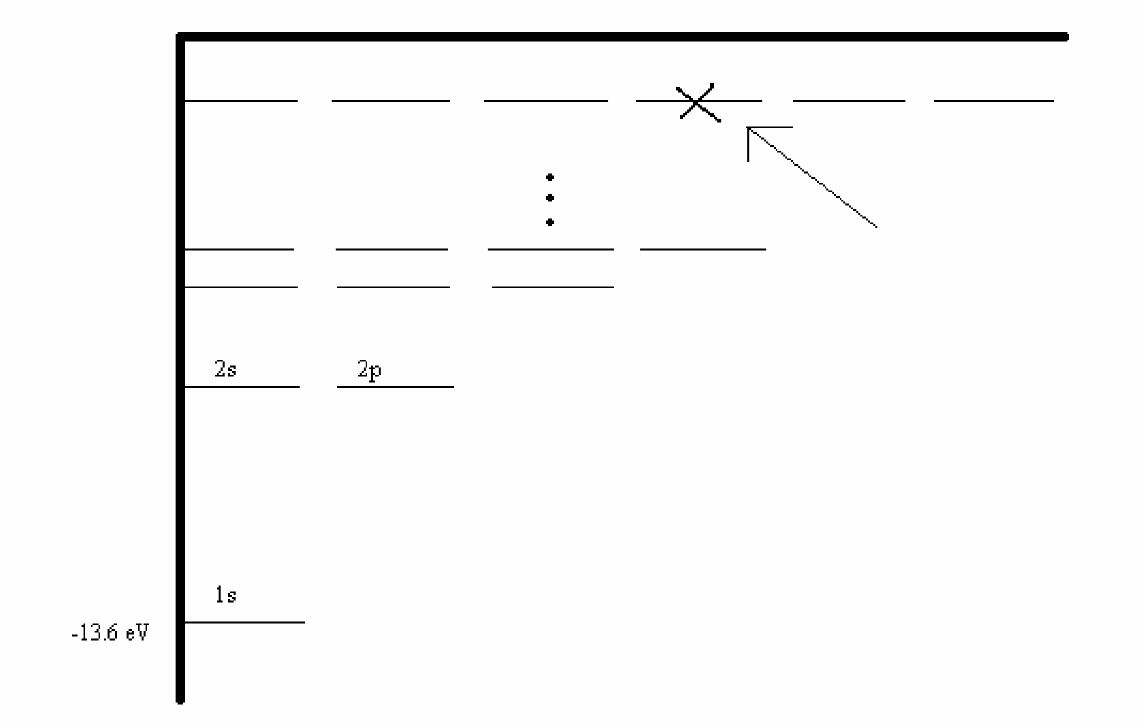
- Bremsstrahlung (free-free): Free electrons and ions collide and emit photons, cooling the region.
- Recombination cooling (bound-free): A free electron can be captured by ionized Hydrogen, taking in energy to bind the electron and cooling the gas.
- Collisional excitation (bound-bound): Atoms in excited bound states can decay into lower states, emitting photons and cooling down the gas.

This paper will focus on the effects of the primary source of cooling for these HII regions, the bound-bound effects of atoms decaying into lower energy states. In the next few sections, we will look at how these effects may be quantized and observed in modern day astronomy.

II. BOUND-BOUND COOLING

After an electron recombines with a hydrogen ion, a highly excited hydrogen atom is created. With very high, energetic electrons, it is possible to have a high n-level and l-level electron. An electron in this state will inevitably decay into lower energy states because high energy states are unstable. Most hydrogen atoms will eventually decay into the stable 1s ground state or the metastable 2s ground state. How it gets there will be the focus of this section.

Let us start with a hydrogen atom excited into a high n, high l state. In the hydrogen spectrum, this places the electron approximately here:



The hydrogen atom will want to decay into a lower energy level state. The rate at which the hydrogen atom will decay is governed by Fermi's Golden Rule stating:

$$w = \frac{2p}{h} \, r \, |H|^2 \tag{1}$$

where w is the rate per unit time of an electron transition. H is the Hamiltonian that couples the wave functions of the states for the electron. r is the density of states for which the electron can take the path from one state to another.

The Hamiltonian can be written as

$$H = \frac{1}{2m_k}(p_k - e_k A)^2 \tag{2}$$

Expanding the quadratic, we get

$$H = \frac{p_k^2}{2m_k} - \frac{p_k e_k A}{m_k} + \frac{e_k^2 A^2}{2m_k} \tag{3}$$

This has the form

$$H = H^0 + H^1 \tag{4}$$

If we neglect the A^2 term, we get our coupling Hamiltonian to be

$$H^1 = -\frac{p_k e_k A}{\mu_k} \qquad (5)$$

It is solved in Heitler that this is simply

$$H^1 = -\frac{e}{\mu}\sqrt{\frac{2\pi\hbar^2 c^2}{k}}\, p e^{ikr} \qquad (6)$$

The wavelength of light emitted $\lambda = 1/k$ is expected to be much larger than the radius r we can neglect the exponential in this Hamiltonian.

We now have the form for the Hamiltonian for (1). To get the transition rate, we need to now look at ρ. The density of states for a particular light quanta k in a particular direction can be written as

$$\rho_k dk = \frac{k^2 dk d\Omega}{(2\pi\hbar c)^3} \qquad (7)$$

Combining (1), (6), and (7), we find the transition rate to be

$$w = \frac{2\pi}{\hbar}\frac{k^2}{(2\pi\hbar c)^3}\left|\frac{e}{\mu}\sqrt{\frac{2\pi\hbar^2 c^2}{k}}\, p\right|^2 \qquad (8)$$

Now, making two quick substitutions

$$k = \hbar\nu \qquad \text{and} \qquad \frac{p}{\mu} = \frac{v}{c} \qquad (9)$$

we can simplify the expression to

$$w = \frac{e^2}{\mu^2}\frac{\nu}{2\pi\hbar c^3}|v|^2 \qquad (10)$$

The velocity is from the time dependent wave equation and has the form

$$v = \dot{x} = \frac{\partial}{\partial t}(x_0 e^{i\nu t}) = i\nu x \qquad (11)$$

which now gives us our final form for the transition rate

$$w = \frac{e^2}{\mu^2}\frac{\nu^3}{2\pi\hbar c^3}|x|^2 \qquad (12)$$

Looking at equation (12) one can notice a few things. First, the only variables remaining in this equation are the frequency of oscillations ν, and the coupled radial integral $|x|^2$. The dependence of the transition rate is on the 3rd power of ν. Recalling from equation (9), the light quanta, or the change in energy in the atom k is proportional to the frequency ν, one can derive that the transition rate will scale with the 3rd power of the change in energy $E_a - E_b$ where a and b represent the states with which the electron is making the transition.

The coupled radial integral $|x|^2$ is a bit more difficult to solve. One may solve it by integrating the radial components

$$\int R_{nl} r R_{n'l'} dr \qquad (13)$$

where

$$R_{nl} = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]}} e^{-\frac{\rho}{2}} \rho^l L_{n-l-1}^{2l+1}(\rho) \qquad (14)$$

$$\rho = \frac{2r}{na_0}$$

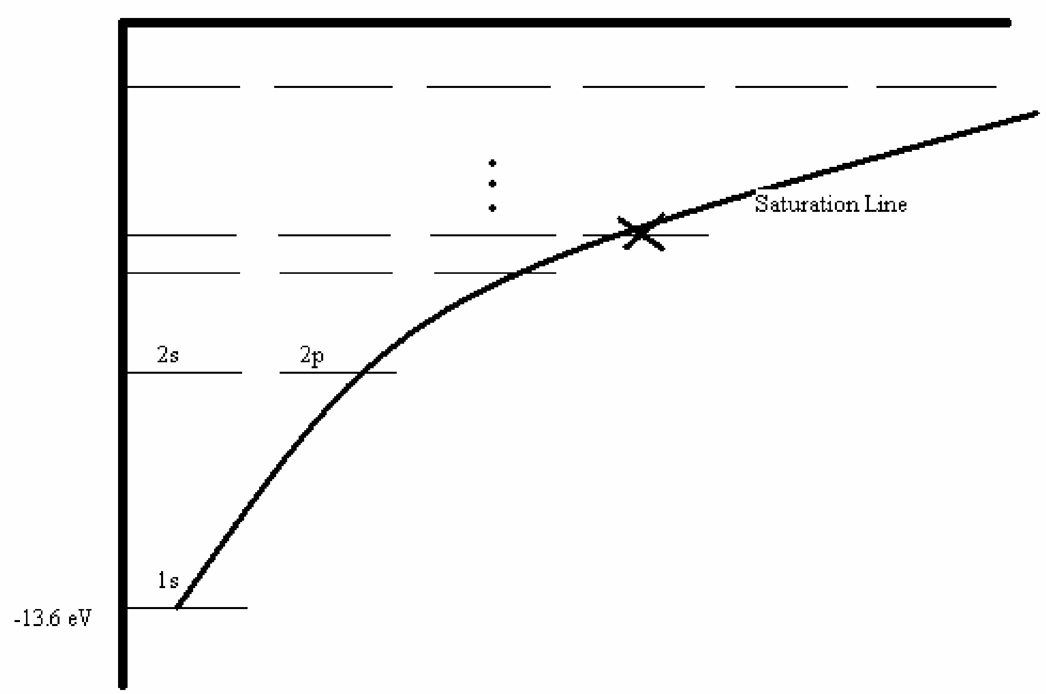
and a_0 is the Bohr radius $5.29x10^{-11}$ meters.

Naively, one may regard the radial integral as the square of the time average position of the electron. The value of this is expected to be somewhere between the time average position of the electron in state a, and correspondingly state b. At high energy levels n>>1, the time average positions are still increasing, but the change in position decreases as n gets larger, we can therefore regard the coupled radial integral to play less of a role in determining which state the electron will transition to.

An excited Hydrogen atom with an electron in the state n=200 and l=100 will decay down towards the stable ground states. In the decay process, there are only a couple of states with which a transition is possible. Selection rule for the states Y_m^l to $Y_{m'}^{l'}$ require that $l' = l \pm 1$ and $m' = m \pm 1$ or 0. This means that the electron will have to go to a state with l=99 or l=101; for the l=99 case, 99<n<200.

III.RESULTS

As observed from the previous section, equation (12), the electron will traverse down the energy levels such that it will reach the saturation line of n=l+1 quite quickly. This means that most excited hydrogen atoms will follow a uniform decay of n' = n-1, $l' = l - 1$ until it drops into the 1s ground state.



On the saturation line, we are guaranteed that all n' = n-1 transitions will occur. Photons that are emitted by such transitions should be very common and plentiful coming from the HII regions. A simple calculation of the approximate energy exhibited by such a transition according to Rydberg's formula is

$$E = 13.6eV * (\frac{1}{n_1^2} - \frac{1}{n_2^2}) \qquad (15)$$

In the previous section, it was shown that the rate of decay per unit time increased as the change in energy increased. At low n-levels, the change in energy increases dramatically, meaning that transitions at the low n-> n-1 levels will occur much faster and much more frequently. If we wish to observe these transitions, it makes sense to pick low n-level transmissions to observe. The Lyman-Alpha transition is the most basic transition of the hydrogen atom. However, since most hydrogen atoms in space are in its ground state, these photons that are released from the HII regions are absorbed right away. As expected, Lyman-Alpha transition emission lines are very rarely observed in Astronomy. The next most basic transition is the n=3 -> n=2 transition. This the Balmer-Alpha transition (otherwise known to Astronomers as H-Alpha). Plugging in 3 for n_1, and 2 for n_2, we get that the energy of this transition is 1.888 eV. This yields a wavelength of $6563\,\mathring{A}$.

This is in the visible red light spectrum. HII regions are known to have very strong H-Alpha lines, confirming that these transitions do take place quite frequently. Similar to how hydrogen atoms decay into its ground state, other gases in the HII regions will experience the same types of transitions. Other typically found lines include:

Line	Wavelength ($\mathring{A}$)
NII	6583
OII	3726 and 3729
OIII	4959 and 5007
SII	6716 and 6731

Together, all these emission lines can produce fabulous beautiful images of the HII regions themselves. There have been many beautiful pictures taken of the Rosette, Horsehead, and Eagle nebulae where these effects are occurring, pictures of these can be found at:

Rosette Nebula: http://www.noao.edu/image_gallery/html/im0557.html
Eagle Nebula: http://www.noao.edu/image_gallery/html/im0050.html
Horsehead Nebula: http://www.noao.edu/image_gallery/html/im0661.html

IV.REFERENCES

Heitler, W. The Quantum Theory of Radiation. Oxford University Press, Oxford, Great Britain. 1944.

Flynn, C. Notes. “Lecture 9: HII regions and planetary nebulae.” Extracted June 2006 from web. http://www.astro.utu.fi/~cflynn/astroII/19.html.

Graham, James. Notes. “Thermal Properties and Line Diagnostics for HII Regions.” Extracted June 2006 from web. http://astro.berkeley.edu/ay216/06/NOTES/ay216_2006_05_HIIThermal.pdf.

McIntosh, Gordon. “A modern physics laboratory activity: Radio astronomical observations of recombination lines.” Accepted by American Institute of Physics October 2001.

Morisset, C., Schaerer, D. et al. “A photoionization model of the compact HII region G29.96-0.02” Received by Astro-ph March 2002.

“Hydrogen Atom” Wikipedia. Extracted June 2006 from web. http://en.wikipedia.org/wiki/Hydrogen_atom.