Sagnac Interferometer

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Final Paper

1. Introduction

Interferometry is the study of the interaction between, specifically the superposition of, electromagnetic waves. When the light is detected after the interaction, a pattern is seen of the construction and destruction of the waves, effectively illustrating the interaction. Information about the waves can be extracted from the resulting interference of the waves which are examined in order to detect very small changes in the waves' properties. This technique is useful in a variety of fields including astronomy, fiber optics, chemistry, quantum mechanics, and particle physics.

An interferometer is a device that employs the concept of superposed waves for the purpose of determining certain properties. Using a laser, two beams, a reference beam and a sample beam, travel along a path of a particular design and undergo an interaction before combining to create an interference pattern. There are many different types of interferometers that are used to measure different properties shown by an interference pattern. They are used in telescopes, guidance systems, and TV holography. Two categories of interferometers are common path and double path interferometers. For a common path interferometer, the two laser beams travel the same path before combining while the beams travel along different paths in a double path interferometer.

One type of common path interferometer, the Sagnac Interferometer, makes use of the Sagnac Effect. The Sagnac Effect uses optical interferometry to measure rotation. An experiment utilizing this effect was first performed in 1913 by Georges Sagnac but the theory was first proposed 16 years earlier with the purpose of measuring the rotation of the Earth. In the experiment, a displacement of the resulting interference fringes is measured, revealing the effect rotation has on the light. Conclusions were also made about the speed of light being independent of the speed of the source. The interferometer that incorporates the Sagnac Effect has evolved into an instrument used in modern technology for many purposes with ring interferometry being a primary technique.

2. Theory

The interference of two waves occurs because the waves superimpose, forming a single wave with an amplitude that's either greater or lower than the initial waves. For a beam of light, many photons are interfering with each other and as the resulting waves are detected, a pattern emerges that illustrates the various properties of the light. The pattern changes with the frequency, phase, and amplitude of the light. An interferometer measures differences in the interference pattern to determine certain properties of the light. This can reveal information about the light source or the effects on the interferometer that caused the shifting of the interference fringes.

In a Sagnac Interferometer, a laser is used whose light is monochromatic (electromagnetic waves of a single frequency) and coherent (correlated phase). Because the beam is of a single frequency, interfering light behaves more predictably and, because it is coherent, a phase shift affects the interference in a way that can be measured. The laser is split into two beams of approximately equal power using a beam splitter (e.g. half-silvered mirror, polarizing beam splitter) and each beam follows a single path directed by mirrors but in opposite directions. Once the beams reach the point where they were split, therefore enclosing an area, they recombine and the resulting beam is detected. The setup as it was originally designed is shown in Figure 1.

A Sagnac Interferometer is usually arranged so that the beams have a triangular or rectangular trajectory using mirrors, or a circular trajectory guided by fiber optics. Rotation of the interferometer changes the paths of the two beams because the position of the point where the beams combine has rotationally shifted relative to the position where the initial beam split. This causes one of the split beams to have a longer travel time than the other because they are travelling in a constant medium and therefore have the same speed. The distance travelled is shifted, as illustrated in Figure 2.

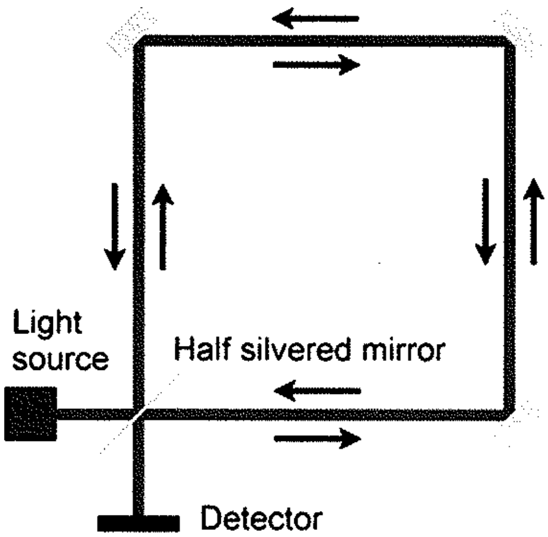

Figure 1. This is an original setup of a Sagnac Interferometer. The half-silvered mirror acts as a beam splitter, resulting in one beam traveling in one direction and the other beam traveling in the opposite direction. The two beams are combined an detected using interferometry where an interference pattern can be analyzed.

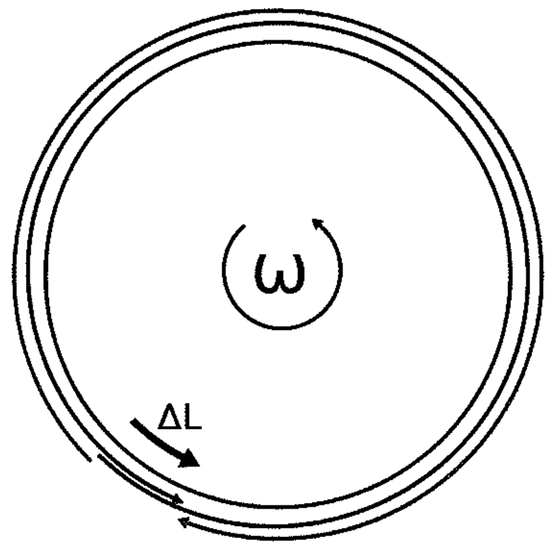

Figure 2. This illustrates the effect that the rotation of the system has on the counter-propagating beams. A beam traveling in the direction of the rotation ultimately has to travel more distance than the beam traveling in the direction opposite the rotation.

The change in travel time of each beam can be, and has been, derived analytically. For this derivation, a circular ring of radius R, rotates at an angular velocity, ω, shown in Figure 2. Another property of this particular system is that the axis of rotation and the vector normal to the enclosed area are parallel. As each beam travels along a common path but in opposite directions, they will be detected after different times of travel and therefore the ring rotates different amounts before the detection of each beam. The time the beam traveling in the direction of rotation takes to travel before being detected, t_1, is first derived using the change in distance caused by the rotating ring in that time, ΔL_1, and is given by

$$t_1 = \frac{2\pi R + \Delta L_1}{v}$$

where v is the speed at which the light is travelling. Light would travel slower, for example, is the glass of the fiber optics than in a vacuum. The distance a point on the ring has moved in time t_1 is

$$\Delta L_1 = R\omega t_1$$

and substituting for ΔL_1 and solving for t_1 yields

$$t_1 = \frac{2\pi R}{v - R\omega} = \frac{2\pi R}{v(1 - \frac{R\omega}{v})}.$$

Performing the same process for the beam travelling in the direction opposite rotation yields a travel time t_2 of

$$t_2 = \frac{2\pi R - \Delta L_2}{v} = \frac{2\pi R}{v(1 + \frac{R\omega}{v})}.$$

Taking the difference between these two times, t_1 - t_2, gives an equation for the time elapsed between when the beam travelling opposition rotation is detected and when the beam travelling in the direction of rotation is detected,

$$\Delta t = t_1 - t_2 = \frac{2\pi R}{v}\left[\frac{1}{1 - \frac{R\omega}{v}} - \frac{1}{1 + \frac{R\omega}{v}}\right] = \frac{4\pi R^2\omega}{v^2}\left[\frac{1}{1 - \left(\frac{R\omega}{v}\right)^2}\right].$$

Because the tangential velocity, $R\omega$, is much less than the speed at which the beams travel, the time difference can be approximated as

$$\Delta t = \frac{4\pi R^2\omega}{v^2}.$$

The area enclosed by the beams is a circle where $A = 4\pi R^2$ and so the equation for the time difference can be expressed in terms of a general area, A, as

$$\Delta t = \frac{4A\omega}{v^2}.$$

To generalize the equation more, for a system where the axis of rotation and the vector normal to the enclosed area are not parallel, the time difference can be further expressed as

$$\Delta t = \frac{4\boldsymbol{A} \cdot \boldsymbol{\omega}}{v^2}.$$

This time difference then causes a shift in the interference fringes. The phase shift, $\Delta\varphi$, is detectable because the light source is monochromatic with a wavelength λ and is given by

$$\Delta\varphi = \frac{2\pi v \Delta t}{\lambda} = \frac{8\pi \boldsymbol{A} \cdot \boldsymbol{\omega}}{\lambda v}.$$

This phase difference is proportional to both the enclosed area and the angular velocity. For very low rotation rates and relatively small enclosed areas, the Sagnac Effect is weakened, causing the beams to exhibit a lock-in effect and, therefore, an interferometer will not measure any phase shift. Lock-in is the tendency to resist splitting into two frequencies but is dependent on the design of the interferometer. A technique known as dithering that applies noise to randomize quantization error can be used to prevent this effect. Beyond a certain threshold, though, a Sagnac Interferometer is sensitive to differences in phase due to rotation.

3. Application

The Sagnac Effect is employed in many applications. One setup uses fiber optics to guide the counter-propagating light beams and a fiber coupler in place of a beam splitter. The fibers can be coiled which multiplies the effective area by the number of loops and, because the time difference and phase shift are dependent on the enclosed area, the effect is multiplied as well. This setup (shown in Figure 3) is called a fiber optic gyroscope and the equation for its phase difference is

$$\Delta\varphi = \frac{2\pi LD}{\lambda v}\omega,$$

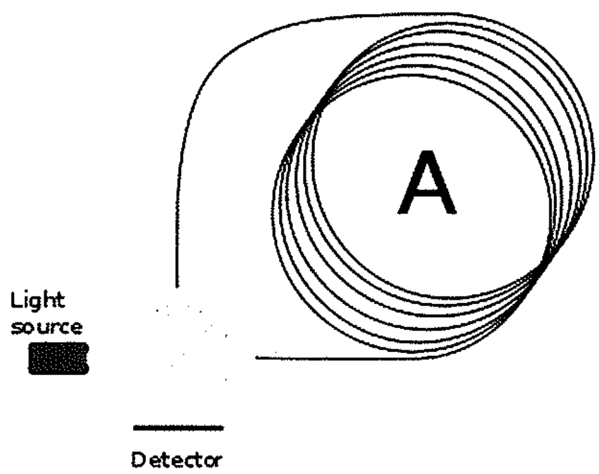

Figure 3. Setup for a Fiber Optic Gyroscope

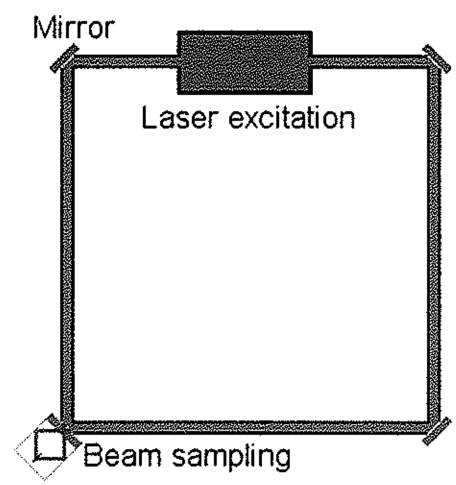

Figure 4. Setup for a Ring Laser Gyroscope

where L is the length of the fiber optic coil and D is its diameter. It is very sensitive to rotation and so the detection of fringe shifts in its interference yields precise calculations on the rotation. A fiber optic gyroscope is also resistant to external forces including vibration, acceleration, and shock. This low maintenance device is useful for technology that utilizes the Sagnac Effect because of its scalability and solid-state construction.

Another such device is a ring laser gyroscope. The setup (shown in Figure 4) involves a resonator cavity that excites the laser, allowing for recirculation that amplifies the effect of rotation. The laser beams are directed around the perimeter of the enclosed area using high quality mirrors. The two counter-propagating waves are recombined and instead of measuring the resulting phase shift, the frequencies can be measured directly which is also proportional to the rotation rate. The frequency difference is given by the equation

$$\Delta f = \frac{4\boldsymbol{A} \cdot \boldsymbol{\omega}}{\lambda P},$$

where P is the perimeter. Accurate measurements of this frequency difference The design of this setup does cause lock-in of the two beams at low rotation rates. Also, resonator size and mirror quality are challenges that limit performance and so a ring laser gyroscope cannot meet certain precision requirements. Gas leaking from the resonator cavity and wearing of the various moving parts diminish the quality of the apparatus over time, making it more high maintenance.

The devices that operate on the principle of the Sagnac Effect are used in modern technology with a wide range of applications from at sea to in space. An instrument sensitive to rotation would be useful for tracking and navigation. An inertial navigation system uses a combination of motion sensing devices to detect linear and rotational motion in order to accurately navigate large distances. Three fiber optic gyroscopes arranged perpendicular to each other would measure rotation in all three special dimensions. An object containing this device would be sensitive to changes in its orientation. There are both commercial and military applications for this type of inertial navigation such as in cars, planes, and ships.

A Sagnac Interferometer like the fiber optic gyroscope is sensitive enough to rotation to detect the Earth's rotation. Technological advancements have improved the quality of ring laser gyroscopes so that they are able to do the same. The ability to accurately measure the rotation rate of the Earth has its own applications. One application is in global positioning systems where small errors are compounded over time. A fiber optic gyroscope can measure the change in position due to the Earth's rotation and use the measurement as a correction for the GPS from the travel time of the signal. This is another use, of many, for an instrument based on a Sagnac Interferometer.

The concept and theory of the Sagnac Effect has been the basis of tests of theories of relativity and absolute rotation. There are also reports on short term sensitivity to atomic matter waves. Sagnac Interferometers have had a large impact on guidance and navigation technology. Systems incorporating both ring laser gyroscopes and fiber optic gyroscopes have become standard due to their precision when measuring the Sagnac Effect.

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