Michael Brewer

PHYS 517: Quantum Mechanics II

Term Paper

Setting up Quantum Gravity

Introduction

One of the major challenges remaining in theoretical physics is the construction of a quantum mechanical theory of gravity. Compared to the other forces, gravity is extremely weak. Consequently, it is difficult to measure gravitational effects on the small scales we generally consider for quantum mechanical interactions. Einstein's general relativity theory of gravity is primarily a geometric theory. From GR we get black holes, but we do not get any way to treat the "singularity" at the center. Hawking radiation and gravitons are quantum features of gravity that are commonly discussed, but they do not appear anywhere in GR, giving us more motivation to produce a quantum gravity theory which is capable of addressing all of these phenomena (assuming they exist) and any other interesting effects that might arise.

So far, no one has been successful producing a quantum theory of gravity. In this paper, we are going to examine a paper written by Johan Noldus (Noldus, 2013) that attempts to build the mathematical and philosophical foundation for a theory of quantum gravity. This foundation is developed from a set of eleven axioms, and the focus of this paper will be reporting those axioms, which set up the geometric and dynamical basis of the theory as well as some of the philosophical basis. Then we will briefly examine a few results of the theory and discuss Hawking radiation.

The Axioms

In the paper the author sets to "axiomatize" a new quantum-gravity-matter theory, laying down the pieces that one would need to develop a functional theory for quantum gravity. We will avoid delving deeply into the mathematical details at any time, because the depth of the mathematics is too much for paper of this size (there is an instance where a single equation is written across six pages), and most of the math is not likely to be very enlightening (or simply not short enough to examine here), instead attempting to explain the purpose of the mathematics involved.

AXIOM 0: "All local one particle Nevanlinna modules ***H***(e_a(x)) are second countable and unitarily equivalent to the local model space ***L***. We do not dispose of an a priori notion of spatiality in M; however we have one in T M and the information contained in any one particle Nevanlinna module generating the universal Fock space K is unitary equivalent to $\oplus^{\infty}{}_{i=1}L$ which still has cardinality $\mathcal{N}_0$. However, the Fock space construction to K is now more complicated since (a) we genuinely have to describe states with an infinite number of particles and (b) we allow for more complicated forms of statistics. We use here the Guichardet construction implying that K has cardinality $\mathcal{N}_0^{\mathcal{N}_0}$ in contrast to what is usually supposed in Quantum Field Theory...The full relational Nevanlinna module however is a dynamical object and not fixed a priori; such as is the case for the spacetime manifold in general relativity." (Noldus, 2013)

AXIOM I: "Manifold structure."
In this axiom, the author develops a set of properties and describes the behavior of the manifold structure and some of the differential geometry that serves as part of the mathematical basis for his theory of quantum gravity. (Noldus, 2013)
AXIOM II: "At each point x of the manifold M, there exists a basic set of particle creation operators $a^{\dagger}{}_{\vec{k}\mathrm{m}\sigma c\pm}(e_b(x), x)$ where $\vec{k}$ is the three momentum with respect to e_j(x), m the inertial mass, σ the spin, ± indicates whether it corresponds to a particle of positive or negative norm respectively and c is a natural index labeling one of the $\mathcal{N}_0$ copies mentioned in AXIOM 0. Moreover, there exists a unique cyclic, generating vacuum state |0, e_a(x), x⟩ on which all creation and annihilation operators act as usual. Moreover, there exists a unique cyclic, generating vacuum state |0, e_a(x), x⟩ on which all creation and annihilation operators act as usual. By convention, F(e_a(x),x) is the local Fock space "generated" by the application of the operators with c = 0 on the vacuum state. However, the rest of K is also ontologically available to the local observer by which I mean that he "knows" about the existence of the particles with c ≠ 0 in the universe but is unable to measure them and therefore cannot tell anything about the

interactions between them. This implies an extremely important subtlety wich should be well understood: this “information” about the rest of the universe must be contained in the energy-momentum and spin tensors evaluated at (x,v^e). However, the local Poincaré algebra only depends upon the restriction of these tensors to $F(e_b(x),x)$.” (Noldus, 2013)

AXIOM III: “There exist two local conserved, non-symmetric energy momentum tensors $T_j^{\mathrm{ab}}\left(x,v^d,e_g(x)\right)$ on TM and anti-symmetric conserved spin tensors $S_{\mathrm{jc}}^{\mathrm{ab}}\left(x,v^d,e_g(x)\right)$ where the conservation laws are respectively

$$\partial_a T_j^{\mathrm{ab}}\left(x,v^d,e_g(x)\right)=0$$

and

$$\partial^c S_{\mathrm{jc}}^{\mathrm{ab}}\left(x,v^d,e_g(x)\right)=0$$

All tensors are normal ordered expressions in terms of the creation and annihilation operators of the whole universe and local particle space respectively.” (Noldus, 2013)

Conservation laws are important tools throughout physics; axiom III deals with setting up the conservation laws in this foundation for quantum gravity.

AXIOM IV: “Having a totally consistent particle interpretation requires amongst others the commutation relations

$$\left[P_1^a\left(e_b(x)\right),{a^\dagger}_{\vec{k}\mathrm{m}\sigma\mathrm{c}\pm}\left(e_b(x),x\right)\right]=k^a{a^\dagger}_{\vec{k}\mathrm{m}\sigma\mathrm{c}\pm}\left(e_b(x),x\right)$$

as the reader can easily convince himself of (actually this equality is en- forced by the multi-particle states). Similar expressions should hold for $P_2^a(x)$ and creation operators corresponding to c = 0. This implies that the “theory” on the tangent space TM_x is a free one which enforces the physical statement that any legitimate Quantum Theory must be asymptotically free.”

(Noldus, 2013)

AXIOM V: "Space-time interactions are kinematically determined by unitary relational operators $U\left(e_a(x), e_b(y), x, y\right)$ including the following conditions:

$$U\left(e_a(x), e_b(y), x, y\right)|\, 0, e_a(x), x\rangle = |0, e_a(y), y\rangle$$

and

$$U\left(e_a(x), e_b(y), x, y\right) a^{\dagger}{}_{\vec{k}\mathrm{m}\sigma\mathrm{c}\pm}\left(e_c(x), x\right) U^{\dagger}\left(e_a(x), e_b(y), x, y\right) = a^{\dagger}{}_{\vec{k}\mathrm{m}\sigma\mathrm{c}\pm}\left(e_c(y), y\right)$$

Moreover, $U^{\dagger}\left(e_a(x), e_b(y), x, y\right) = U\left(e_a(y), e_b(x), y, x\right)$ and we must demand the group law to hold

$$U\left(e_b(y), e_c(z), y, z\right) U\left(e_a(x), e_b(y), x, y\right) = U\left(e_a(x), e_c(z), x, z\right)$$

which can be interpreted as a trivial homology condition." (Noldus, 2013)

The author then further discusses the trivial homology.

AXIOM VI: "We need a principle of local Lorentz *covariance* since the dynamics should be covariant with respect to local changes on *M* in the local reference frames $e_b(x)$ (hence, we need the notion of a quantum spin connection). Let me start by saying something about transformation laws in general: the Lorentz transformations depend upon $e^b(x)$ and therefore we write $U(\Lambda(x),e^b(x))$. Quantum mechanically, all we require for a unitary transformation T from one reference frame to another is that $T(\Gamma(x), \Lambda^a{}_b(x)e^b(x))T(\Lambda(x),e^b(x)) = T(\Gamma(x)\Lambda(x),e^b(x))$." (Noldus, 2013)

The author goes on to examine a few different viewpoints that follow from this requirement.

AXIOM VII: "The only way our local particle notions can couple to space- time is by means of a vielbein $e^a{}_\mu(x,v^a)$ and the classical aspect of gravity is fully contained in this symbol and the differential operator $D_\mu(x,v^a)$." (Noldus, 2013)

Then, to begin developing the dynamical content of the theory, Axiom VII continues by covering the mathematical construction of objects equivalent to the Einstein and Spin tensor and setting this equal to the expectation values of the local energy momentum tensor $T^{\mathrm{ab}}\,(x, v^a)$ and spin tensor $S_c^{\mathrm{ab}}\,(x, v^a)$.

AXIOM VIII: "The universal equations of motion for the unitary potential $U(e_b(x),x)$ and Hermitian quantum gauge field $A_\mu(e_b(x),x)$ are much easier to write down and it is easy to prove that they necessitate the point of view of -at least- an indefinite Clifford Hilbert module. Indeed, the most general equation for $U(e_b(x),x)$ must satisfy the following conditions: (a) it transforms covariantly under quantum local Lorentz transformations (b) coordinate invariant (c) preserves the unitarity relationship. From (c), one derives that the equation must be first order in the derivatives and from (b) one concludes one has to contract the covariant derivative ∇_μ with the vielbein $e^\mu{}_a$. To make this equation generally covariant, we need the gamma matrices, that is the Clifford algebra." (Noldus, 2013)

Axiom VIII serves to develop the matter dynamics for the unitary potential and Hermitian quantum gauge field described in the axiom.

AXIOM IX: This axiom talks about "consciousness" which operates in both terms of local particle notions and also in terms of "quasi-local" particle states.

"Nevertheless, perception is relative to these entities and there is something in this world which apart from being dynamical itself, recognizes the dynamics of shapes even though the fundamental materialistic theory does not know shapes and therefore could not even define what it means that they change." (Noldus, 2013)

So, in this axiom, the author explains the reasons why a theory of consciousness is important to include in a theory of quantum gravity (the shortest explanation being that consciousness has an effect on the dynamics in the theory that needs to be accounted for).

AXIOM X: This an extension of axiom IX, and addresses the development of a dynamical measurement theory. (Noldus, 2013)

AXIOM XI: Here the author notes that he doesn't have a mathematical reason to force the classical cosmological constant to be zero in this theory, but argues on physical ground that it should be zero, noting that there could be "time dependent" effective cosmological field generated by quantum fluctuations in the matter field as long as the average value of the effective cosmological field is zero. (Noldus, 2013)

Applications

Working with linearized equations following from axiom VII and looking at the Newtonian limit, the author develops a few application-oriented results.

BLACK HOLES: A conventional black hole is an extremely compact object where its entire mass is concentrated at a single point. These objects have a characteristic radius called the "event horizon" inside which nothing, including light, can propagate outwards. The author's work with linearized equations in Newtonian limit produces a family of solutions (Noldus, 2013):

$$g\left(r,\theta,v^1,v^2,v^3\right) = y\left(\ln(r)+v^2,v^3\right)z\left(\theta - v^1,v^3\right)$$

He then notes that this class of solutions does not contain the Kerr or Schwarzschild spacetimes which we use to describe conventional black holes in general relativity. This doesn't prevent the existence of black hole-like objects within the theory, but would require that either the objects are not perfectly dark or that they are impermeable from both sides of the event horizon. (Noldus, 2013)

FORCE FORM: Continuing the development in the Newtonian limit from the above family of solutions, we can produce a formula for the gravitational force:

$$\vec{F} = -\mathrm{GM}\,/\left(r_0\ln^2\left(r\,/r_0\right)r\right)\,\vec{e}_r$$

where r_0 is some reference length. What's interesting about this result is that for $r > r_0$, this yields a stronger gravitational force than in Newtonian gravity, providing us with a possible way to explain "dark matter" without requiring any matter to be present. Conversely, when we have $r < r_0$, we have a gravitational force weaker than the Newtonian form. The author notes shortly after presenting this that the diversity of different axisymmetric solutions supported under his theory is sufficient to solve all the problems Einstein gravity faces. (Noldus, 2013)

Hawking Radiation

One of the only connections between quantum physics and general relativity that is reasonably well established in physics today is the existence of Hawking Radiation. Although Hawking's calculation was much more rigorous, the simplified derivation commonly used in introductory general relativity textbooks (Schutz, 2011) is as follows: suppose we have a black hole with an event horizon at coordinate position $r = R$ (take $r = 0$ to be the center of the black hole). From the uncertainty principle, $\Delta t \Delta E \geq \hbar/2$, we note that we can temporarily produce virtual particle pairs which will quickly annihilate, with lifetimes in accordance with the uncertainty principle. Since these particles have energy, they will therefore also carry momentum, and thus we have an uncertainty in their position given by $\Delta x \geq \hbar/(2\Delta p)$. Suppose then that instead of these virtual particles being produced in free space, one is produced at a position $R+\Delta x$ and the other at $R-\Delta x$. The particle at $R-\Delta x$ cannot propagate outwards (because it is inside the event horizon), and will be unable to annihilate with the other particle, which is allowed to escape towards $r = \infty$. These escaped particles are Hawking radiation.

There is no experimental evidence for Hawking radiation (the redshift from climbing out of the black hole gravitational potential is quite large, so it is not surprising that we cannot detect the emission), and the author goes into some length considering the physical interpretation of the mathematical basis for Hawking radiation, but ultimately seems to accept Hawking radiation. (Noldus, 2013)

Conclusion

The axioms described may serve as a basis for a theory of quantum gravity, but, generally speaking, a great deal of work remains to more thoroughly develop and expand the theory at a mathematical level, or at least enough work so that we can examine real physical observations. General Relativity, as one of its big tests, was able to account for the deviation in the procession of Mercury's orbit from Newtonian gravity. An analogous proving ground for this new theory would perhaps be to see if we can produce a force law, either the form derived in the Newtonian limit $\vec{F} = -\mathrm{GM} \big/ \left(r_0 \ln^2 \left(r / r_0 \right) r \right) \vec{e}_r$ or something higher which can remove the need for dark matter in our theories. Some of the areas where the author sees future development are: tensorial and spinorial quantum mechanics, topological quantum manifolds, canonical differentiable structure, realizations of non-commutative continuum manifolds, and examining other views for construction solutions of differential equations, among other topics. (Noldus, 2013)

The philosophical discussions of consciousness discussed in his paper are primarily concerned with observers and observations. We know from quantum mechanics that the act of observing a system has an effect on the state of the system. Therefore, it is important that any quantum theory of gravity is built with the mechanisms to address this situation, and although we have avoided discussing the particulars of Noldus' philosophy in this area, we should not ignore the relevance of such a component in the theory.

Developing new theories is generally a massive mathematical and philosophical challenge in physics. This statement is especially true when trying to combine a very macroscopic phenomenon like gravity with a phenomenon that is generally very microscopic like quantum mechanics. We look forwards to the day when we have a more complete view of the universe, but until that day we look with interest on ways today's physicists attempt to resolve our current theoretical problems.

References

1. Noldus, Johan. “Foundations of a theory of quantum gravity.” May 2013. <http://arxiv.org/abs/1101.5113v5>
2. Schutz, Bernard. “A First Course in General Relativity.” 2nd Edition, Cambridge University Press, 2011.