Efficiency in Solar Cells

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December 15, 2013

Abstract

The theoretical efficiency limit is calculated with some attention paid to the laws of physics ruling the limiting behavior of the system. Beginning with single gap solar cell systems, moving onward to tandem cells (solar cells containing several p-n junctions), and ending with solar cells with an intermediate energy band gap (created through an impurity in the system) progress towards maximum efficiency is revealed and the implications this may have on future energy consumption is briefly remarked upon.

Alternate sources of energy are vital to the continuation of the human race. Conventional energy production is derived mainly through non-renewable sources. Of the total energy produced in the United States of America in 2011 82% was produced from oil, natural gas, and coal. 8% came from nuclear power. Only 9% of the total energy produced was from renewable sources such as geothermal, wind, or solar.[1] Every year as humanity expands in population, in life expectancy, so too does the amount of energy we require continue to expand with it. Simply put, as the nonrenewable sources dwindle it is imperative to seriously look further into alternate, sustainable and renewable power sources. Research into maximizing the efficiency of solar cells is not a knew phenomenon, by as early as 1954 the potential of the silicon solar cell had begun to be explored in the scientific literature. [2] And it was in 1960 that the classical analysis of single-gap systems by Shockley and Quisser was published [3], an efficiency analysis that to this day remains the standard for measuring against when exploring new systems in the hope to maximize solar cell efficiency. Starting with Shockley and Quisser's classical analysis this paper will attempt to step through some of the significant leaps in progress we have made in increasing the efficiency in solar cells.

1 Single Gap Solar Cells

When studying the efficiency of solar cells it is important to understand exactly what it is happening. With that in mind, there are two terms that are important to distinguish between . The *semiempirical limit* of efficiency (calculated as a function of the energy gap) is based up empirical values for constants of the system while the *detailed balanced limit* is a theoretically justified limit.[3] At the time of Shockley and Queisser, efficiency of solar cells were based solely on the semiemperical limit. The walk through, following Shockley and Queisser's steps, finds the maximum possible efficiency by deriving the theoretically justified limit, the detailed balanced limit. Since the detailed balanced limit is significantly higher than the semiemperical limit, at times up to 50% higher, the difference between the two gave scientists hope that real progress could be made in increasing the efficiency of pre-existing solar cells and bringing the experimentally found semiemperical limit up to to match the newly found theoretical limits. [3]

Maximum Efficiency of A Solar Cell

For the efficiency definition used in thermodynamic engines, the maximum efficiency corresponds with zero entropy creation, which then leads to the Carnot efficiency limit. The efficiency definition in solar cells however is different and maximum efficiency operation does not correspond with reversible operation (zero entropy) but gives rise to appreciable entropy creation.[4] The thermodynamic efficiency limits of Carnot can not be reached in solar cells; different approaches must be used when it comes to calculating the theoretical efficiency limits of solar cells.

The *ultimate efficiency hypothesis* states that each photon with energy greater than hν_g produces one electronic charge q at a voltage of $V_g = h\nu_g/q$, where v_g is the cut off frequency in a device that utilizes a photoelectric process.[3] (See Fig. 1 for a schematic of a generalized device which utilizes a photoelectric figure. This set up holds true for all configurations of solar cells, from single gap to tandem.) Keeping this theory in mind, a derivation for the maximum efficiency of a solar cell can be formed. In theory the efficiency of a solar converter can be brought to the thermodynamical limit: $(T_s - T_c)/T_c$ by using reflectors. However a simple planar solar cell with out any method to concentrate the radiation cannot hope to even approach this limit. The limit it can approach depends on:

- its energy gap, E_g
- the angle subtended by the sun
- the angle of incidence of radiation
- absorption coefficient of solar energy and other degrading factors which in principle can be treated as if they approach unity

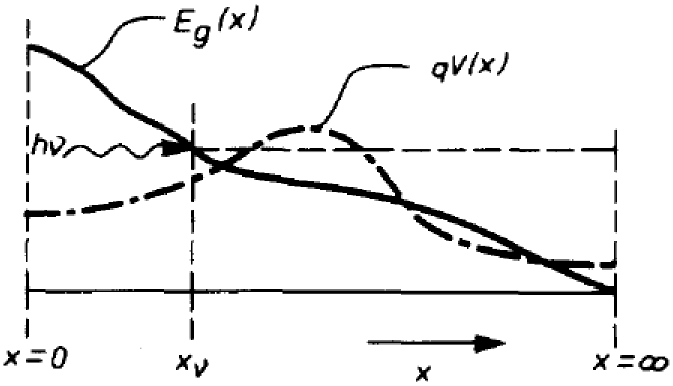

Figure 1: Schematic representation of a general photovoltaic structure.The band gap E_g(x) decreases monotonically to zero from left to right. The bias voltage V(x) varies arbitrarily with the x-coordinate. Radiation of temperature T_s (or T_c) is incident from the left; from the right radiation of temperature T_c is incident. T_c is the cell temperature, T_s the sun temperature.[4]

It is the fraction of recombination between holes and electrons that result in radiation, and it is this radiative recombination that determines the detailed balance limit for efficiency. Thus efficiency is the ratio of power delivered from the cell to the the incident solar power collected. In order to carry out the calculation to determine the limit of efficiency, five processes must be considered: (1) generation of hole-electron pairs by the incident solar radiation, (2) the radiative recombination of hole-electron pairs with resultant emission of photons, (3) other nonradiative processes which result in generation, (4) recombination of hole-electron pairs, and (5) removal of holes from the p-type region and electrons from the n-type region in the form of a current I which withdraws hole-electron pairs at a rate I/q.[3]

In accordance to the given processes, the ultimate efficiency is

$$u(x_g) = hv_g Q/P_{sun} = \frac{x_g \int_{x_g}^{0} x^2 dx/(e^x - 1)}{\int_0^\infty x^3 dx/(e^x - 1)} \tag{1}$$

The Detailed Balance Limit of Efficiency

In view of the maximum output of the system, which is obtained by using a voltage V that produces a maximum IV value, the efficiency can be expressed as a function dependent upon four variables

$$\eta = \eta(x_g, x_c, t_s, f) \tag{2}$$

where $x_g = E_g/kT_s$ and $x_c = T_c/T_s$ are both ratios, t_s is the probability that a photon with $h\nu < E_g$ incident on the surface will produce a hole-electron pair, and f is the quantity which takes into account all other parameters such as those involving transmission of radiative recombination (f_c) and the solid angle subtended by the sun.

$$\begin{aligned} \eta(x_g, x_c, t_s, f) &= I_{max}V_{max}/P_{inc} \\ &= t_s u(x_g)\nu(f, x_c, x_g)m(\nu x_g/x_c) \end{aligned} \tag{3}$$

Where t_s is the probability that a photon will produce an electron-hole pair; $u(x_g)$ is the ultimate efficiency in accordance with the ultimate efficiency hypothesis; $\nu(x_g, x_c, f)$ is the ratio of the open-circuit voltage to the energy gap of the cell; and $m(\nu x_g/x_c)$ is the impedance matching factor, which is a function of the ratio of the open-circuit voltage to thermal voltage for the cell. To summarize: the maximum efficiency of a single gap solar cell is the electrical power out of the cell divided by the power of the incident solar energy falling on the cell.

2 Infinite Tandem Cells

The search for increased efficiency in solar cells led to a repeating of the characteristics of the maximum efficiency device (first covered in the Shockley and Queisser paper) but extrapolating one step further to an infinite tandem structure of p-n cells.[4] In an infinite tandem structure the bandgap decreases monotonically but in infinitesimal steps from ∞ to 0, and each individual cell with band gap $E_g = h\nu_g$ is considered ”selectively black” (i.e. it acts as a selective black body) for all all $\nu = \nu_g$. (See Fig. 2) [5]

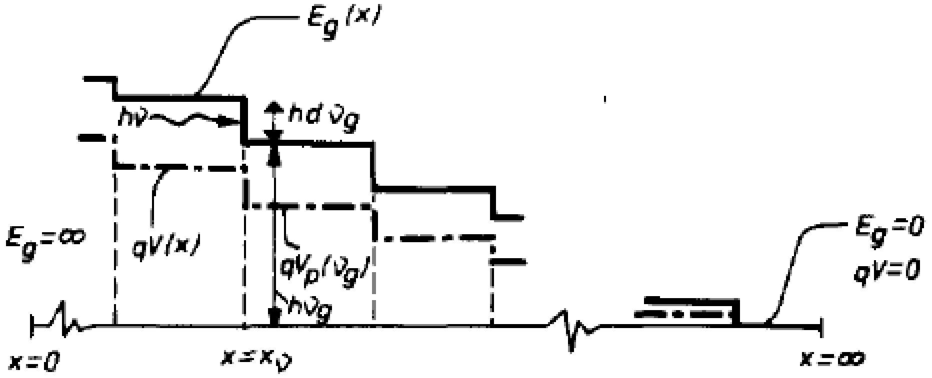

Figure 2: Maximum efficiency photovoltaic structure: the band gap decreases monotonically in infinitesimal steps, from infinity to zero.

The efficiency is once again the electrical power out of the cell divided by the power of the incoming solar energy.[4] Calculating the electrical power output by the infinite sum of solar cells in tandem from the current and the voltage difference, an upper limit of the power emerges where

$$P \leq \frac{2\pi}{c^2} \int_0^\infty q\nu^2 V_p(\nu)[n_s(\nu) - n_o(V_p)]d\nu \tag{4}$$

Notice that the maximum power point of $V_p[n_s - n_o]$ depends on the frequency ν. The incoming power of the sun can be calculated to be

$$\begin{aligned} P_s &= \frac{2\pi h}{c^2} \int_0^\infty \nu^3 n_s(\nu) d\nu \\ &= \frac{2\pi^5 k^4}{15c^2h^3} T_s^4 \end{aligned} \tag{5}$$

And thus the maximum efficiency is:

$$\eta = \frac{P}{P_s} = \frac{15h^3}{\pi^4 k^4 T_s^4} \int_0^\infty q\nu^2 V_p(\nu)[n_s(\nu) - n_o(V_p)]d\nu \tag{6}$$

3 Solar Cells with Intermediate Energy Levels

A third possible configuration, created with the sole purpose of increasing the detailed balanced limit, is that of a solar cell with an intermediate energy level. Fig. 3 presents such a structure. It contains the usual semiconductor valence band and conduction band but, in addition, there is an intermediate band placed between the two. [6]

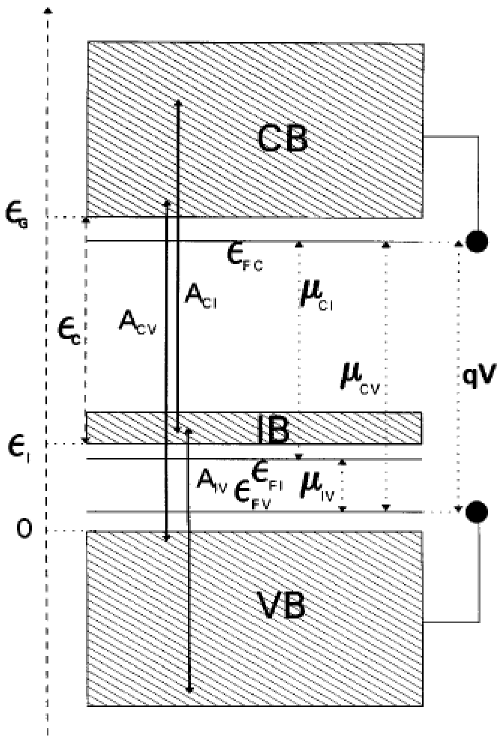

Figure 3: Band diagram of a solar cell with an intermediate band.

In addition to the common absorption of photons by electron transitions between the valence and the conduction band, with the addition of an intermediate band there will also be absorption due to transitions between the valence band and the intermediate band and between the intermediate band and the conduction band. [6]

Just as in the seminal work by Shockley and Queisser an I-V equation can be derived and used to calculate the cell efficiency. The derivation is not given in this paper however the results speak quite elegantly of the difference in efficiency an intermediate energy level makes.

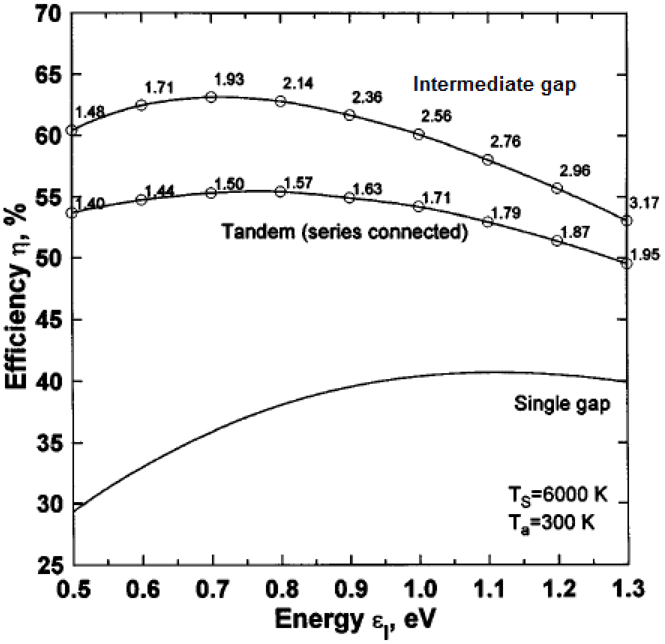

Figure 4: Efficiency limit for a solar cell with an intermediate band, for a two-terminal ideal tandem cell, and for a cell with a single band gap.

In Fig. 4, looking at the graphical representation of the results from Luque and Marti's work, it is clear that introducing an intermediate energy level in a traditional single gap solar cell can dramatically increase its maximum efficiency.

4 Comparison

The ultimate goal in this research is to end up with a highly efficient yet relatively cheap and easy to make solar cell. The table below gives the highest found efficiency rates for several different types and configurations of solar cells.

Type of Solar Cell	Maximum Efficiency (%)
Single Cell	40.7
Two Cells in Tandem	55
Cell w/ Intermediate Level	63.1
Infinite Cells in Tandem	86.8

While the infinite cells in tandem have the highest maximum efficiency they are also the most unrealistic. It is the cell with intermediate band, which can reach an efficiency of 63.1% instead of the more traditional 40.7% of the Shockley-Queisser model, that at appears to be the most efficient and still relatively practical solar cell model to make. [6]

References

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