

Alice in Stretch & SqueezeLand: 16 Lorenz Map

August 12, 2012

Chapter Abstract

Alice in
Stretch &
SqueezeLand:
16 Lorenz
Map

Chapter
Summary-01

Lop-01

Lop-02

Lop-03

Lop-04

Lop-05

Lop-06

Lop-07

Lop-08

Lop-09

Lop-10

Lop-11

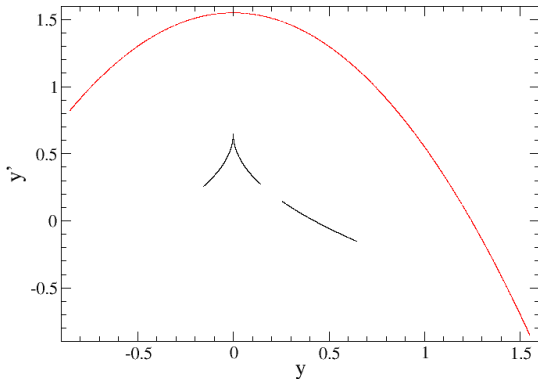
Lop-12

Formation order and stability properties of orbits created in the Lorenz map are determined from the properties of corresponding orbits in an image map related to the Knife Map.

Forcing Diagram - Horseshoe

Return Maps for Chaotic Attractors

$k=2, a=1.55$ and $k=1/2, a=0.65$



Return Map Approximations

The Rossler return map is well approximated by the following maps:

$$x' = \lambda x(1 - x)$$

$$x' = a - x^2$$

$$x' = 1 - \mu x^2$$

$$x' = 1 - \left| \frac{x - m}{w} \right|^2$$

Image of Lorenz Return Map

The image of the Lorenz return map is well approximated by the following maps:

$$y' = b - |y|^{1/2}$$

$$y' = 1 - \mu|y|^{1/2}$$

$$y' = 1 - \left| \frac{y - m}{w} \right|^{1/2}$$

Class of Lopsided Maps

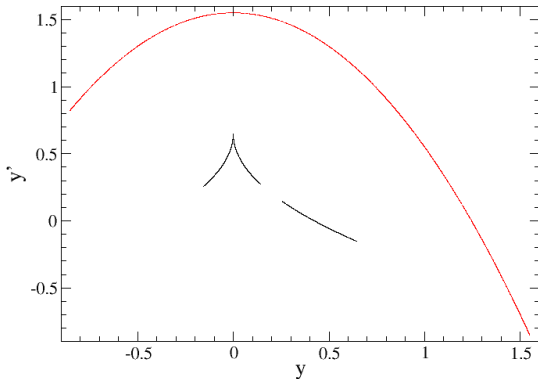
$$x' = f(x; k, a) = 1 - \left| \frac{x - m}{w} \right|^k$$

- 1 Zero crossings at $x = +1$ and $x = a$, $-1 \leq a \leq 0$
- 2 Maximum at $m = \frac{1+a}{2}$
- 3 Half-width $w = \frac{1-a}{2}$
- 4 $m + w = 1$

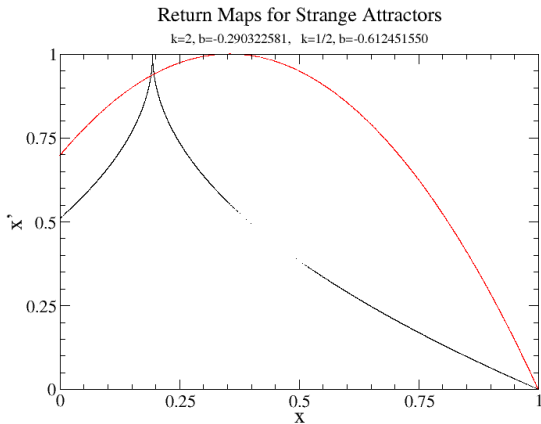
Forcing Diagram - Horseshoe

Return Maps for Chaotic Attractors

$k=2, a=1.55$ and $k=1/2, a=0.65$

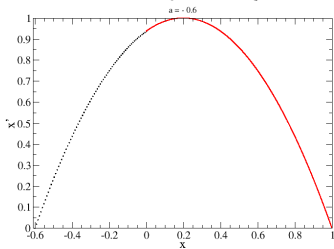


Forcing Diagram - Horseshoe

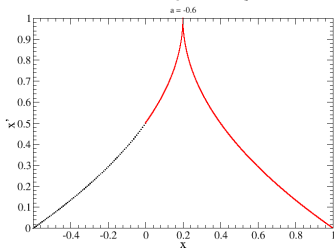


Map Comparisons

Modified Logistic Return Map



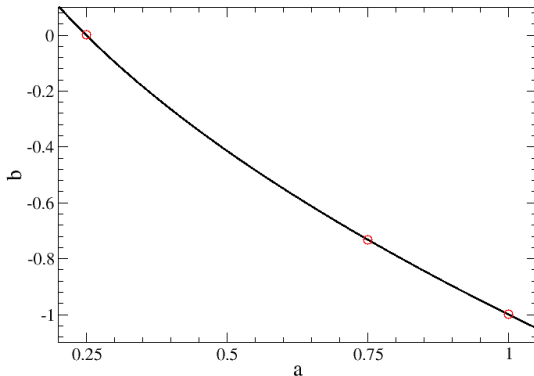
Lorenz-image Return Map



Forcing Diagram - Horseshoe

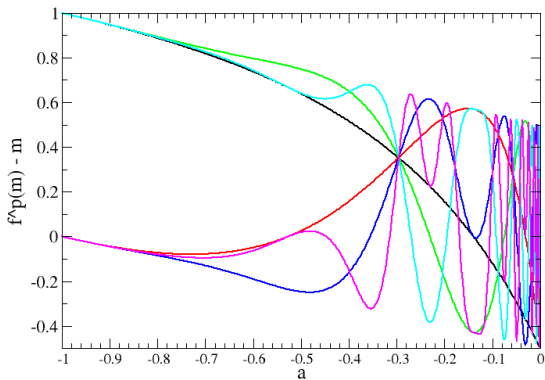
Transformation Between Control Parameter Values

$$y' = a - by^{1/2} \quad y' = 1 - l(x-m)/w^{1/2}(1/2)$$



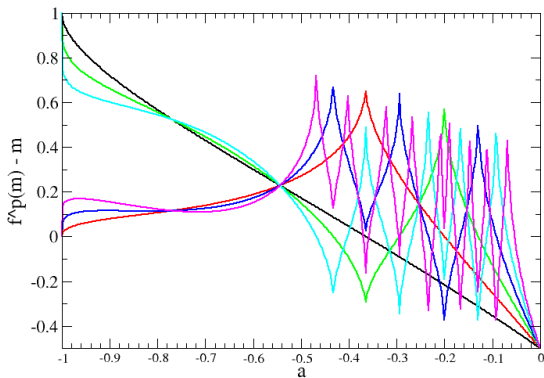
Forcing Diagram - Horseshoe

Superstable Orbits for Logistic Map



Forcing Diagram - Horseshoe

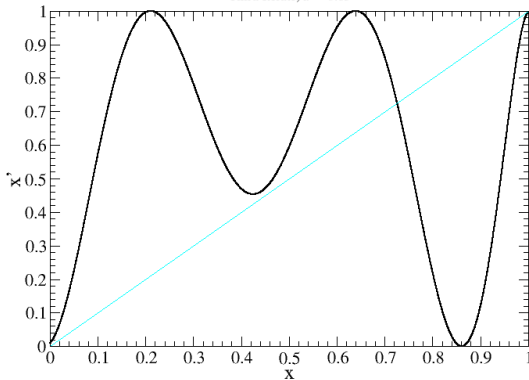
Homoclinic Orbits, Lorenz-Image Map



Forcing Diagram - Horseshoe

Modified Logistic Map

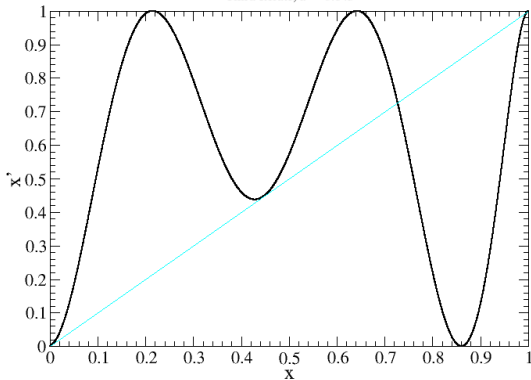
Third Iterate, $a = -0.15$



Forcing Diagram - Horseshoe

Modified Logistic Map

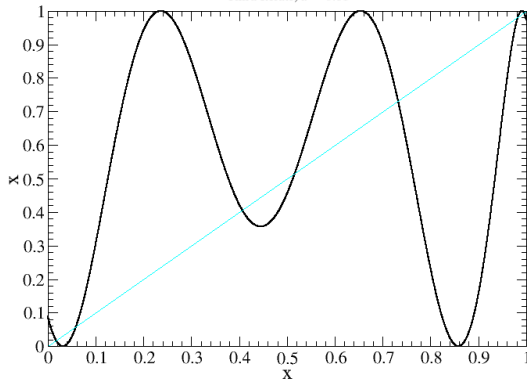
Third Iterate, $a = -0.143$



Forcing Diagram - Horseshoe

Modified Logistic Map

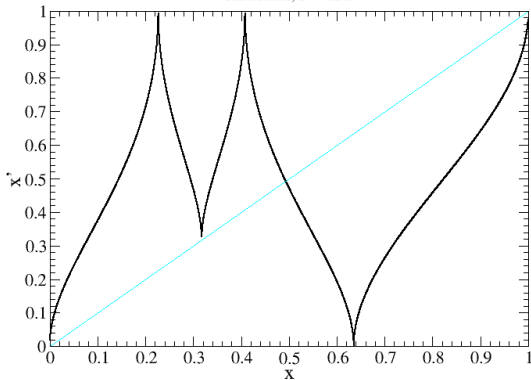
Third Iterate, $a = -0.11$



Forcing Diagram - Horseshoe

Lorenz-Image Map

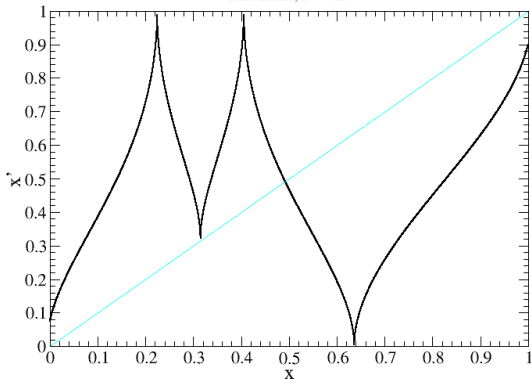
Third Iterate, $a = -0.365$



Forcing Diagram - Horseshoe

Lorenz-Image Map

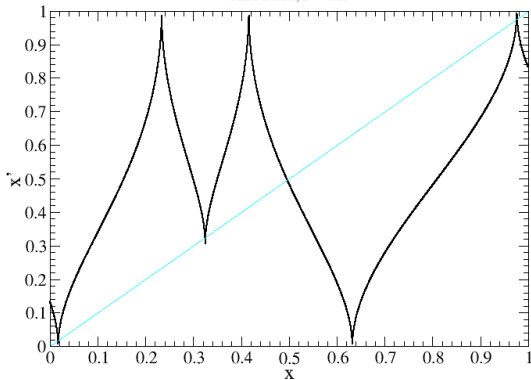
Third Iterate, $a = -0.37$



Forcing Diagram - Horseshoe

Lorenz-Image Map

Third Iterate, $a = -0.35$



Scaling

- Logistic: SNB Period 3 = scaled version SNB of M.
- Renormalization theory applies.
- U Sequence

- Knife: S-SNB Period 3 = scaled version S-SNB of K.
- Renormalization theory applies.
- U^{-1} Sequence

Topological Organization

The knife and logistic maps are suspensions of flows. Corresponding orbits (identical names) in each suspension are organized identically.

Identical lifts of the logistic and knife maps lead to identical covering orbit organization.

The mysteries of orbit organization in flows with $g > 1$ are the same for stretch-and-fold and for tear-and-squeeze mechanisms.