

# Alice in Stretch & SqueezeLand: 13 Quantizing Chaos

August 12, 2012

# Chapter Abstract

Alice in  
Stretch &  
SqueezeLand:  
13  
Quantizing  
Chaos

Chapter  
Summary-01

Quantizing  
Chaos-01

Quantizing  
Chaos-01b

Quantizing  
Chaos-01c

Quantizing  
Chaos-02

Quantizing  
Chaos-03

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Chaos-04

Quantizing  
Chaos-05

The global torsion of an attractor inside a torus can be changed by integer values,  $n$ , in a simple way.

This involves a simple global diffeomorphism.

Local diffeomorphisms are used to produce  $q$ -fold covers.

Classical-like statistics (average energy, spin angular momentum) depend on the quantization indices  $n$ ,  $p/q$  in the expected intuitive way.

## Rotating the Attractor

$$\frac{d}{dt} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_1(X, Y) \\ F_2(X, Y) \end{bmatrix} + \begin{bmatrix} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = R\mathbf{F}(R^{-1}\mathbf{u}) + R\mathbf{t} + \Omega \begin{bmatrix} -v \\ +u \end{bmatrix}$$

$$\Omega = n \omega_d$$

$$q \Omega = p \omega_d$$

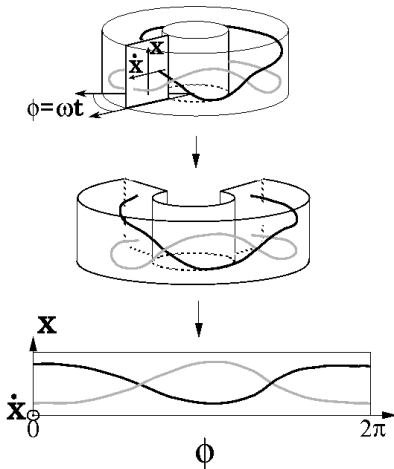
Global Diffeomorphisms

Local Diffeomorphisms

( $q$ -fold covers)

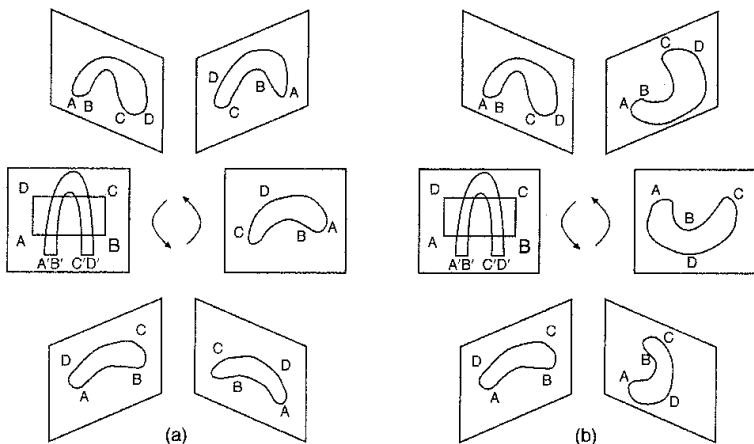
# Another Visualization

## Cutting Open a Torus



# Satisfying Boundary Conditions

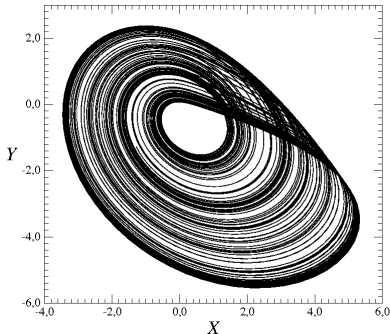
## Global Torsion



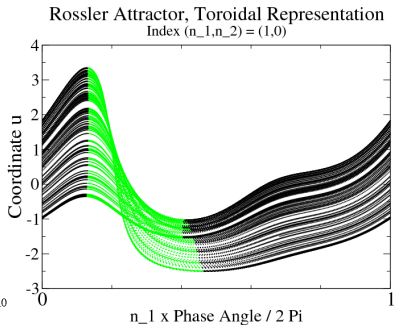
# Two Phase Spaces: $R^3$ and $D^2 \times S^1$

## Rosler Attractor: Two Representations

$R^3$



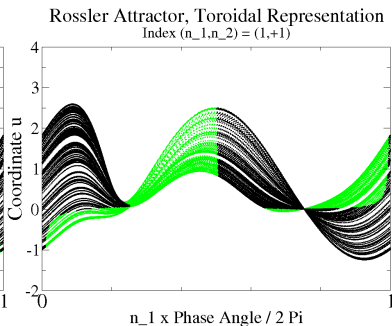
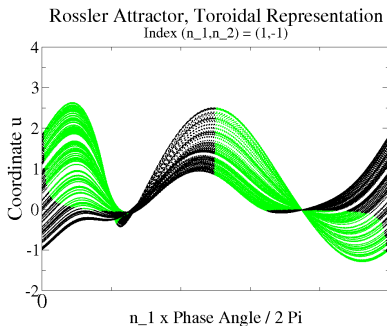
$D^2 \times S^1$



# Other Diffeomorphic Attractors

## Rossler Attractor:

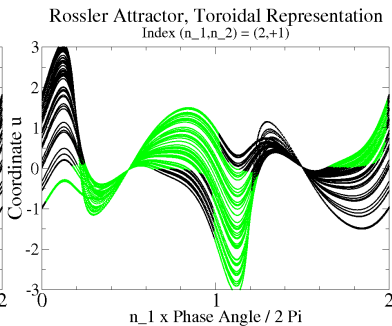
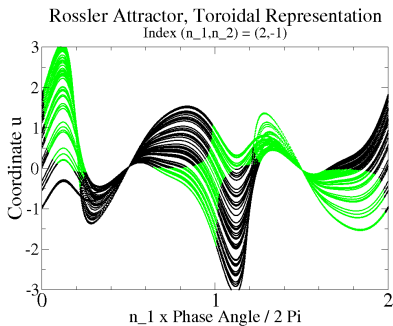
### Two More Representations with $n = \pm 1$



# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

### Two Two-Fold Covers with $p/q = \pm 1/2$

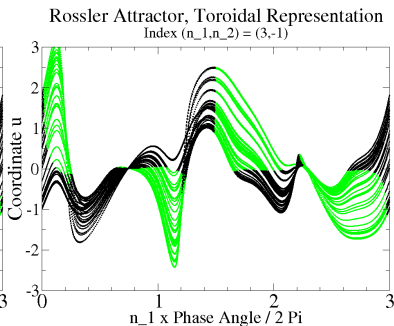
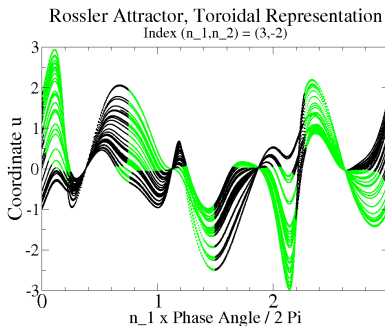




# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

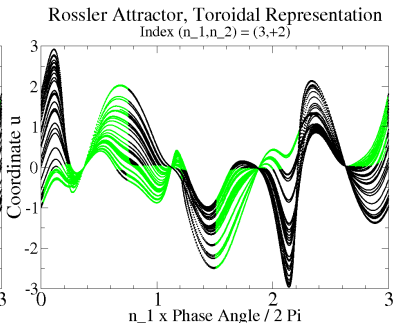
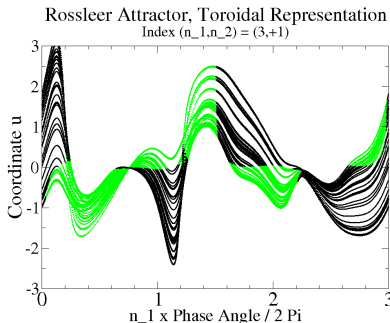
### Two Three-Fold Covers with $p/q = -2/3, -1/3$



# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

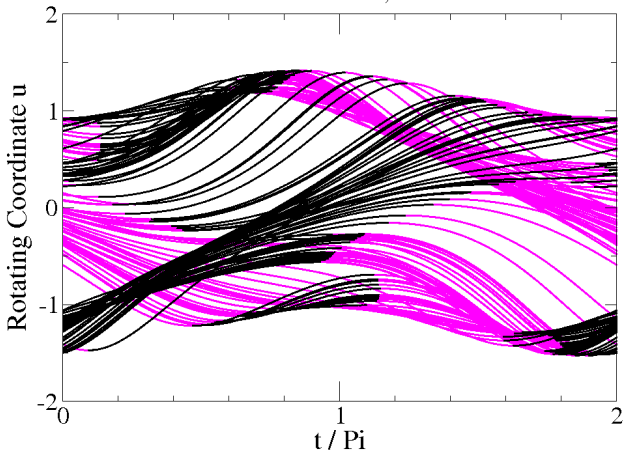
And Even More Covers (with  $p/q = +1/3, +2/3$ )



# Representations of Duffing Attractor

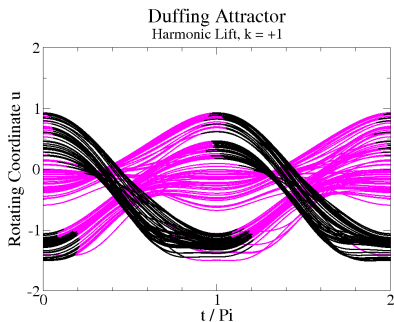
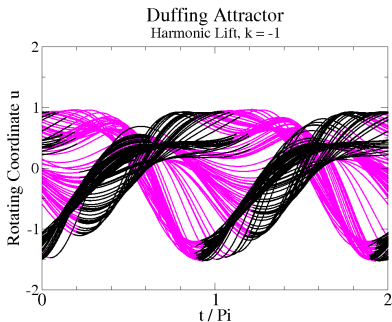
## Duffing Attractor, Toroidal Representation

Duffing Attractor  
Harmonic Lift,  $k=0$



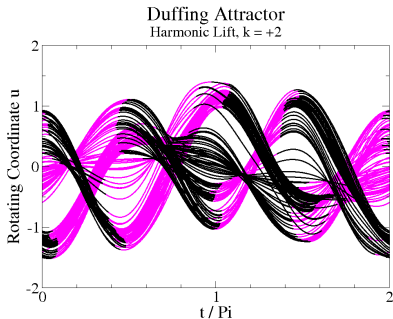
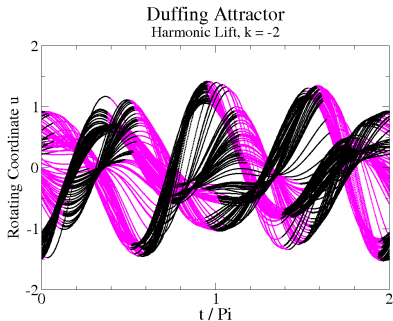
# Representations of Duffing Attractor

## Duffing Attractor, Rotation by $\pm 1$



# Representations of Duffing Attractor

## Duffing Attractor, Rotation by $\pm 2$



## Angular Momentum and Energy

$$L(0) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau X dY - Y dX \quad K(0) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \frac{1}{2} (\dot{X}^2 + \dot{Y}^2) dt$$

$$L(\Omega) = \langle uv - vu \rangle \quad K(\Omega) = \left\langle \frac{1}{2} (\dot{u}^2 + \dot{v}^2) \right\rangle$$

$$= L(0) + \Omega \langle R^2 \rangle \quad = K(0) + \Omega L(0) + \frac{1}{2} \Omega^2 \langle R^2 \rangle$$

$$\langle R^2 \rangle = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau (X^2 + Y^2) dt = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau (u^2 + v^2) dt$$

## Energy and Angular Momentum

### Diffeomorphic, Quantum Number $n$

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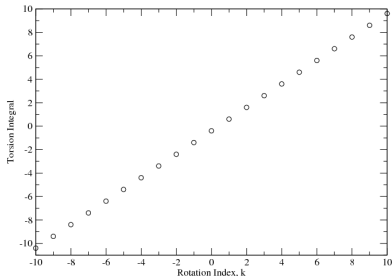
Quantizing  
Chaos-02

Quantizing  
Chaos-03

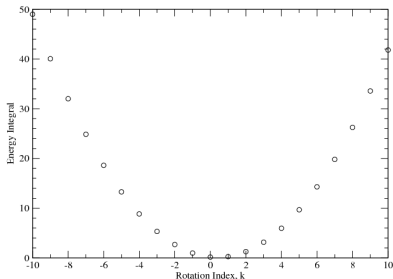
Quantizing  
Chaos-04

Quantizing  
Chaos-05

Torsion Integral

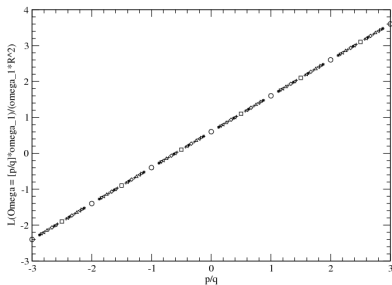


Energy Integral



## Energy and Angular Momentum Subharmonics, Quantum Numbers $p/q$

Torsion Integral



Energy Integral

