Alice in Stretch & SqueezeLand: 13 Quantizing Chaos

August 12, 2012

Chapter Abstract

The global torsion of an attractor inside a torus can be changed by integer values, n, in a simple way.

This involves a simple global diffeomorphism.

Local diffeomorphisms are used to produce q-fold covers.

Classical-like statistics (average energy, spin angular momentum) depend on the quantization idncies n, p/q in the expected intuitive way.

Rotating the Attractor

$$\frac{d}{dt}\left[\begin{array}{c} X \\ Y \end{array}\right] = \left[\begin{array}{c} F_1(X,Y) \\ F_2(X,Y) \end{array}\right] + \left[\begin{array}{c} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{array}\right]$$

$$\left[\begin{array}{c} u(t) \\ v(t) \end{array}\right] = \left[\begin{array}{cc} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{array}\right] \left[\begin{array}{c} X(t) \\ Y(t) \end{array}\right]$$

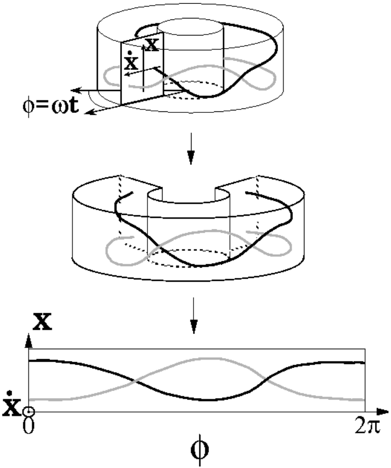
$$\frac{d}{dt}\left[\begin{array}{c} u \\ v \end{array}\right] = R\mathbf{F}(R^{-1}\mathbf{u}) + R\mathbf{t} + \Omega \left[\begin{array}{c} -v \\ +u \end{array}\right]$$

$$\Omega = n\ \omega_d \qquad\qquad q\ \Omega = p\ \omega_d$$

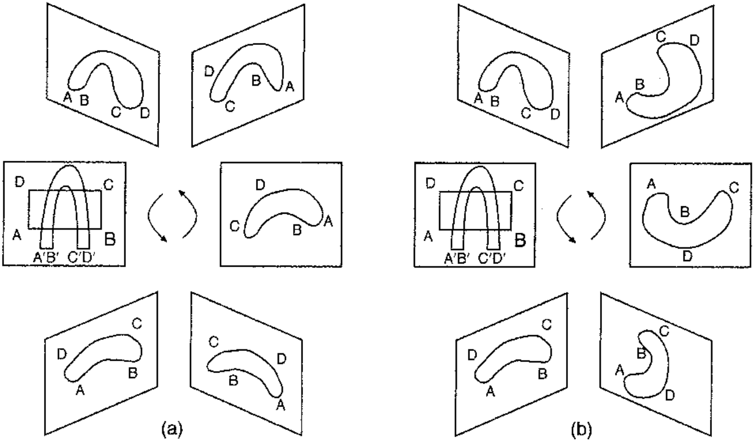
Global Diffeomorphisms

Local Diffeomorphisms (q-fold covers)

Cutting Open a Torus

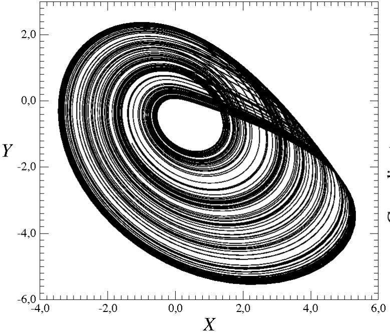


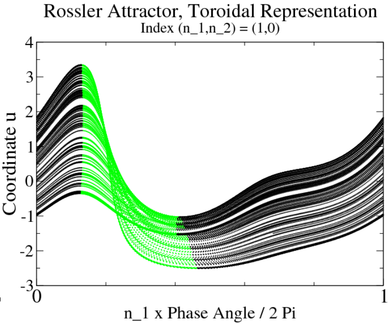
Global Torsion



Rossler Attractor: Two Representations

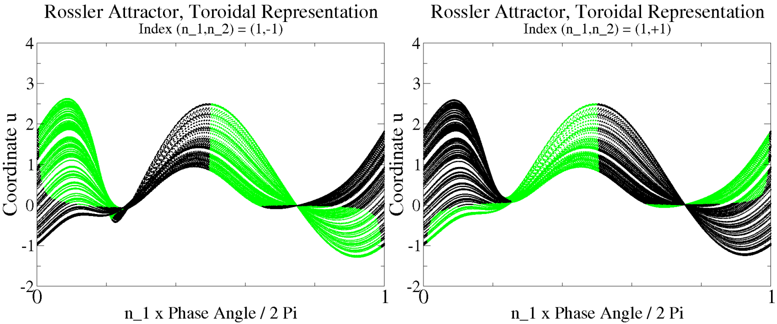
R^3 | $D^2 \times S^1$





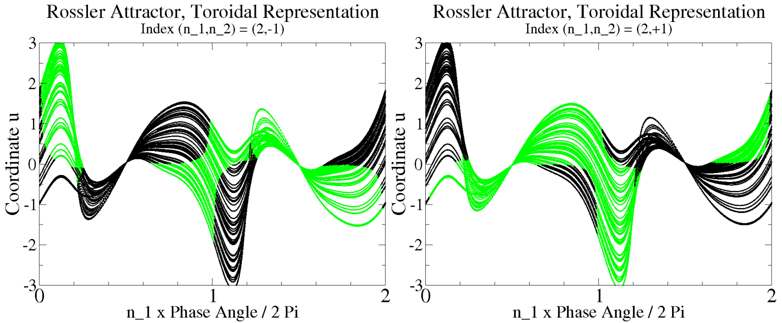
Rossler Attractor:

Two More Representations with $n = \pm 1$



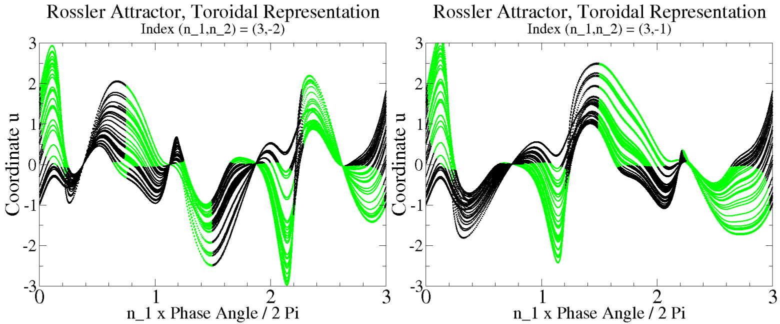
Rossler Attractor:

Two Two-Fold Covers with $p/q = \pm 1/2$



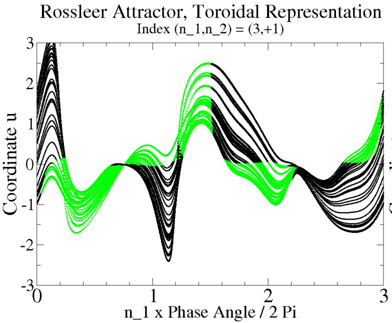
Rossler Attractor:

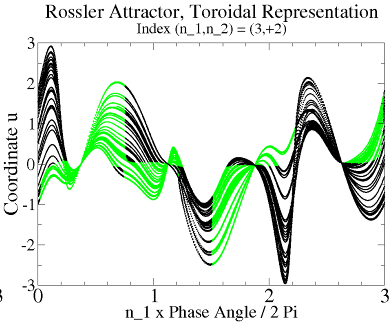
Two Three-Fold Covers with $p/q = -2/3, -1/3$



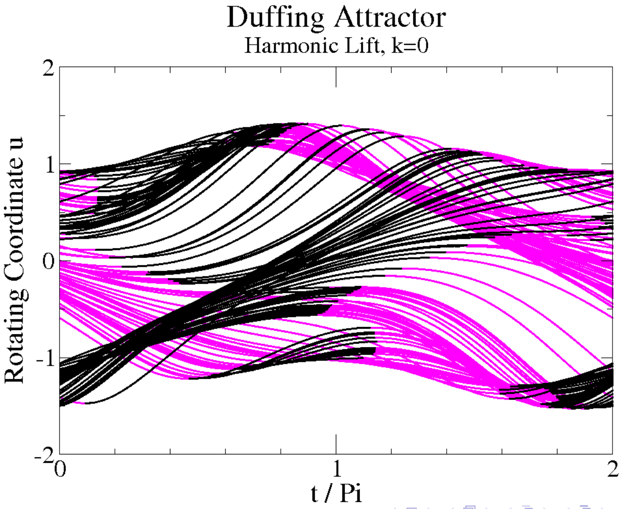
Rossler Attractor:

And Even More Covers (with $p/q = +1/3, +2/3$)

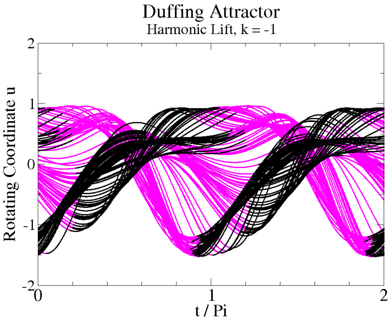


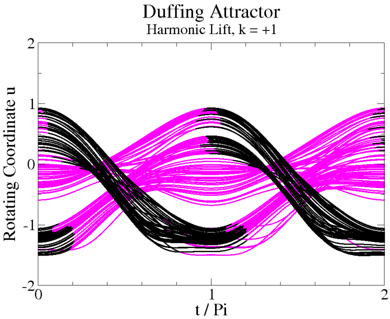


Duffing Attractor, Toroidal Representation

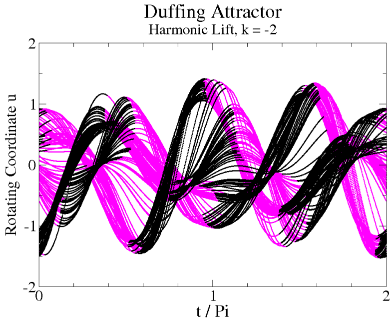


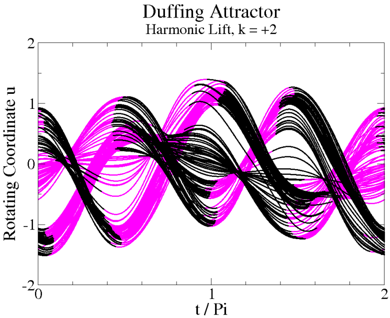
Duffing Attractor, Rotation by ± 1





Duffing Attractor, Rotation by ± 2





Angular Momentum and Energy

$$L(0) = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau XdY - YdX \qquad K(0) = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \frac{1}{2}(\dot{X}^2+\dot{Y}^2)dt$$

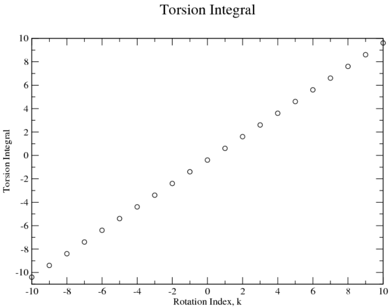
$$L(\Omega) = \langle u\dot{v} - v\dot{u}\rangle \qquad K(\Omega) = \langle \frac{1}{2}(\dot{u}^2+\dot{v}^2)\rangle$$

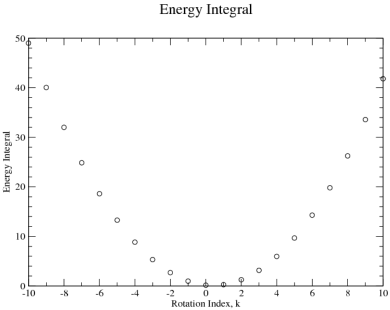
$$= L(0) + \Omega\langle R^2\rangle \qquad = K(0) + \Omega L(0) + \frac{1}{2}\Omega^2\langle R^2\rangle$$

$$\langle R^2\rangle = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau (X^2+Y^2)dt = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau (u^2+v^2)dt$$

Energy and Angular Momentum

Diffeomorphic, Quantum Number n





Energy and Angular Momentum

Subharmonics, Quantum Numbers p/q