Alice in Stretch & SqueezeLand: 11 Bounding Tori

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# Alice in Stretch & SqueezeLand: 11 Bounding Tori

August 12, 2012

#### Chapter Abstract

Alice in Stretch & SqueezeLand: 11 Bounding Tori

Each strange attractor in  $\mathbb{R}^3$  is contained in a 3D submanifold in  $\mathbb{R}^3$ .

Chapter Summary-01

The boundary of this 3D submanifold is a torus.

Tori-01

Tori are classified by *genus*, and bounding tori are also dressed by the flow induced on the surface.

Tori-0

Bounding tori are classified by permutation group elements.

Tori-0!

They are also built up by an Aufbau Principle.

Fori-08

The global Poincaré surface of section of a genus-g bounding torus consists of g-1 disconnected disks.

Tori-0

Tori-10



#### Perestroikas of Branched Manifolds

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#### Constraints

Branched manifolds largely constrain the 'perestroikas" that forcing diagrams can undergo.

Is there some mechanism/structure that constrains the types of perestroikas that branched manifolds can undergo?

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#### Perestroikas of Branched Manifolds

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## Constraints on Branched Manifolds

"Inflate" a strange attractor

Union of  $\epsilon$  balls around each point

Boundary is surface of bounded 3D manifold

Torus that bounds strange attractor

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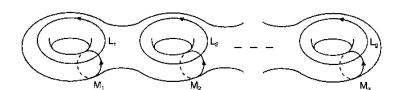
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#### Torus and Genus

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## Torus, Longitudes, Meridians



Tori are identified by genus g and dressed with a surface flow induced from that creating the strange attractor.

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#### Flows on Surfaces

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## Surface Singularities

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Flow field: three eigenvalues: +, 0, -

Vector field "perpendicular" to surface

Eigenvalues on surface at fixed point: +, -

All singularities are regular saddles

$$\sum_{s.p.} (-1)^{\text{index}} = \chi(S) = 2 - 2g$$

# fixed points on surface = index = 2g - 2

Singularities organize the surface flow dressing the torus

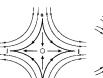
#### Flows in Vector Fields

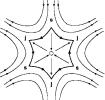
## Flow Near a Singularity



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### Some Bounding Tori

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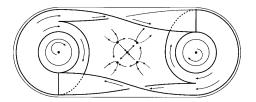
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#### Torus Bounding Lorenz-like Flows

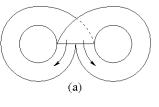


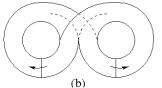
#### Canonical Forms

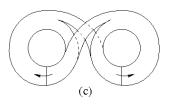
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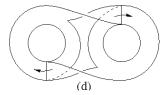
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## Twisting the Lorenz Attractor









## Constraints Provided by Bounding Tori

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Two possible branched manifolds in the torus with g=4.

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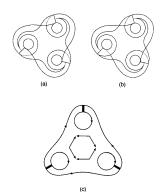
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### Labeling Bounding Tori

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## Labeling Bounding Tori

Poincaré section is disjoint union of g-1 disks.

Transition matrix sum of two g-1  $\times$  g-1 matrices.

Both are g-1  $\times$  g-1 permutation matrices.

They identify mappings of Poincaré sections to P'sections.

Bounding tori labeled by (permutation) group theory.



### Some Bounding Tori

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## Bounding Tori of Low Genus

TABLET B

Snumeration of canonical forms up						
g	m		) n1n2ng-1			
1	1	(0)	1			
3 4 5	2	(2)	11			
4	3	(3)	111			
5	4	(4)	1111			
5	3	(2,2)	1212			
5	5	(5)	11111			
6	4	(3,2)	12112			
7	6	(6)	111111			
7	5	(4,2)	112121			
7	5	(3,3)	112112			
7	4	(2,2,2)	122122			
7	4	(2,2,2)	131313			
8	?	(7)	1111111			
8	6	(5,2)	1211112			
8	6	(4,3)	1211121			
8	5	(3,2,2)	1212212			
8	5	(3,2,2)	1 221 221			
8	5	(3,2,2)	1313131			
9	8	(8)	111111111			
9	7	(6,2)	11111212			
9	7	(5,3)	11112112			
9	7	(4,4)	11121112			
9	6	(4,2,2)	11122122			
9	ð	(4,2,2)	11131313			
9	ð	(4,2,2)	11212212			
9	6	(4,2,2)	12121212			
9	6	(3,3,2)	11212122			
9	6	(3,3,2)	11221122			
9	ō	(3,3,2)	11221212			
9	ō	(3,3,2)	11311313			
9	5	(2,2,2,2)	12221222			
9	5	(2,2,2,2)	12313132			
9	5	(2,2,2,2)	14141414			

#### **Exponential Growth**

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## The Growth is Exponential

TABLE I: Number of canonical bounding tori as a function of genus, g.

g	N(g)	g	N(g)	g	N(g)
3	1	9	15	15	2211
4	1	10	28	16	5549
5	2	11	67	17	14290
ð	2	12	145	18	3 <b>6</b> 824
7	5	13	3 <b>6</b> 8	19	96347
8	ð	14	870	20	252927

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#### Motivation

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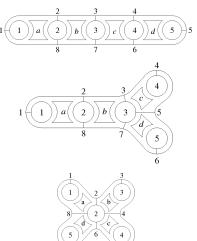
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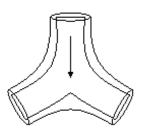
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## Some Genus-9 Bounding Tori



## Aufbau Princip for Bounding Tori

# Alice in Streetch & Squeezel and: Aufbau Princip for Bounding Tori





These units ("pants, trinions") surround the stretching and squeezing units of branched manifolds.

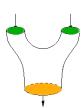


Bounding Tori

## Aufbau Princip for Bounding Tori

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Any bounding torus can be built up from equal numbers of stretching and squeezing units



- Outputs to Inputs
- No Free Ends
- Colorless

#### Poincaré Section

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#### Construction of Poincaré Section

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## Aufbau Princip for Bounding Tori

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Application: Lorenz Dynamics, g=3

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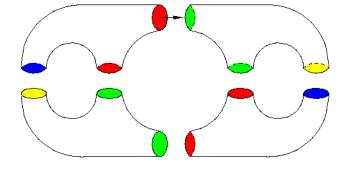
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## Represerntation Theory Redux

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#### Representation Theory for g > 1

Can we extend the representation theory for strange attractors "with a hole in the middle" (i.e., genus = 1) to higher-genus attractors?

Yes. The results are similar.

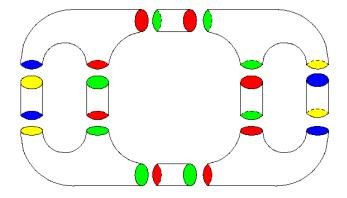
Begin as follows:

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## Aufbau Princip for Bounding Tori

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Application: Lorenz Dynamics, g=3



### **Embeddings**

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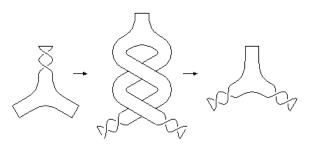
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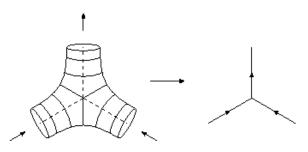
## **Embeddings**



### **Embeddings**

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#### Redcution to Networks



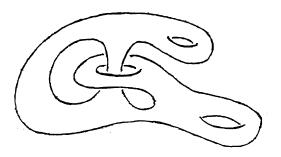
Equivalent to embedding a specific class of directed networks into  $R^3$ 

### Extrinsic Embedding of Bounding Tori

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## Extrinsic Embedding of Intrinsic Tori



A specific simple example.

Partial classification by links of homotopy group generators. Nightmare Numbers are Expected.

### Creating Isotopies

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# Equivalences by Injection Obstructions to Isotopy

Index	$R^3$	$R^4$	$R^5$
Global Torsion	$Z^{\otimes 3(g-1)}$	$Z_2^{\otimes 2(g-1)}$	-
Parity	$Z_2$	-	-
Knot Type	Gen. KT.	-	-

In  $R^5$  all representations (embeddings) of a genus-g strange attractor become equivalent under isotopy.

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