Alice in Stretch & SqueezeLand
The Marvels of Topology and Chaos

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Abstract

Suppose you have data from a physical system that is behaving chaotically. What do you do? How do you analyze these data? What should you look for? What is the mechanism that generates chaos?

For a large class of systems an algorithm now exists for addressing each of these questions successively and successfully. We will go through the steps of this algorithm, showing how each works using experimental data and pointing out the connection with topology. In the process we will develop a classification scheme for strange attractors.

Outline

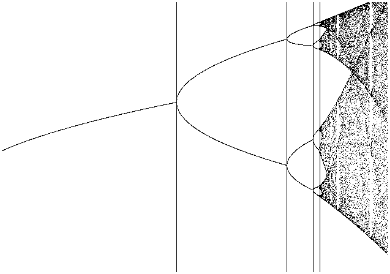
Background

J. R. Tredicce

Can you explain my data?

I dare you to explain my data!

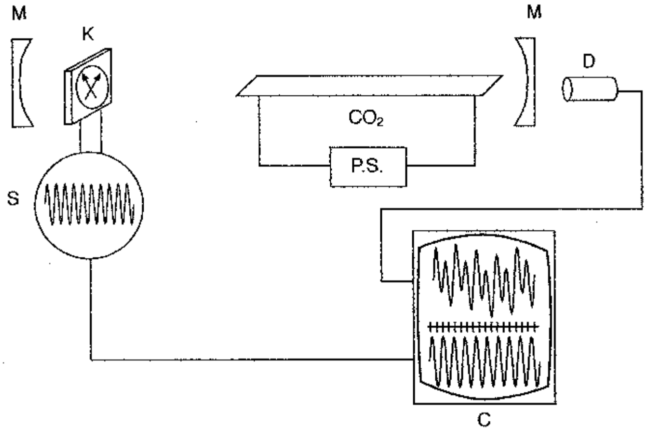
Where is Tredicce coming from?



Feigenbaum:

$$\alpha = \quad 4.66920\ 16091\ \ldots\ldots$$
$$\delta = -2.50290\ 78750\ \ldots\ldots$$

Laser with Modulated Losses Experimental Arrangement



Original Objectives

Construct a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Result

There is now a classification theory.

1. It is topological
2. It has a hierarchy of 4 levels
3. Each is discrete
4. There is rigidity and degrees of freedom
5. It is applicable to R^3 only — for now

The 4 Levels of Structure

- **Basis Sets of Orbits**
- **Branched Manifolds**
- **Bounding Tori**
- **Extrinsic Embeddings**

What Have We Learned?

1. Cover and Image Relations
2. Continuations: Analytical, Topological, Group
3. Cauchy Riemann & Clebsch-Gordonnery for Dynamical Systems
4. "Quantizing Chaos"
5. Representation Theory for Dynamical Systems

What Do We Need to Learn?

1. Higher Dimensions
2. Invariants
3. Mechanisms