Nonlinear Dynamics

PHYS 750

Problem Set # 9
Distributed March 1, 2005
Due March 10, 2005

Undergraduates: Do problems 1 and 3.
Graduates: Do problems 1, 2, and 3.

Topological Analysis of Chaos

1. The Rössler equations are

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c)\end{aligned}$$

a. Integrate these equations for $(a, b, c) = (0.398, 2, 4)$.

b. Within the data set that you generate, locate segments $(x(t), y(t), z(t))$ that are good surrogates for five low-period orbits, with period $p \leq 5$.

c. Plot these surrogates. Show where they fail to close up.

d. Compute a table of linking numbers for these orbits (there are 10 nontrivial entries).

2. Locate the five orbits 1, 01, 001, 0001, 00001 on the branched manifold shown below. Compute a table of linking numbers for these orbits.

3. You will receive an e-mail from me with an attached data file. The file consists of one column of length 10,000.

a. Plot this data file.

b. Construct a three-dimensional embedding of the data.

c. Plot the x-y projection of this embedding.

d. Locate five low-period orbits in this data set.

e. Identify each by a symbol name.

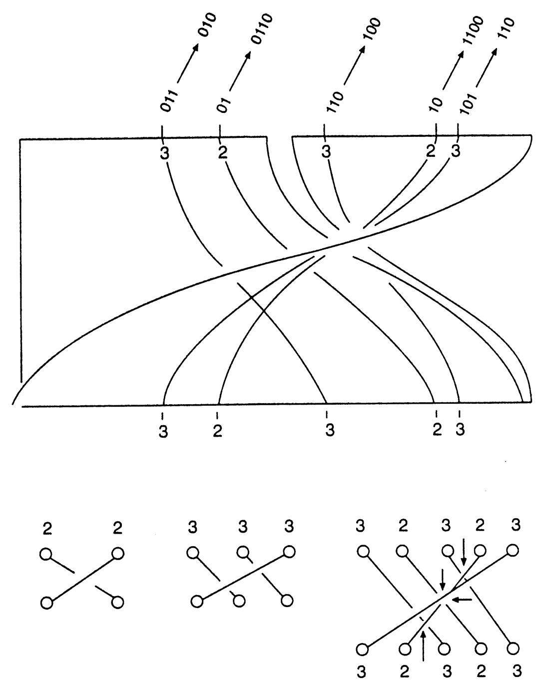

Figure 1: The interesting part of a Smale horseshoe branched manifold, showing the location of two periodic orbits on it. The location of the orbits on the branched manifold is determined using kneading theory, and the linking numbers of the orbits are computed by counting signed crossings and dividing by 2.

f. Compute a table of linking numbers for these orbits.

g. Identify the mechanism that gives rise to this chaotic data set. (**Remark:** This mechanism is the one that is responsible for generating chaotic behavior in more than half of all real physical systems that have been analyzed so far.)