Nonlinear Dynamics

PHYS 750

Problem Set: Optional
Distributed February 24, 2005
Not Due Whenever

1. The return map that you obtain from the Rössler attractor with canonical control parameter values is part of a parabola. It looks like the logistic map $x' = \lambda x(1-x)$, except that the left-hand edge of the parabola does not go all the way down to zero.

The actual form of the return map can be approximated by adding a linearly decreasing term to the logistic map:

$$x' = f(x;\lambda,\alpha) = \lambda x(1-x) + \alpha(1-x)$$

These terms can be combined to the functional form

$$f(x;\lambda,\alpha) = (\lambda x + \alpha)(1-x)$$

This family of maps does not satisfy the conditions for unimodal maps of the interval, since one of the conditions is that $f(0) = f(1)$. Therefore the intuition gained from study of the logistic map may be wrecked.

a. Show that this function has a maximum at

$$x = \frac{\lambda - \alpha}{2\lambda}$$

b. Show that the value of f at the maximum is $(\lambda+\alpha)^2/4\lambda$. We require that $f(x_{\text{max}}) = 1$. Why?

c. What constraint does this condition put on the two parameters λ and α? Solve for λ as a function of α and show

$$\lambda = (2-\alpha) + 2\sqrt{1-\alpha}$$

d. When $\alpha = 0$ every possible trajectory (sequence of 0's and 1's) can be found. In particular there is a full spectrum of unstable periodic orbits. As α increases from 0 some of these orbits must be destroyed. The earliest orbits that are destroyed have long sequences of 0000 .. 00 in them. Why? (Hint: look at the cobweb diagram for this return map, and start an orbit off at $x = 0$. How many iterates can it have before it reaches the maximum?

e. Identify values α_n, below which orbits with n successive 0's can exist, and above which they cannot. This is one version of the "pruning" idea: orbits are pruned away as some control parameter is varied.