Nonlinear Dynamics

PHYS 471, 571

Problem Set # 5
Distributed Feb. 12, 2015
Due February 19, 2015

Undergraduates: Problems 1, 3 and 4.
Graduates: Problems 2, 3 and 4.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Rössler Equations: The Rössler equations are

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \tag{1}$$

For the value of the control parameters use $(a, b, c) = (0.398, 2.0, 4.0)$.

a. Find the fixed points.

b. Determine the stability of each fixed point.

c. Integrate these equations. Use as initial conditions $(x, y, z) = (1, 1, 1)$ and allow transients to die out before beginning to record data. Provide a plot, orientation optional.

e. Use only your computed x values. Create a three-dimensional embedding $\mathbf{y}_i = (x_i, x_{i+\tau}, x_{i+2\tau})$, where τ is some "delay". Explore various values of τ until you find a "nice" value. Plot the projection of this embedding on the $x_i, x_{i+\tau}$ plane.

f. Is it "like" the original Rössler attractor? Words and feelings, please.

2. Lorenz Equations: The Lorenz equations are

$$
\begin{aligned}
\dot{x} &= \sigma(-x+y) \\
\dot{y} &= Rx - y - xz \\
\dot{z} &= -bz + xy
\end{aligned} \tag{2}
$$

For the value of the control parameters use $(R, \sigma, b) = (28, 10.0, 8/3)$.

a. Find the fixed points.

b. Determine the stability of each fixed point.

c. Integrate these equations. Allow transients to die out before beginning to record data. Provide a plot, orientation optional.

e. Use only your computed x values. Create a three-dimensional embedding $\mathbf{y}_i = (x_i, x_{i+\tau}, x_{i+2\tau})$, where τ is some "delay". Explore various values of τ until you find a "nice" value. Plot the projection of this embedding on the $x_i, x_{i+\tau}$ plane.

f. Is it "like" the original Lorenz attractor? Words and feelings, please.

3. The Feigenbaum attractor lives at the accumulation point of a period-doubling cascade. It is a two-scale attractor. The scale factors are $\lambda_1 = 1/\alpha$ and $\lambda_2 = 1/\alpha^2$, where $\alpha = 2.50290\ 78750\ 95892\ \cdots$. Compute its dimension (box counting dimension).

4. Symbolic Dynamics: Write out the symbolic name of the periodic orbits 4_2 (undergraduates) and 5_3 (graduate students).

a. Relate each point on the trajectory of your orbit with a rational fraction. For simplicity use the always unstable orbit 0^k11, $k = 2$ or $k = 3$.

b. Show that these rational fractions map to each other under the tent map.

c. Choose one of these rational fractions to identify your orbit (use the smallest).

d. Order the points on your orbit along the real line. Identify how they map into each other under forward iteration.

e. Construct a "return map" for the 4 (5) points on your orbit.

f. Use this return map to compute the topological entropy of your orbit.