Nonlinear Dynamics

PHYS 471, 571

Problem Set # 8
Distributed March 7, 2013
Due March 14, 2013

Undergraduates: Problem 1.
Graduates: Problems 2.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Linking Numbers for a Model Flow: A branched manifold with two branches has been used to model the nonlinear dynamics exhibited by the Belousov-Zhabotinskii chemical reaction (and also many other physical systems that exhibit simple stretch-and-roll type chaos). Assume that the position of points on an orbit as it passes through the "branch line" (or "joining line") is governed by the tent map.

a. Locate the three points of the period three orbit $3_1 = (001)$ along the branch line. Draw a picture.

b. Ditto for the orbits $4_2 = (0001)$ and $5_1 = (01011)$. Draw pictures.

c. "Connect the dots". (It is useful to use different colors.) Count the number of times the segments of the period-three orbit cross the segments of the period-four orbit. Divide by 2 to determine the linking number of these two orbits.

d. Ditto for 3_1 with 5_1 and 4_2 with 5_1.

e. Place your results in a table of linking numbers for these three orbits.

2. Linking Numbers for the Rössler Attractor: The Rössler equations are

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \tag{1}$$

For the value of the control parameters use $(a, b, c) = (0.432, 2.0, 4.0)$.

a. Integrate these equations and allow transients to die out before beginning to record data. Provide a plot, orientation optional.

b. Record and plot successive intersections with the halfplane $y = 0, x < 0$.

c. Create a return map onto this Poincaré section.

d. Use the method of close returns to find three orbits $3_1 = (001)$, $4_2 = (0001)$, and $5_1 = (01011)$. Take care to ensure that your encoding is correct.

e. Plot each as a closed orbit in the three-dimensional phase space.

f. Compute linking numbers for each of the three pairs. Present your results in a table of linking numbers.