Nonlinear Dynamics

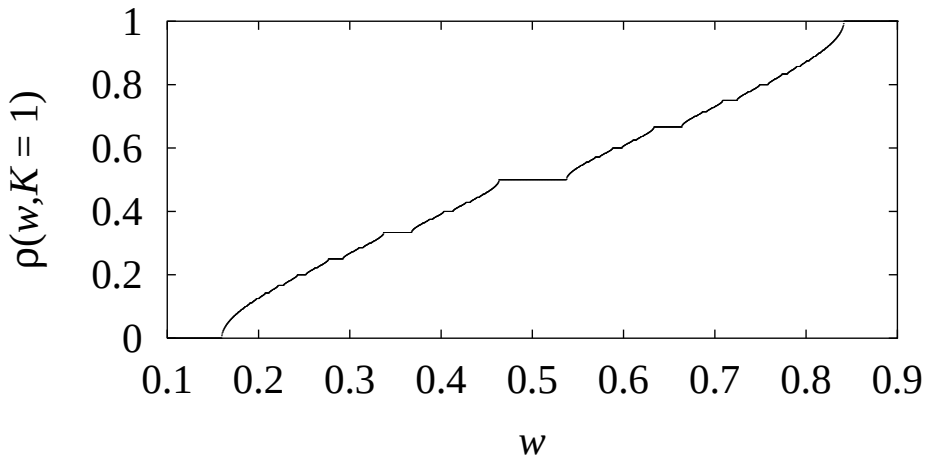
PHYS 471, 571

Problem Set # 7
Distributed Feb. 28, 2013
Due March 7, 2013

Undergraduates: Problems 1 and 2
Graduates: Problems 1, 2 and 3.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Devil's Staircase: Construct the "Devil's Staircase" for the circle map by plotting $\rho(\omega, K)$ vs. ω for $K = 1$ (Notes, Eq. (7.3)). Use

$$\theta_{n+1} = \theta_n + \omega + \frac{K}{2\pi} \sin 2\pi\theta_n \qquad \text{mod } 1$$



2. Saddle-Node Bifurcation Bounds: Construct the p^{th} iterate of the circle map and find the range of values of ω between a lower value at which a saddle-node bifurcation creates a pair of periodic orbits with rotation number $1/p$ and a larger value at which a(n inverse) saddle-node bifurcation destroys this resonance.

Undergraduates: $p = 2$

Graduates: $p = 3$

3. Farey Sequence: Between circle map resonances identified by rational fractions p/q and p'/q' there is a "fattest resonance" identified by the Farey fraction $\frac{p+p'}{q+q'}$ when $\det \begin{bmatrix} p & p' \\ q & q' \end{bmatrix} = \pm 1$. Construct the Farey resonance tree to level 5 starting with the resonances 0/1 and 1/1.