Nonlinear Dynamics

PHYS 471, 571

Problem Set # 6
Distributed Feb. 21, 2013
Due February 28, 2013

Undergraduates: Problem 1.
Graduates: Problems 1 and 2.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. First Return Maps: The Rössler equations are

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \tag{1}$$

For the value of the control parameters use $(a, b, c) = (0.398, 2.0, 4.0)$.

a. Integrate these equations. Use as initial conditions $(x, y, z) = (1, 1, 1)$ and allow transients to die out before beginning to record data. Provide a plot, orientation optional.

b. Record and plot successive intersections with the halfplane $y = 0, x < 0$.

c. Create a return map onto this Poincaré section.

d. Use the method of close returns to find "periodic orbits" of period p, $p = 1, ...5$. Be sure to explain carefully how you did this.

e. For each of these periodic orbits provide the symbol sequence (up to cyclic permutation).

e. Plot two of these periodic orbits. Explain what you are doing.

f. Find the periodic orbit of largest topological entropy.

g. Compute the topological entropy of the Rössler attractor for these control parameter values.

2. Analytic Aid: Construct an analytic approximation to the first return map created in part **c.** of the first problem. You can use any method that works for you.

a. Plot the analytic approximation on top of the "experimental" return map.

b. Create the second- and third-return maps using your analytic approximation.

c. Use the second- and third-return maps of part **b.** to locate period two- and period-three orbits. Give the symbolic name for each orbit.

d. Use the coordinates you discover as initial conditions for integration, and check of you really have found period-two and -three orbits.

e. What is the orbit of highest topological entropy you can find? What is he called?

f. Compute its topological entropy.

g. Compare with **1f, 1g**.