Nonlinear Dynamics

PHYS 471, 571

Problem Set # 5
Distributed Feb. 12, 2013
Due February 19, 2013

Undergraduates: Problems 1 and 3.
Graduates: Problems 2 and 4.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Rössler Equations: The Rössler equations are

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \tag{1}$$

For the value of the control parameters use $(a, b, c) = (0.398, 2.0, 4.0)$.

a. Find the fixed points.

b. Determine the stability of each fixed point.

c. Integrate these equations. Use as initial conditions $(x, y, z) = (1, 1, 1)$ and allow transients to die out before beginning to record data. Provide a plot, orientation optional.

d. Record and plot successive intersections with the halfplane $y = 0, x < 0$.

2. Lorenz Equations: The Lorenz equations are

$$\begin{aligned} \dot{x} &= \sigma(-x + y) \\ \dot{y} &= Rx - y - xz \\ \dot{z} &= -bz + xy \end{aligned} \tag{2}$$

For the value of the control parameters use $(\sigma, b) = (10.0, 8/3)$.

a. Find the fixed points as a function of $R \geq 0$

b. Determine the stability of each fixed point.

c. Integrate these equations for $R = 28.0$. Allow transients to die out before beginning to record data. Provide a plot, orientation optional.

d. Record and plot successive intersections with the plane $z = R - 1$.

3. Conservative Map: The mapping

$$\begin{aligned} x' &= x^2 - y \\ y' &= x + a \end{aligned} \tag{3}$$

describes an area-conserving system, since $J = \partial(x', y')/\partial(x, y) = 1$.

a. Compute $J = \partial(x', y')/\partial(x, y)$ and show that it is equal to $+1$ for all a.

b. Set $a = -0.4224$. Choose an initial condition, allow transients to die out, and record about 100 iterations.

c. Repeat step **b.** about 40 times with different initial conditions.

d. Plot all iterates from step **c.** on a single plot.

e. Have you "reproduced" the figure that accompanies Feigenbaum's paper in *Los Alamos Science*?

4. Henon-Heiles Equations: These equations represent one of the simplest nonintegrable systems possible, and have been used in an astrophysical context to model planar motion of a star around the center of the galaxy (See Henon-Heiles / Wolfram for exciting plots.). Both Henon and Heiles are astrophysicists.

$$\begin{aligned} \ddot{x} &= -x - 2xy \\ \ddot{y} &= -y + y^2 - x^2, \qquad \text{or} \\ V(x, y) &= \tfrac{1}{2}(x^2 + y^2) + x^2 y - \tfrac{1}{3}y^3 \end{aligned} \tag{4}$$

a. Set $E = \frac{1}{12}$ *or* $E = \frac{1}{8}$ and integrate these equations. Take an initial condition near the origin and allow transients to die out. Record the values of $(y, \dot{y})$ whenever $x = 0$. Record about 100 points.

b. Repeat step **a.** about 50 times.

c. Plot all the data recorded in **b.** on a single graph.

d. Can you discern ellipses and regions of chaotic behavior?