Nonlinear Dynamics

PHYS 471, 571

Problem Set # 3
Distributed Jan. 24, 2013
Due January 31, 2013

Undergraduates: Problems 1, 2, 3 and 4a.
Graduates: Problems 1, 2, 3 and 4a,b,c.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Bifurcation Diagram: Construct a bifurcation diagram for the logistic map $y' = a - y^2$ for $-\frac{1}{4} < a \leq 2$.

2. Escape Clause: Set $\lambda = 4.1$ in the map $x' = \lambda x(1-x)$. Choose uniformly spaced initial conditions in the range $(0, 1)$, count the number of iterates it takes for an iterate to become negative. Bin this number. Plot the binned distribution.

3. Caustics: Choose uniformly spaced initial conditions in the range $x \in (0, 1)$ for $x' = \lambda x(1-x)$. Plot $f^{(3)}(x; \lambda)$ for the map $1 < \lambda \leq 4$. Say something useful about the structure of this plot. (Words like *singularity* are welcome.) Class questions about what to calculate and how to plot are welcome.

4. Orbit Order: Plot caustics for the fifth and sixth iterations of the logsitic map $y' = a - y^2$.

a. Predict the relative order in which the (three) period five windows and the five period-six windows appear in the bifurcation diagram.

b. Compare with the results of Problem #1.

c. Determine the control parameter values a at which the three period five orbits are superstable (Newton's method or divide and conquer are recommended).

5. Lyapunov Exponent: Construct and plot the Lyapunov exponent for the map $y' = a - y^2$. Estimate the Lyapunov exponent at $a = 2$. Say something useful about the (negative) spikey structure of this plot.