Nonlinear Dynamics

PHYS 471, 571

Problem Set # 2
Distributed Jan. 15, 2013
Due January 24, 2013

Undergraduates: Problems 1, 2 and 4a.
Graduates: Problems 1, 3 and 4b.
In these problems you will use predictability (or lack) to test whether a data set is deterministic or not.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

Theory: If a system is deterministic, if you know the past behavior you can predict the future behavior.

Divide a data set into two halves. The first half (x_i, $i = 1 \cdots$) will be the "data base" or "learning set." The second half (y_j, $j = 1 \cdots$) will be the "testing set." The idea (to be tested) is: If you know the value of y_j you can predict the value of y_{j+1}. Do this as follows: Define the difference $d(i,j) = |x_i - y_j|$. Keep j fixed and scan over the "data base" (i.e., i) to find the x-value (x_k) in the data base that is closest to y_j (i.e., $\min_i d(i,j) = d(k,j) = |y_j - x_k|$). Use the next value in the data base, x_{k+1}, as the predictor for the next value y_{j+1} after y_j in the testing set. Now compare the two "next values" y_{j+1} with x_{k+1}. Compare by computing the difference $y_{j+1} - x_{k+1}$. Repeat for all measurements in the test set. Bin the differences and plot the histogram. The shape of the binned differences indicates whether the data set comes from a predictable or stochastic source.

1. Construct 10,000 "uniform random numbers" on $[0,1]$ as in Problem Set #1. Divide into two halves and carry out the procedure outlined above. How does this compare with what you expect? In fact, what is it that you expect?

Carry out a χ^2 test of the binned output against the triangular distribution that you expect for stochastic data. Do you reject or fail to reject that this data set is stochastic?

2. Construct 10,000 "logistic numbers" using the form $x' = \lambda x(1-x)$ of the logistic map. Choose $\lambda = 4$. Divide into two halves and carry out the procedure outlined above. How does this compare with what you expect? In fact, what is it that you expect? Carry out a χ^2 test of the binned output against the "delta function" distribution that you expect for deterministic data. Do you reject or fail to reject that this data set is deterministic?

3. Construct 10,000 scalar values using the Henon map:

$$\begin{aligned} x' &= a - x^2 + z \\ z' &= bx \end{aligned} \tag{1}$$

Set $a = 1.4, b = 0.3$. Record 10000 values of the x variable (only). Begin recording after transients have died out. Divide into two halves as above.

a. Carry out the test for determinism as described above for the logistic map. Does this work or not? Why? (Before answering 'why' do part **b.**)

b. Sometimes two initial conditions are required as initial values for a (two-dimensional) deterministic set of equations. Modify the test described above as follows. Choose *two* successive points in the test set: y_{j-1}, y_j. Construct the measure $d(i,j) = |y_{j-1} - x_{i-1}| + |y_j - x_i|$. For fixed j search over all values of the index i in the data base to find the minimum value $\min_i d(i,j) = d(k,j) = |y_{j-1} - x_{k-1}| + |y_j - x_k|$. Use x_{k+1} as the predictor of y_{j+1}. Construct and bin the difference $y_{j+1} - x_{k+1}$. Repeat for all values in the test set. Plot this histogram. Look at it. Is this data set deterministic? What is the dimension of dynamical system: How many components does an initial condition require?

4. $f(x;\mu)$ is a family of maps of $x \to x' = f(x;\mu)$ with a single maximum at a critical point x_c, where $f'(x_c;\mu) = 0$. Plot the k^{th} iterate of the critical point vs. μ for $k = 1, 2, 3, 4, 5, 6, 7, 8$.

a. $f(x;\mu) \to \lambda x(1-x)$, $x_c = \frac{1}{2}$, $0 < \lambda \leq 4$.

b. $f(x;\mu) \to a - x^2$, $x_c = 0$, $-\frac{1}{4} < \lambda \leq 2$.