QUANTUM MECHANICS III

PHYS 518

Problem Set # 4
Distributed: Oct. 25, 2010
Due: Nov. 3, 2010

The Hamiltonian describing an electron in a time-dependent external magnetic field is

$$H = -\mu \cdot \mathbf{B} = g\frac{e}{mc}\frac{\hbar}{2}(B_0\sigma_z + B_1\cos(\omega t)\sigma_x) = \frac{1}{2}\hbar\omega_0\sigma_z + \frac{1}{2}\hbar\omega_1\cos(\omega t)\sigma_x \quad (1)$$

Here g is the electron gyromagnetic ratio, $-e$ is the electron charge, $\frac{ge}{mc}B_0 = \omega_0$, etc. for the computations below choose $\omega_0 = 5$ and $\omega_1 = 1$

1. Assume the electron starts off ($t = 0$) in the ground state $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Integrate the equations of motion for the spin-up and spin-down amplitudes a and b ($\begin{pmatrix} a \\ b \end{pmatrix}$) for the first few periods. Search over ω to find the "resonance frequency". Plot $|a|^2 + |b|^2$ and $|a|^2 - |b|^2$ at the resonance frequency as a function of time during this interval. If $|a|^2 + |b|^2$ does not remain 1 during this time, what did/should you do? (Cosmopeople, MDpeople, think symplectic integration.)

2. Instead of computing the time evolution of the state function, compute the time evolution of the density operator. First, write $\rho = \frac{1}{2}(I_2 + \sigma \cdot \mathbf{a})$ where $\mathbf{a} = \langle\sigma\rangle$. Then find the equation of motion for $\mathbf{a}$. It is a classical-like equation. Estimate the resonance frequency from this equation. Integrate this equation numerically for several periods at the estimated resonance frequency. Plot the analogs of $|a|^2 + |b|^2$ and $|a|^2 - |b|^2$. Also plot $\langle\sigma_x\rangle$ during this interval and compare the result with $|\text{trig. function}|^2$, where an obvious trig function is guessed.