# QUANTUM MECHANICS III

# PHYS 518

## Problem Set # 1

## Distributed: Sept. 24, 2010

## Due: Oct. 1, 2010

Itoh derives the Hamiltonian for a multielectron atom (with $n$ electrons) by writing down the terms involving a single electron, the "$j$"-th (1a - 1h) and then the terms involving the electron-electron interactions (2a - 2g).

The one-electron terms involve four space-time fields, generically $G(x,t)$. One pair of fields includes the scalar potential $\Phi(x,t)$ and the vector potential $\mathbf{A}(x,t)$, which are the time- and space- like parts of the four-vector potential $A_\mu(x,t)$ that describes the electromagnetic field. The other pair of fields, $\mathbf{E}(x,t), \mathbf{B}(x,t)$, are the space-like and time-like parts of the antisymmetric second order tensor $F_{\mu.\nu}(x,t)$ that are the electromagnetic fields.

Itoh decomposes each of these four fields into an extrinsic part and an intrinsic part.:

$$G(x,t) \rightarrow G_{\text{ext}}(x,t) + G_{\text{int}}(x,t) \tag{1}$$

The extrinsic part interacts with electron $j$ (Itoh's notation) but is independent of the other $n-1$ electrons. The intrinsic part describes the contribution of the other $n-1$ electrons to the field $G$.

In deriving the single particle contributions to the Hamiltonian only the extrinsic fields enter. To derive the particle-particle interaction the intrinsic part of each of the four fields must be constructed and included.

Write a description of this derivation that would be suitable for inclusion in a graduate level Quantum Mechanics text, or suitable for submission to a Physics journal. [**I will not accept a list of equations without explanation of your derivation!**] Your treatment must describe this process. You may begin by paraphrasing and expanding on the introductory comments above. You must then write down the other $n-1$ electrons' contribution to: $\Phi, \mathbf{E}, \mathbf{B}, \mathbf{A}$. Since these particles are moving, you should talk about "effective" $\mathbf{E}$ and $\mathbf{B}$ fields: $\mathbf{E}_{\text{eff}} = \gamma(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B})$ and $\mathbf{B}_{\text{eff}} = \gamma(\mathbf{B}-\frac{\mathbf{v}}{c}\times\mathbf{E})$. Acknowledge that since $\beta = \mathbf{v}/c$ is "small" (one hopes) $\gamma \simeq 1$. The velocity is really the difference between the electron $k$ and $j$ velocities, and these are to be replaced by their momentum differences. You are absolved from deriving the Thomas precession fraction $\frac{1}{2}$ that migrates from (1g) to (2d). You should judiciously neglect higher-order (than $\mathcal{O}(\alpha^4)$) terms: Itoh describes how if you read him carefully. If you can't find his hints, use your own judgment/intuition.

My Astro folks (Austen, Crystal) have an extra load. You must also express the "single particle" term $-\mu \cdot \mathbf{B}_{\text{ext}}$ in terms of the nuclear spin $J$ and the nuclear gyromagnetic ratio, as well as the electron spin and gyromagnetic ratio.

Please: not too long, not too short. Just right (Goldilocks). Say what's essential. Write down key equations. Skip inessential derivation details. Keep in mind that if you do this well your writen description of this derivation will fall into the hands of this year's crop of new graduate students. A good job will make you famous but it won't make you rich.