QUANTUM MECHANICS III

PHYS 518

Problem Set # 3

Distributed: October 20, 2008

Due: October 30, 2008

1. Wave Packet Spreading:

$$\psi(x,0) = \frac{e^{-(x/a)^2/2}}{\sqrt{a\sqrt{\pi}}} \qquad \psi(x,t) = ?$$

2. Coherent States: If the initial state vector of the harmonic oscillator is the coherent state $|\psi(0)\rangle = |z\rangle$, show that the state remains coherent, and the time evolution of $|\psi(t)\rangle$ corresponds to a classical orbit in the classical phase space (q,p). (This is Problem 19.2, p. 581 in Ballentine.)

3. More on Oscillators: Consider a one-dimensional harmonic oscillator of angular frequency ω_0 that is perturbed by the time-dependent potential $W(t) = bx\cos(\omega t)$, where x is the displacement of the oscillator from equilibrium. Evaluate $\langle x\rangle$ by time-dependent perturbation theory. Discuss the validity of the result for $\omega \simeq \omega_0$ and for ω far from ω_0. (Problem 12.10, pg. 368.)

4. Atomic Perturbation: A hydrogen atom is placed in a time-dependent homogeneous electric field, of magnitude $\mathbf{E}(t) = A\tau/(t^2+\tau^2)$. Note that the total impulse of the force is independent of τ. If at $t = -\infty$ the atom is in its ground state, calculate the probability that at $t = +\infty$ it has been excited to the first excited state. (Problem 12.11, pg. 368.)

5. Berry Phase: Calculate the geometrical phase of the wave function of a charged particle in a box when the box is adiabatically transported around a magnetic flux tube that does not enter the box. See Fig. 12.4, pg. 366 of Ballentine. (Problem 12.12, pg. 369.)

6. Nuclear Precession: An electron is in a magnetic field $\mathbf{B} = B_z\hat{\mathbf{k}} + B_x(t)\hat{\mathbf{i}}$. Here $B_x(t) = B_0, 0 \le t \le \tau$ and $B_x(t) = 0$ otherwise. The electron is in its ground state for $t < 0$. How large must τ be for the electron to be in a state in the equatorial plane precessing about the z axis for $t > \tau$?