QUANTUM MECHANICS I

PHYS 516

Problem Set # 6
Distributed: March 9, 2016
Due: With the Final Exam

1. Neutrinos with 3 Flavors: Assume neutrinos come with three flavors. The flavor eigenstates are not energy eigenstates. The three types of states are related by a 3×3 unitary transformation (neutrino mixing matrix):

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

Use $\sin^2 2\theta_{12} = 0.861$; $\sin^2 2\theta_{23} = 0.97$; $\sin^2 2\theta_{12} = 0.092$ (N.B: $\sin\theta_{13} \to \sin\theta_{13}e^{-i\delta}$ and we have set $\delta = 0$ for this problem set. There is one additional phase matrix that can be sandwiched between the last 3×3 matrix above and the mass column vector. It is a phase matrix

$$\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which carries physical signifficance only if neutrinos are Majorana particles. You may ignore this matrix for the purposes of thes problem.

a. Assume that a decay produces a flavor-1 neutrino. Resolve this flavor state into energy states.

b. Propagate the energy states forward in time from $t = 0$ to arbitrary time T.

c. Assume a detector has been set up a distance $L = cT$ away to detect neutrinos with any flavor. Compute the probability *amplitude* for detecting neutrinos each flavor. Use $E_3 = 6E_2 > 0, E_2 = 4E_1$ and T large enough so your plots show interesting things.

d. Compute the probability for detecting neutrinos of each flavor.

e. Do the probabilities sum to +1? (**Hint:** they better!)

2. Angular Momentum: Write down the eigenstates of J^2 and J_z, with $\mathbf{J} = \mathbf{L} + \mathbf{S}$ for $l = 2$ and $s = \frac{1}{2}$. Identify the eigenvalues of each eigenvector.