QUANTUM MECHANICS I

PHYS 516

Problem Set # 1

Distributed: Jan. 8, 2016

Due: January 15, 2016

1. Scaling: Bohr computed the energy level spectrum of the hydrogen atom using the Old Quantum Theory, Heisenberg did the same using Matrix Mechanics, and so did Schrödinger using Wave Mechanics. They all derived

$$E_N = \frac{E_g}{N^2} \qquad E_g = -\frac{1}{2}mc^2\alpha^2/N^2 \qquad\qquad R_N = Na_B \quad a_B = \hbar^2/me^2 \quad (1)$$

Here $N = 1, 2, 3, ...$ is the principal quantum number, $-e$ is the charge on the electron, m is the mass of the electron (actually electron-proton reduced mass), $\alpha = e^2/\hbar c = 1/137.036...$ is the fine structure constant, and a_B is the Bohr radius. $E_1 = -13.6eV$ and $a_B = 0.529 \times 10^{-8}cm$.

For each of these pairs compute the binding energy and the size (diameter):

System	Energy	Size
hydrogen atom: p^+e^- (nonrelativistic)	13.6 eV	1.058 Å(diam.)
He$^{\mathrm{II}}$:		
mu-mesic atom: $p^+\mu^-$		
pi-mesic atom: $p^+\pi^-$		
positronium: e^+e^-		
muonium: $\mu^+\mu^-$		
pionium: $\pi^+\pi^-$		
Muonic atom: $Pb^{82+}\mu^-$		
Si exciton: $\epsilon = 11.9, m_e = 0.8m, m_h = 0.4m$		
GaAs exciton: $\epsilon = 12.5, m_e = 0.07m, m_h = 0.4m$		

El$^{\mathrm{I}}$ is neutral Element, and El^{n+1} is Element without n of its electrons. For excitons the electron (m_e) and hole (m_h) effective masses are given as multiples of the free electron mass. Recall that the mass m used in expressions for the hydrogen atom properties is the proton-electron reduced mass.

2. Relativistic Schrodinger Equation: Schrödinger solved the relativistic problem before he proposed his nonrelativistic equation. You will do that here

a. Write down the relativistic equation for a spinless electron in the presence of a spinless proton.

b. Use separation of variables to "get rid of" the angular dependence.

c. Use the useful transformation $R(r) = \frac{1}{r} f(r)$ and write down the radial equation in terms of the unknown function $f(r)$.

d. Compare this equation with an equation in Table 22.6 from Abramowitz and Stegen. What do you conclude?

e. Show

$$E(n, l, \alpha) = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{\left(n + \frac{1}{2} + \sqrt{(l + \frac{1}{2})^2 - \alpha^2}\right)^2}}} \tag{2}$$

f. What effect does relativity have on the "Bohr radius" of an electron?

g. Expand the expression for the energy of a relativistic electron without spin in powers of the fine structure constant α up to and including order six. Compare the first correction to the rest energy with the nonrelativistic spectrum.