Mathematical Physics II

PHYS 502

Problem Set # 3
Distributed January 15, 2016
Due January 22, 2016

1a. Estimate the scaling exponent for the biological data shown in Fig. 1.

1b. Estimate the scaling exponent for the astrophysical data shown in Fig. 2.

2. $f_1(\mathbf{x}) = e^{-(\mathbf{x}-\mathbf{a_1})^2/2\sigma^2}$, $f_2(\mathbf{x}) = e^{-(\mathbf{x}-\mathbf{a_2})^2/2\sigma^2}$. Evaluate

$$\int f_1(\mathbf{x})V(\mathbf{x})f_2(\mathbf{x})d\mathbf{x}$$

and approximate the result in terms of the potential and its first and second derivative evaluated as some cleverly chosen point.

3. Convince me that you can approximate the first and second derivatives of a function on a line, $f(x)$, by evaluating the function at discrete points along the line spaced a distance Δ apart, and that

$$\frac{df}{dx}|_{x_i} \simeq \frac{f(x_i+\Delta)-f(x_i-\Delta)}{2\Delta}$$

$$\frac{d^2f}{dx}|_{x_i} \simeq \frac{f(x_i+\Delta)-2f(x_i)+f(x_i-\Delta)}{\Delta^2}$$

For other numerical approximations of derivatives, including ∇ and ∇^2, see the pictures in Chapter 25 of Abramowitz and Stegun.

4. Can this integral be evaluated analytically? If so, what is it?

$$f = 2z\ e^{z^2}\ \log z + \frac{e^{z^2}}{z} + \frac{\log z - 2}{\left[(\log z)^2 + z\right]^2} + \frac{(2/z)\log z + 1/z + 1}{(\log z)^2 + z}$$

Can this integral be evaluated in closed form:

$$f = 2z\ e^{z^2}\ \log z + \frac{e^{z^2}}{z} + \frac{\log z + 2}{\left[(\log z)^2 + z\right]^2} + \frac{(2/z)\log z + 1/z + 1}{(\log z)^2 + z}$$

Web search coordinates: *Risch algorithm.*

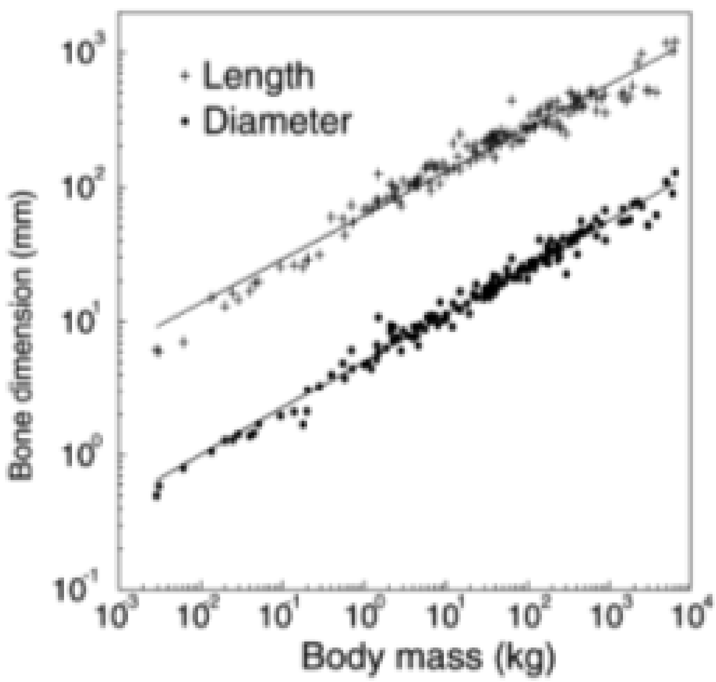
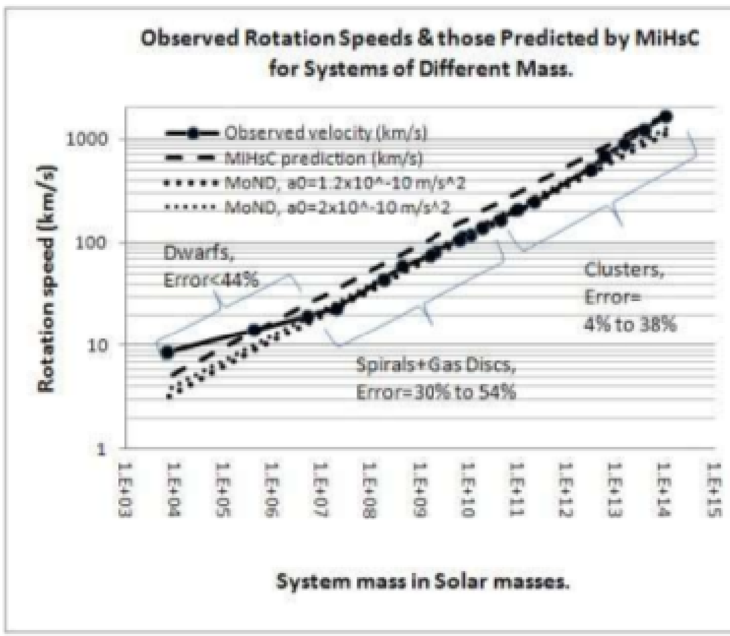


Figure 1: (Two data sets that were used to test Galilean scaling in biological creatures.



A comparison of the observed rotation speeds in km/s (black dots) with the predictions of MoND (dotted) and MiHsC (dashed) for galaxies and galaxy clusters of increasing baryonic mass (in Solar masses). Credit: M.E. McCulloch

Figure 2: (Several data sets used to discern information about “dark matter”.